Mathematical and Physical Basis of Medical Biophysics

Chapter 4

Mechanics –statics and dynamics

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Kinematics-Dynamics

- Kinematics: deals with movements of bodies without investigating the cause of the movement
- Dynamics: handles the causes of the different forms of movement
  - study of forces acting between bodies
  - study of the effect of forces on motion

Medical relevance of dynamics

- Sports medicine
- Orthopedics
- Physical therapy
- Audiology
- Dentistry

Different interactions between bodies

- Gravitation
- Friction
- Electric attraction/repulsion
- Magnetic attraction/repulsion
- Nuclear forces
- …etc

Interaction: they exert forces on each other

Force

The ´force´ describes the strength of interaction

Effect of force:

- change in motion (more conventional)
- or change in shape

We can measure (and define) the force by these changes.

Force

$$F = m \cdot a$$

F: force $\left[\text{N=Newton=}\frac{kg \cdot m}{s^2}\right]$

m: mass [kg]

a: acceleration $\left[\frac{m}{s^2}\right]$

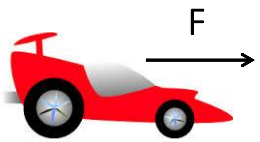
Direction of *F* equals the direction of *a,* but not necessarily that of the speed (vector quantities)

Problem IV/2

A racecar (m=1500 kg) accelerates from rest with uniform acceleration. It reaches the 100 km/h velocity in a time interval of 3,1 s.

a) Calculate the force that is accelerating the car.

b) Calculate the distance in which the car reaches the 100 km/h velocity.



Newton's laws of motion

- I. The law of inertia
- II. Fundamental law of dynamics
- III. Law of equal action and reaction

Newton's first law

- an object will remain at rest or in uniform motion in a straight line unless acted upon by an external force
- objects will remain in their state of motion unless a force acts to change the motion: inertia

Only in an ‚inertial reference frame'

It is a special case of the Second law.

Newton's second law

$$F = m \cdot a$$

- The acceleration of an object and the force acting on the object are proportional to each other
- If more force act on the object at the same time:

$$\sum F = m \cdot a$$

$\sum F$: vector sum of forces, net force, resultant force

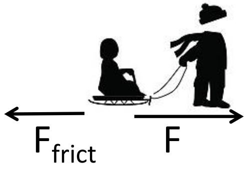
Problem IV/5

A father is pulling a sled for 5 seconds starting from rest with a constant force of 105 N. The mass of the sled together with the child on it is 25 kg. The friction force acting on the sled is 15 N.

a) Calculate the acceleration of the sled.

b) What will be the final velocity of the sled after 5s?

c) How far can dad pull the sled during this time?



Problem IV/6.

A man is pulling a sled with constant velocity (*m* = 20 kg).

The rope suddenly breaks. The sled slides on for 6,1 seconds with uniform deceleration and stops after 9,2 m.

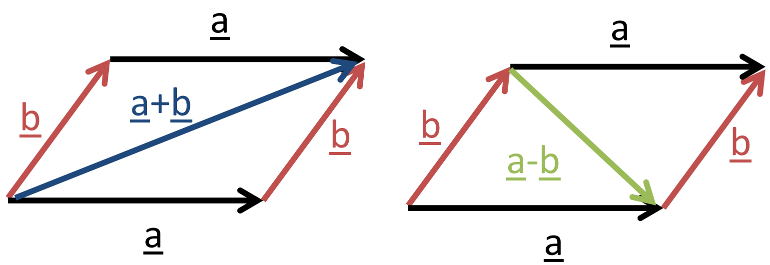
a) Calculate the velocity of the sled at the moment when the rope breaks.

b) Calculate the acceleration of the sled (in fact it is deceleration).

c) Calculate the force that slows the sled.

Resultant force, net force

The parallelogram law gives the rule for vector addition of vectors a and b. The sum a+b of the vectors is obtained by placing them head to tail and drawing the vector from the free tail to the free head.



Newton I. – Newton II.

- The first law is a special case of the second:

In equilibrium: $\sum F = 0 \rightarrow a = 0$

If $a = 0$

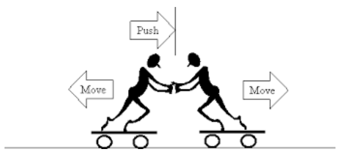
- velocity remains constant (‚uniform motion')
- velocity is zero (‚remains at rest')→STATICS

Newton's third law

- When one body exerts a force on a second body, the second body simultaneously exerts a force on the first body equal in magnitude and opposite in direction

$$F_{12} = -F_{21}$$

- Forces always appear in pairs
- Action is always followed by reaction (symmetry of interactions)



Newton's third law

A truck driving down the road runs into a bug flying the other way.

a) Which object has greater force exerted on it?

Both objects have the same small force exerted on them.

b) Which object will change its motion the most?

The bug changes its motion significantly more because of its smaller mass.

$$F_{12} = -F_{21}$$

$$_{m}a = m_{a}$$

Limits of newtonian mechanics

Newtonian mechanics is not applicable in any situation:

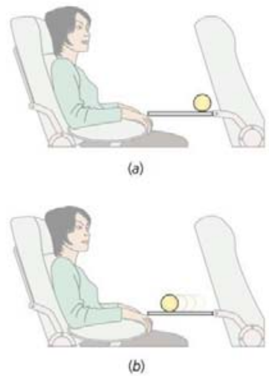
1) velocities near speed of light →special relativity
2) atomic-sized bodies→quantum mechanics
3) non-inertial reference frame
   (e.g. accelerating aircraft)

Inertial reference frame

In a straight flying aircraft a tennis ball is placed on the folding table, which remains at rest relative to the aircraft.

If the pilot accelerates the aircraft the ball rolls suddenly towards the sitting person.

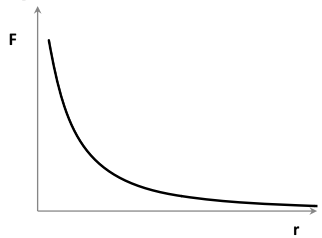
Accelerating (or rotating) aircraft: non-inertial system
Ground or aircraft flying with constant speed: in good approximation inertial systems



Quelle: Tipler, Paul A.; Mosca, Gene; Kommer, Christoph (2015): Physik. Für Wissenschaftler und Ingenieure. 7. Aufl. Hg. v. Jenny Wagner. Springer Spektrum

Law of universal gravitation

$$F = G\frac{m_1 m_2}{r^2}$$



F: gravitational force [N]

G: gravitational constant $=6.67\cdot10^{-11}\left[\frac{m^3}{kg\cdot s^2}\right]$

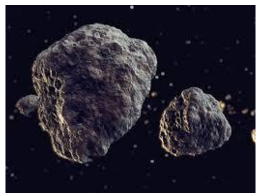
m_1: mass of first object [kg]

m_2: mass of second object [kg]

r: distance between the objects[m]

Problem IV/8.

Calculate the gravitational force between two asteroids (masses are 200 000 and 300 000 tons) when they pass by each other at a distance of 2 km.



Gravity

- a special case of the gravitational law
- accelerates an object falling freely towards the Earth

$$F_{Gravity} = G\frac{m_{Earth} \cdot m_{object}}{{r_{Earth}}^2} = m \cdot g$$

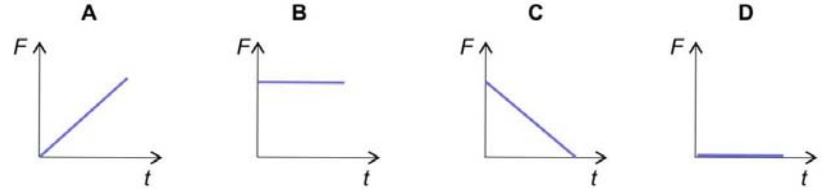
gravitational acceleration (g):

$$g = G\frac{m_{Earth}}{{r_{Earth}}^2} \approx 9.81 \left[\frac{m}{s^2}\right]$$

Earth is not spherical, g is greater near the Equator

Problem IV/13 a

The figures show the change in force as a function of time:



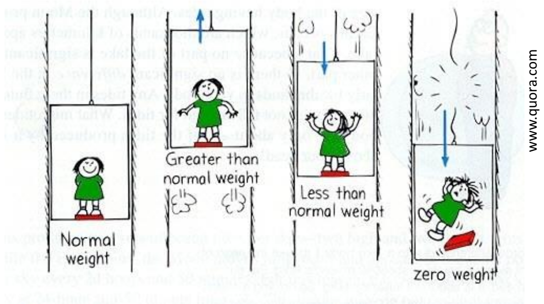
We throw a ball straight up. Which figure shows correctly the change of gravity acting on the ball?

Weight – F_{grav} – mass

- Weight (W): force with which an object pressing or pulling the surface that holds it

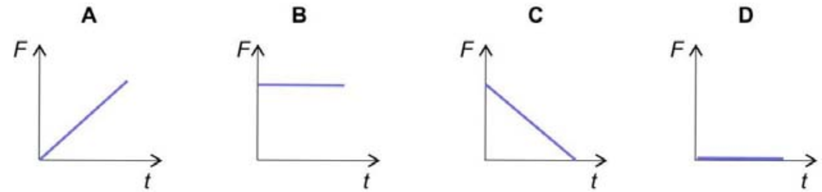
In equilibrium: $F_{grav} = W = m \cdot g$

Free falling: $F_{grav} = m \cdot g$, but W=0 (weightlessness)



Problem IV/13 c

The figures show the change in force as a function of time:



A ball falls freely towards the ground. Which figure shows correctly the change of the ball's weight?

Hooke's law

$$F = -k \cdot x$$

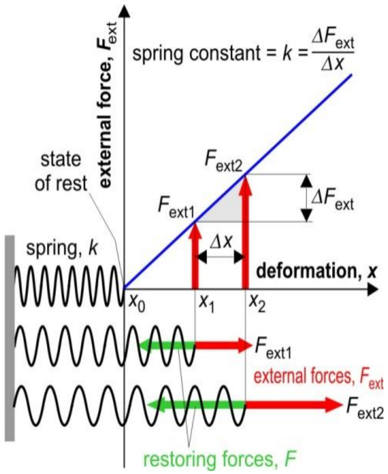
F: restoring force [N]

(-): opposite direction to the extension

k: spring constant $\left[\frac{N}{m}\right]$

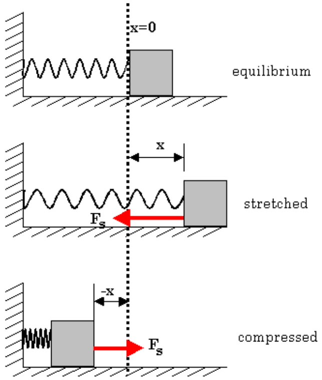
depends on the material and dimensions of the spring

x: extension, deformation [m]



Hooke's law

- The same formula can be applied for compression.



$$F = -k \cdot x$$

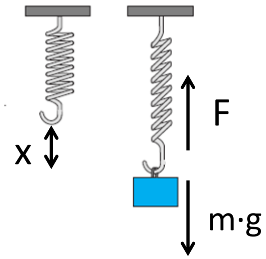
Applies approximately for tendons and ligaments in human body if they are not too heavily loaded.

Problem IV/11

We hang a mass of 2 kg on a spring.

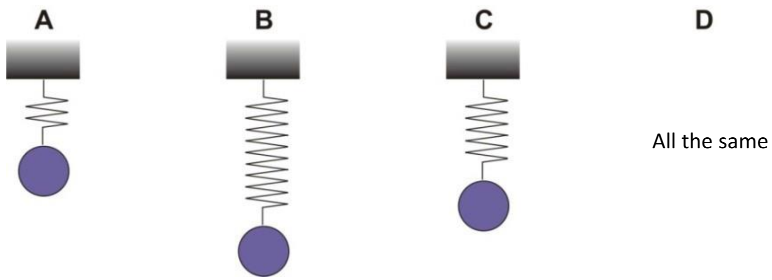
Its extension after equilibrium is 25 cm.

Calculate the spring constant.



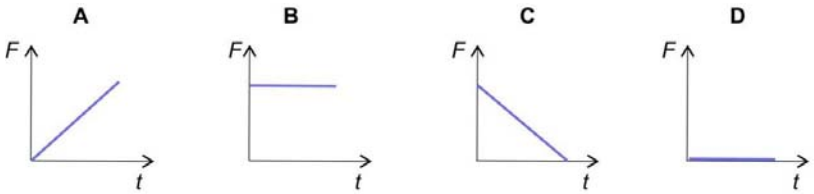
Problem IV/12

All the spring shown in the figures are extended by 10 %, when we hang the same mass on them. Which spring has the highest spring constant? Or do all have the same spring constant?



Problem IV/13 b

The figures show the change in force as a function of time:



We slowly compress a spring uniformly. Which figure shows correctly the change of spring force during compression?