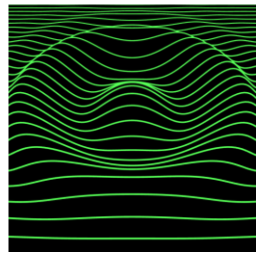
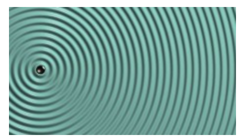
Wave optics

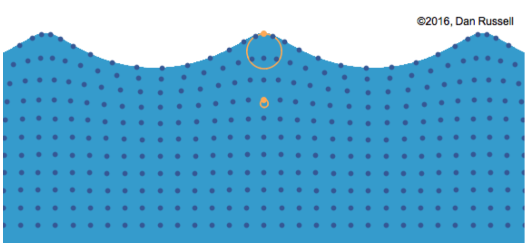
Waves are everywhere, not just in optics!

G.Schay 2016

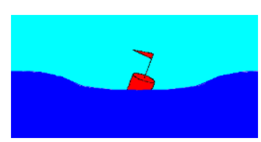




Waves are seen most often in water:



Waves can be described by the wave equation, which relates
the motion of individual parts of the medium to the observed wave.

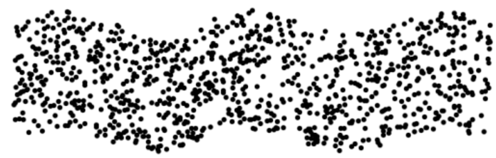


It is important to note, that as the waves propagate, the parts
of the medium (here the water molecules) stay "in place", which means
there is no net transport of material.

Longitudinal waves:
propagation direction is parallel to the “motion”

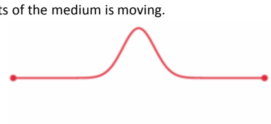
Moving surface (wave “source”)

Transversal wave – such as light, or sound in some cases in solids



Transversal: wave propagation is perpendicular to the “motion”

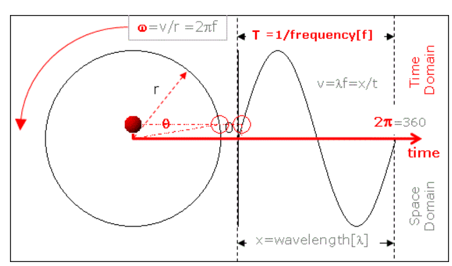
As an observation we can say that as waves travel or **propagate** the change in the state of the parts of the medium is moving.



Here the “state of the parts” simply means the deflection of the string at a given point.

The wave is moving forward (propagating) with a speed of “v”, any point’s position is dependent on both space and time.

Most important motion type is the harmonic motion. (produced by a harmonic oscillator)



The wave equation is a bit complicated:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}.$$

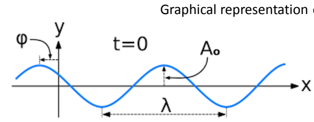
We take the change of any property (here "u") in time (du/dt) and also in space (du/dx), but we need to take the change of the change (d^2u/du^2), and these are linked by the propagation speed (or other named phase velocity) (here as "c").

A simple solution for u(x,t) is:

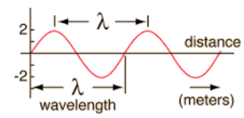
$u(x,t) = A * \sin(k*x + \omega*t + \phi)$
where
A is the amplitude of the wave, k is the wavenumber, and ω is the angular frequency

$\omega = 2\pi f$, where f = 1/T [Hz], while T is the period time.
$\omega = c*k$ defines the wavenumber, which can be written as $k = 2\pi/\lambda$.
here λ is the wavelength.

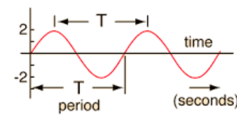
Graphical representation of the solution



Snapshot at a given time

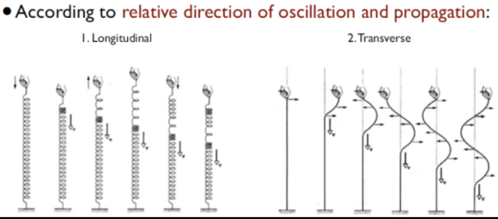


Time evolution at a given point



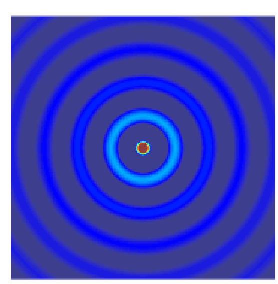
Different types of waves

- According to source:
  1. Mechanical: elastic deformation propagating through elastic medium
  2. Electromagnetic: electric disturbance propagating through space (vacuum)
- According to propagation dimension:
  1. One-dimensional (rope)
  2. Surface waves (pond)
  3. Spatial waves (sound)
- According to relative direction of oscillation and propagation:

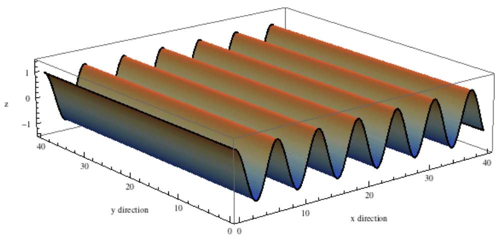


Point source: radiating in all directions along a sphere.

This is a transverse wave example, longitudinal is also possible.

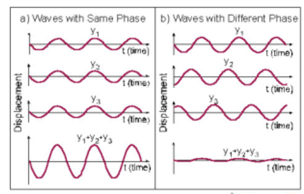


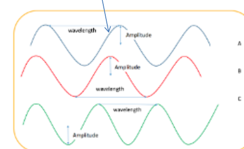
Plane wave (again transversal)



Waves can be coherent or incoherent.
Coherent waves have the same frequency and a constant phase difference

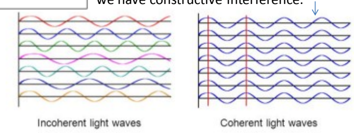
Only coherent waves can have a stable interference pattern,
incoherent waves on average sum up close to 0.





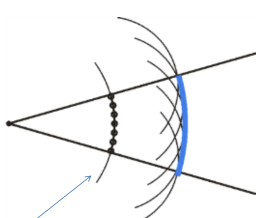
If the phase difference is 0 (or 1,2,3.. $*2\pi$),
we have constructive interference.

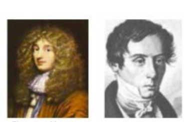
Constructive interference:
the resulting amplitude (A_{sum})
after summation is maximal.
Destructive: $A_{sum} = 0$



The Huygens-Fresnel principle describes the propagation
of waves, it can be used to explain most of the experimental results
(but not the quantum-mechanical ones!)

In short: every point at the wavefront acts as a new sperical point source,
and the resulting new wavefront can be computed as a superposition of all of the
waves generated in this way.




Christiaan Huygens (1629-1695)
Augustin-Jean Fresnel (1788–1827)

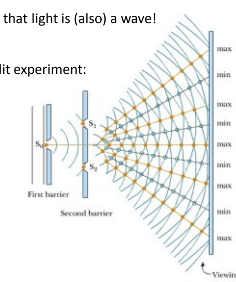
Wavefront: a surface containing points affected in the same way by a wave at a given time.

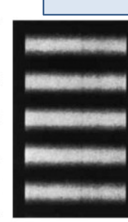
Some experiments, observations, which can only be understood
with the help of wave theory.

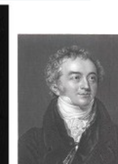
Young's two-slit experiment
Diffraction on grating
Interference patterns

this proves that light is (also) a wave!

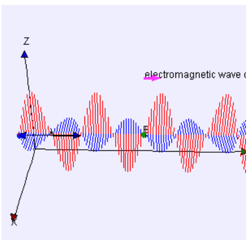
The two-slit experiment:

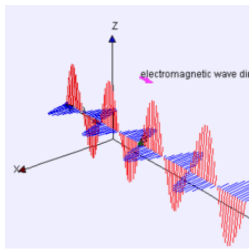





Thomas Young

Since we have interference, we must assume light is a wave.
If so, then we have a wave equation for it. Since it is electro-magnetic, we have two oscillating quantities: electric field strength (**E**) and magnetic field strength (**B**).





For EM waves we have two equations, and the wave can travel to x,y,z directions, so the equations are a bit even more complicated:

$$\frac{\partial^2 \mathbf{E}}{\partial t^2} - c_0^2 \cdot \nabla^2 \mathbf{E} = 0$$
$$\frac{\partial^2 \mathbf{B}}{\partial t^2} - c_0^2 \cdot \nabla^2 \mathbf{B} = 0$$

Here the ∇ means the $d^2/d\ldots^2$ in all directions

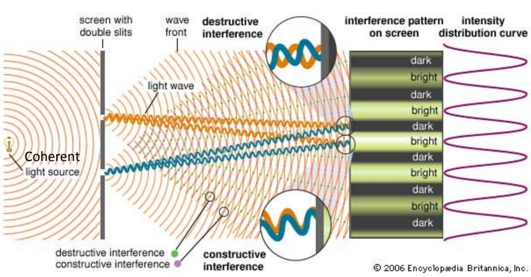
The solution is again a sine or cosine wave:

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}_0 \cos(\omega t - \mathbf{k}\cdot\mathbf{r} + \phi_0)$$
$$\mathbf{B}(\mathbf{r},t) = \mathbf{B}_0 \cos(\omega t - \mathbf{k}\cdot\mathbf{r} + \phi_0)$$

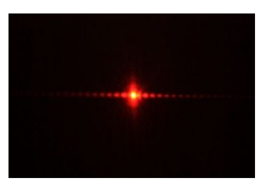
At any point of the observation we have to add all of the incoming sine waves, and that gives the net value of **E** and **B**.

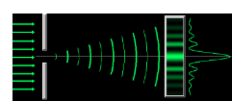
Remember: incoherent waves add up to practically 0, while coherent ones can add up from 0 to a maximum, depending on the phase difference.

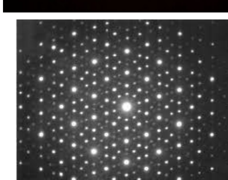
The explanation of the periodic profile : interference of waves, Huygens principle.

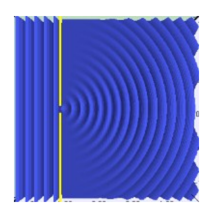


Diffraction and interference patterns with coherent light

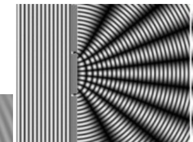


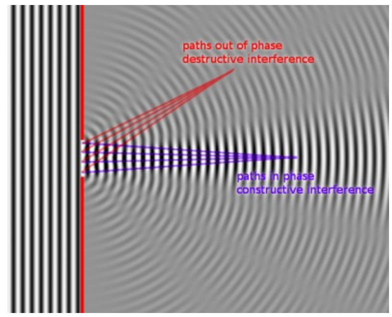


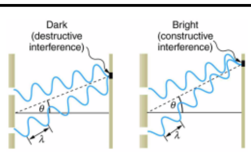


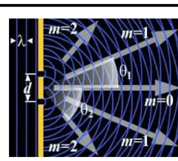


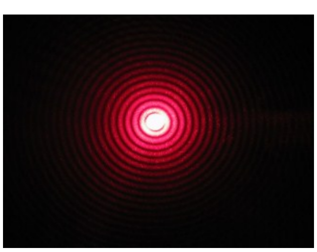
To understand the patterns we need to calculate the phase for each wave at the screen.

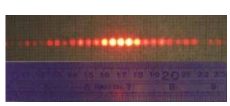




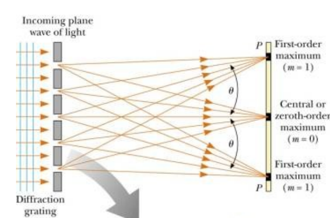






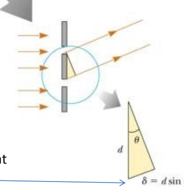


Diffraction on optical grating:



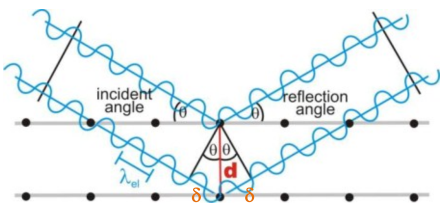
Due to the path difference of δ a phase difference of $2\pi*\delta/\lambda$ arises between waves.

(n.B.: during a length of λ, the time it takes for the light to travel is T, under which the change in the phase is exactly 2π)



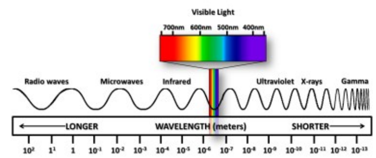
Constructive interference is only possible if the phase difference is 0,1,2,3,… * 2π. This means that δ has to be 0,1,2,3,… * λ

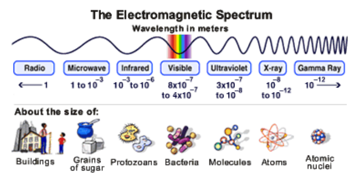
On a reflection grating the concept is the same, but the phase difference (due to **path difference 2*δ**) is twice as much as on a transmission grating.



Applications:

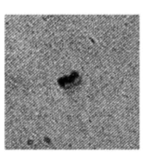
Optical grating can separate the different frequencies -> recording of a spectrum

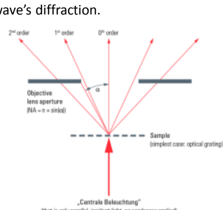


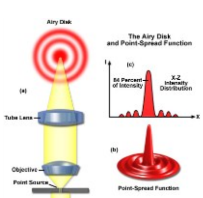


Resolving power of a microscope is limited by the wave's diffraction.

Abbe's principle:
There is **only** an image formation in the microscope **if** at least the first order diffracted waves enter the objective lens.





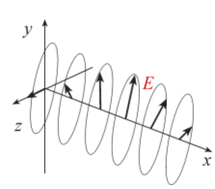


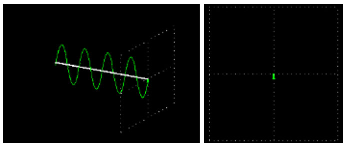
$$d = \frac{\lambda}{2n \sin\alpha}$$

Polarization

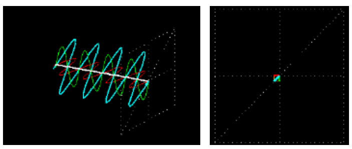
Linearly polarized light

Circular or elliptical polarization

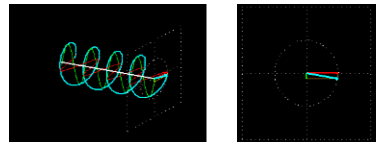




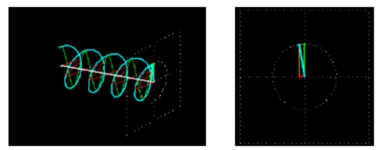
Summation of two waves with different linear polarization



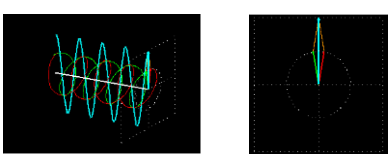
Summation if there is a phase difference, now 90 degrees



The amplitudes are the same, so the resulting electrical field vector rotates in time.
The direction of the rotation depends only on the phase difference (+ or – 90 deg.)



We can even sum two circularly polarized lights:

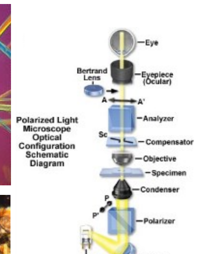


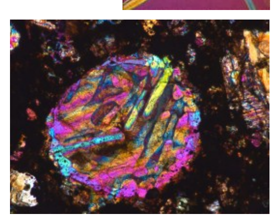
Application: polarimetry.
Chiral molecules turn the polarization direction when light passes through a solution, it can be used to determine the concentration of the solution.
(see lab practice of polarimetry)

For more animations, go to: http://cddemo.szialab.org/

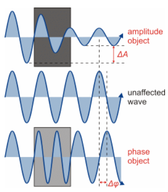
Polarization microscopy

Colors: due to the dependence of specific rotation on the wavelength of light.

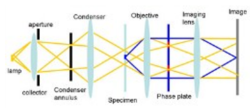




Phase contrast microscopy

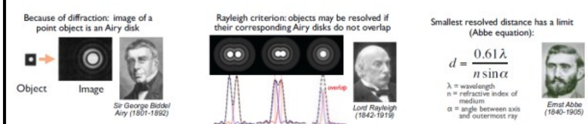


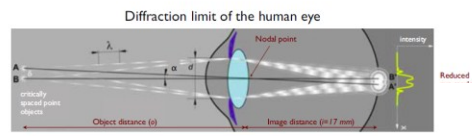
The phase object is transparent,
but the refractive index is different,
so the speed of light is different,
which builds up a phase difference during
the way through the object.
This is not observable to the human eye, but
can be made visible with the phase-contrast
microscope, which creates intensity changes
from the phase changes.



It creates the contrast (amplitude change) by coherently re-interfering a reference, or surround beam (S), with a diffracted beam (D) from the specimen. The S beam is shown in yellow, and the D beam is shown in blue.

Diffraction is also present in the human eye:

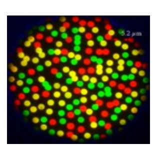


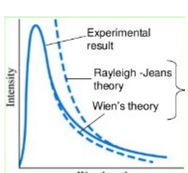


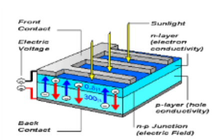
For a detailed explanation, and more figures: see the "optics of the eye" lab manual!

There are some experiments, observations which still can not be explained alone by the wave theory

Photoelectric effect
Quantum dots
Fluorescence
Lasers
Black body radiation



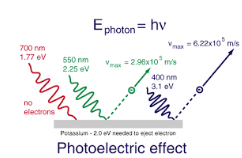




-> **light is a wave AND a particle (this leads us to quantum physics)**

The photoelectris effect:

If we shine light onto some material
(often metals) then we might observe
the release of an electron.
This however *does NOT depend on the intensity* of the light,
but instead **solely on the frequency!**



From simple waves, since energy is flowing in a wave, after a given time the electron should have gathered the required energy to split free of the atom. If we increase the intensity, so the time required for this would become shorter, and we would see more electrons coming
out of the surface in a given time-range. Instead, we see absolutely no electrons independently of the intensity at some wavelength, but some electrons at a shorter wavelength. This contradicts the wave theory.

Solution (Einstein): The energy flow in light is not a smooth constant flow, but instead energy is distributed in "packets" of $\varepsilon = h*f$, where h is the Planck's constant, and f is the frequency.

This theory says, that while light often “behaves” as a wave, it also has some type of particle nature, which is manifested in the existence of a finite energy, which is closely related to the frequency of the wave. This energy is called the “photon energy”, and the term “photon” reflects the particle-type of behavior of light.

The particle-type is also manifested in the impuse of the photon, although it has no resting mass.

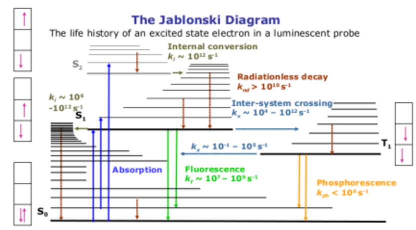
$$E = \hbar\omega = h\nu = \frac{hc}{\lambda}$$

$$\boldsymbol{p} = \hbar\boldsymbol{k},$$

Where $\hbar = h/2\pi$

$$p = \hbar k = \frac{h\nu}{c} = \frac{h}{\lambda}.$$

Fluorescence, phosphorescence, lasers also need the photon picture, if we want to understand what is happening. (this will come later)



- Fluorescence is observed if $k_f \sim> k_i + k_x$
- The time a molecule spends in the excited state is determined by the sum of the kinetic constants of all deexcitation processes