



## Ranks

**Rank:** numerical or ordinal data belonging to a value in data series sorted according to a certain rule.

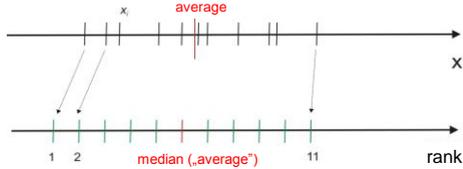
**Ties:**  
In the case of same values every value replaced by the average of the ranks.

e.g.:

- lieutenant
- major
- colonel
- etc.

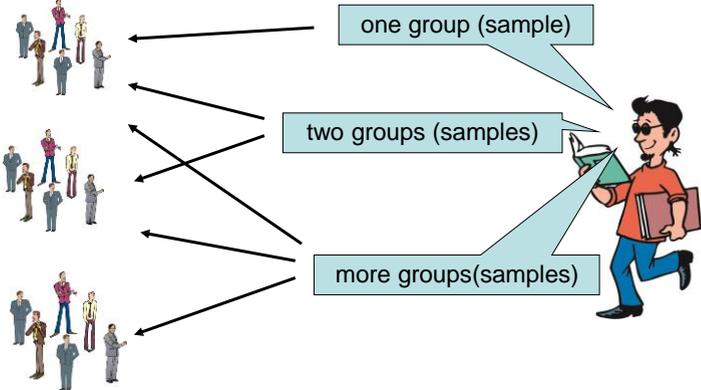
value:	1.2	2	2	3.5	4
rank:	1	2,5	2,5	4	5

## the „average” of the ranks is the median



Rank scale is an equidistance one!  
The median plays the same role as the average.

## On the base of the question



## Summary table of the most frequently used tests

	parametric	non-parametric
<b>one group</b>	1-sample t-test,	Wilcoxon's signed-rank test, sign-test
<b>two groups</b>	2-sample t-test	Mann-Whitney U-test
<b>more groups</b>	ANOVA	Kruskall-Wallis test

## Examination in one group

Question: On the base of the sample the parameter of the population may be a given value?

**parametric**

$\mu = ?$

Nullhypothesis:  $\bar{x} = \mu_0$

1-sample t-test

**non-parametric**

median = ?

Nullhypothesis: the median is a given value

Wilcoxon's signed-rank

## 1-sample t-test

**Example:** The medicine is effective or not?



**Nullhypothesis:** not!  $\mu_0 = 0$ . But the average is not 0!

Sample	Average
1.	-0.2 °C
2.	-1 °C
3.	-1.5 °C



If the difference is big, it seems to be more probable to be non-random!

## What is big difference?

What is the measure of the difference?

**Standard error:** the average deviations of the averages from the  $\mu$ .

**memo:**  $(\bar{x} \pm s_{\bar{x}})$  ~ 68% - confidence interval.

So, we must compare the difference to the standard error!

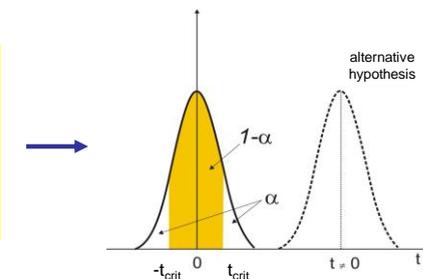
## t-value

$$t = \frac{\bar{x} - \mu_0}{s_{\bar{x}}}$$

Compare the difference to the standard error!  
(frequently  $\mu_0 = 0$ )

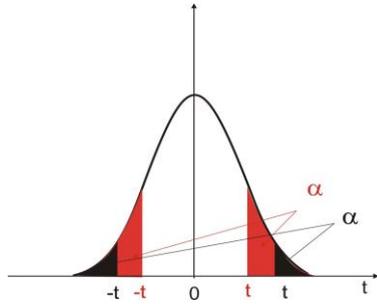
The averages fluctuate around the  $\mu_0$  so the t-values deviate around the 0.

(providing, that the nullhypothesis is true!)



## Why is the t-value better than the original value?

We are able to calculate the probabilities on the base of this distribution!!! (Student- or t-distribution)



It only describes the **random deviations** of the t-values!

The shape of the distribution depends on the no. of elements.

## Degree of freedom (d.f.)

I think 3 numbers! (sample)

The average of them: 8! (information!)



they must be!

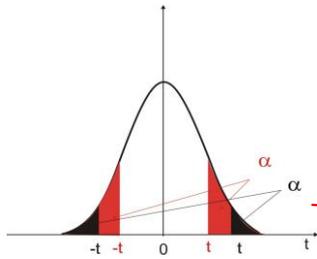
3, 12, 8 or 5, 7, 11 etc.

d.f. = n

3, 12, **9** or 5, 7, **12** etc.

d.f. = n-1

## The t-table



t-table

d.f.	significance level			
	0.1	0.05	0.02	0.01
1	6.31	12.7	31.8	63.7
2	2.92	4.3	6.96	9.92
3	2.35	3.18	4.54	5.84
4	2.13	2.78	3.75	4.6
5	2.02	2.57	3.37	4.03

Different  $t_{crit}$  values belong to different significance levels.

d.f.: n-1

## Decision the base of t-table

t-table

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d.f.: n-1

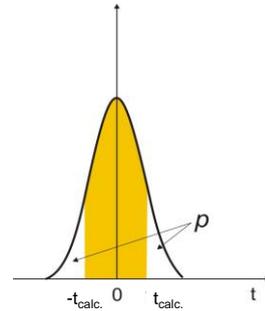
Select an appropriate significance level!

If  $\geq 2.78$  reject, if smaller accept the nullhypothesis.



## Decision using computer

I am able to integrate!!!

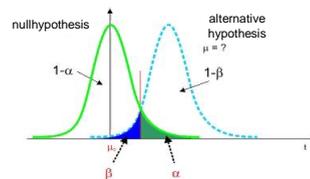
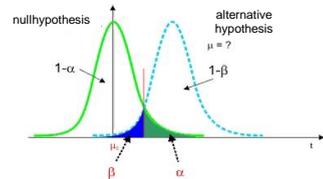


$p$ : probability, that the  $t_{calculated}$  is so large randomly.

## Decision

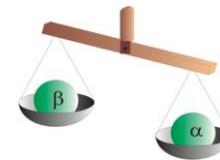
- 1. If the probability of the random deviation is small ( $p(|t| \geq t_{crit}) \leq 5\%$ ) – **reject** the null hypothesis.
- 2. If the probability of the random deviation is large ( $p(|t| \geq t_{crit}) > 5\%$ ) – **accept** the null hypothesis.

## Error?



## The error

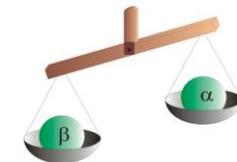
Reject the null hypothesis



**I. type error**

$\alpha$  is the measure of the error. Smaller  $p$  is better.

Accept the null hypothesis



**II. type error**

$\beta$  is the measure of the error. Larger  $p$  is better.

## Condition for 1-sample t-test

- Task: Decision about the  $\mu$  on the base of one sample.
- The variable must have **normal distribution**.



What can we do if it isn't true?

## Wilcoxon's signed-rank test

Example: Is there any effect of an entertaining movie on the patients? ( The numbers are scores)

n	before	after	Diff.
1	2	2	0
2	0	1	1
3	3	2	-1
4	2	4	2
5	1	3	2
6	3	3	0
7	1	4	3
8	1	5	4
9	5	2	-3
10	4	4	0

Normal distribution?



## Ranking

Sort the absolute values of the differences (without 0-s)! Let the sign of ranks be same then the differences!  
Calculate the averages and sd of signed-ranks!

Diff.	absolute value	rank	Signed-rank
0	0		
1	1	1.5	1.5
-1	1	1.5	-1.5
2	2	3.5	3.5
2	2	3.5	3.5
0	0		
3	3	5.5	5.5
4	4	7	7
-3	3	5.5	-5.5
0	0		



## The nullhypothesis

There is no effect of the movie!

The median = 0!  
The deviation is random!

$$H_0: \mu_0 = 0$$

$$H_1: \mu_0 \neq 0$$



## known distribution



$$t = \frac{\bar{R} - 0}{s / \sqrt{n}}$$

If n is enough large!  
(small sample: base distribution of ranks)

$\bar{R}$  - the average of the signed-ranks  
s - the standard deviation of them

Remember!  
„average” of the ranks = median



## Decision

This is known!!!

Of course! This is similar to the 1-sample t-test!!!



## Paired t-test

If the data may be paired according to a rule!

Observation on the same person, paired organ (e.g. kidney).

Rare, on the base of viewpoints (age, profession, etc.).

Look at: decreasing the fewer.



Experimental design

## Experimental design

test from previously collected data?

practical order:  
Question → experimental design → calculation.



Several problems may arise.  
e.g. a few data are suitable only.



## „real” 1-sample t-test

Is it possible that the  $\mu$  is equal to a value?

$$t = \frac{\bar{x} - \mu_0}{s_x}$$



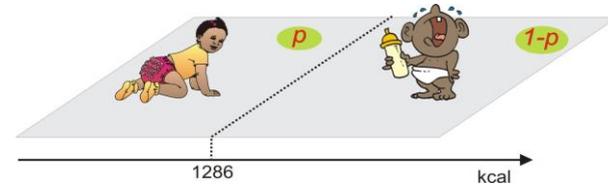
Rare case.



## sign-test

Example: Energy uptake in the population of 2-year old children.

**Question:** May be the median (This derives from an another test) is 1286 kcal?  
**Nullhypothesis:** median = 1286 kcal (deviation is random).



## Test

Small sample

$$P(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

binomial distribution

Large sample

$$z = \frac{|x - np| - 1/2}{\sqrt{np(1-p)}}$$

standard normal distribution

x – no. of children below 1286 kcal.

n – no. of children in test.

p – probability, that randomly smaller (look at: binomial distribution)

## Decision

Calculate the probability of the random deviation. (binomial, or standard normal distribution)

End of this part!

