Test in two goups

Question: May the samples derive from the same population? May the parameters of the two populations be the same?

parametric

$\mu_1 = \mu_2$?

Nullhypothesis: $\mu_1 = \mu_2$

2-sample t-test

non-parametric

Nullhypothesis: same.

Mann-Whitney U-test

Why is not a paired test?

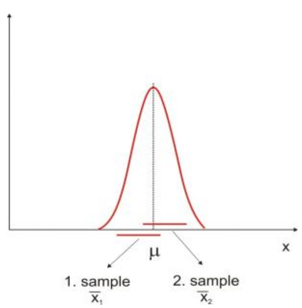
	A			B			C	
172	184	12	162	170	8	162	184	22
180	172	-8	165	172	7	165	180	15
165	180	15	172	175	3	172	175	3
184	175	-9	180	180	0	180	172	-8
162	170	8	184	184	0	184	170	-14
	Avg	3.6		Avg	3.6		Avg	3.6
	Sd	**11.33**		Sd	**3.782**		Sd	**15.11**
	Se	5.066		Se	1.691		Se	6.757
	t	0.711		t	2.129		t	0.533
	p	0.517		p	0.1		p	0.622

different s_d!

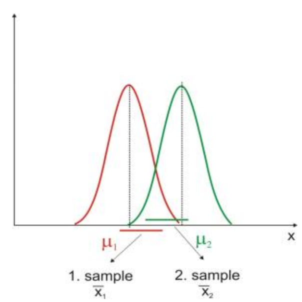
arbitrary paired values!

Two-sample t-test

one population
(the deviation of the averages is random)



two populations
(the deviation of the averages is not random.)



Standard error

$$s_1 = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n_1 - 1}} \qquad s_2 = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n_2 - 1}}$$

$$s_{\bar{x},1} = \frac{s_1}{\sqrt{n_1}} \qquad s_{\bar{x},2} = \frac{s_2}{\sqrt{n_2}}$$

Common standard error: the weighted average of the two standard errors.

$$s_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{Q_1 + Q_2}{n_1 + n_2 - 2}} \cdot \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

2-sample t-test

$\bar{x}_1 \neq \bar{x}_2$

?

It may be random (nullhypothesis) or non-random (alternative hypothesis).
Known distribution is necessary!

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s^*} \sqrt{\frac{n_1 \cdot n_2}{n_1 + n_2}}$$

$$s^* = \sqrt{\frac{Q_1 + Q_2}{n_1 + n_2 - 2}}$$

Test

The t-vaue is same!

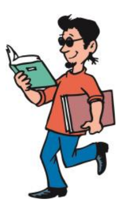
How much is the d.f.?

d.f. = n_1 + n_2 -2

($(n_1-1)+(n_2-1)$)

Conditions for the test

- Task: comparison of two **independent** samples.
- The quantity has **normal distribution**.
- The sd-s are **same** in the goups.

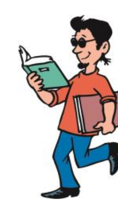


This is new!
How is it proved?

Test for standard deviations

How can I do?

Nullhypothesis: the two standard deviations are the same and the difference is random (sampling error).

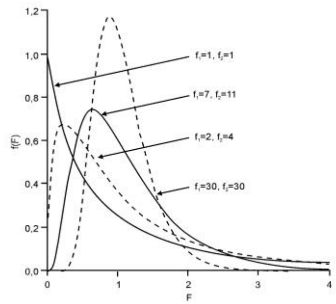


It is similar to the hypothesis testing!

F-test

A so-called F-distribution belongs to the nullhypothesis.

$$F = \frac{s_1^2}{s_2^2}$$



Degree of freedom:

nominator: n_1-1
denominator: n_2-1

Degree of freedom

Using a computer it is not so important.
Using F-table always the higher value is in the nominator.
(F>=0 and d.f. depends on the situation.)

Which variance is in the nominator?

Decision

- 1. If the probability of the random deviation is small ($p \le \alpha$) –we ***reject*** the nullhypothesis.
- 2. If the probability of the random deviation is high ($p > \alpha$) – we ***accept*** the nullhypothesis.

If the two standard deviations are not the same!

Correction:

Correction of the degree of freedoms.

Correction of the t-values.

Mann-Whitney U-test

Example: Is the painkiller effective?

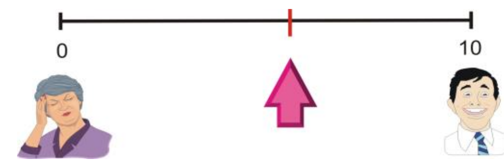
It has a serious pain.

How can we measure the effect?

Experiment

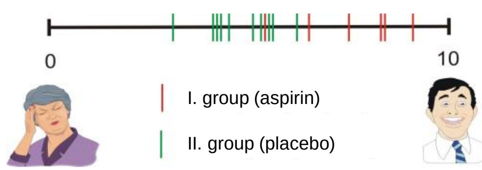
I. group:
(case)
aspirin is used

II. group:
(control)
placebo isused
(without agent)



This is an arbitrary scale.

Results



Value	3.1	4.1	4.2	4.3	4.5	5.1	5.3	5.4	5.5
Rank	1	2	3	4	5	6	7	8	9
Value	5.6	6.2	6.2	6.5	7.5	8.3	8.3	8.4	9.1
Rank	10	11.5	11.5	13	14	15.5	15.5	17	18

The nullhypothesis

The medicine is not effective.

The 2 groups belong to the same population.

(The „medicine” is really a placebo.)

The sum of the ranks (Gauss story)

Add the numbers from 1 to 100!

It is easy to calculate!

1 + 100 = 101
2 + 99 = 101 …

$$\sum_{i=1}^{n} i = \frac{n}{2}\cdot(n+1)$$

Sum of the ranks

T – the sum of the ranks in the I. group, in the case of random deviation. The expected value is:

$$n_1 \cdot \frac{n_1+n_2+1}{2}$$

(n_1 element, their average = $(n_1+n_2+1)/2$)

Nullhypothesis: the deviation from this is random.

Small n: U-distribution describes the probability of the random deviation.

The transformation

(if n is enough large)

T – the sum of the ranks in the I. group. The expected value in the case of random distribution is:

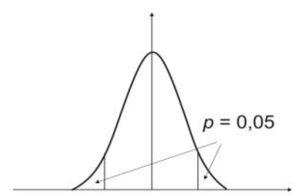
$$n_1 \cdot \frac{n_1+n_2+1}{2}$$

z has standard normal distribution.

$$z = \frac{T - n_1(n_1+n_2+1)/2}{s}$$

$$s = \sqrt{\frac{n_1 \cdot n_2 \cdot (n_1+n_2+1)}{12}}$$

Decision



standard normal distribution

The calculated *z*-value: 3.24.

Higher then 1.96.

Conclusion: we reject the nullhypothesis.

Calculated p-value < 0.1%.

The conclusion is same.