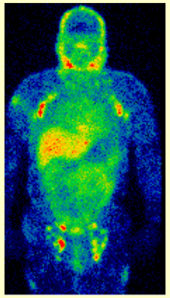


Nuclear radiation,

radioactive isotopes

Constituents of atoms

Particle	Symbol	Rest Energy (MeV)	Relative Charge*	Mass (kg)	Relative Mass (AMU)**
electron	e	0.51100	1-	9.11×10^{-31}	$5.4858X10^{-4}$
proton	p	938.272	0	$1.6726X10^{-27}$	1.0072765
neutron	n	939.566	1+	$1.6749X10^{-27}$	1.0086649

* electrons have an electric charge of -1.602×10^{-19} C

**The atomic mass unit is defined as 1/12 of the carbon (^{12}C)atom

Nuclear notation

Mass number
A = Z + N

$^{A}_{Z}X$ $^{27}_{13}Al$

Chemical symbol for the element

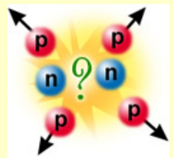
Atomic number = Number of protons

N = number of neutrons

Nuclear stability

- There are very large *repulsive electrostatic forces* between protons

  These forces should cause the nucleus to fly apart



- $\Delta M = [Zm_p + (A\text{-}Z)m_n] - M(A,Z)$ (?)

It must be that a different type of force exists within the nucleus

1911 Rutherford, Geiger and Marsden

Nuclear stability

$$\Delta M = [Zm_p + (A\text{-}Z)m_n] - M(A,Z)$$

Instead of two separate conservation laws, a single conservation law states that the sum of mass and energy is conserved. Mass does not magically appear and disappear at random. A decrease in mass will be accompanied by a corresponding increase in energy and vice versa.

$$\Delta E = \Delta Mc^2$$

Another, short-range force is present, called the *nuclear force* (Rutherford, 1911

Nuclear stability

- Another, short-range force is present, called the *nuclear force* (Rutherford, 1911)

This is an attractive force that acts between all nuclear particles

The nuclear attractive force is stronger than the Coulomb repulsive force at the short ranges within the nucleus

$$\Delta E = \Delta Mc^2$$

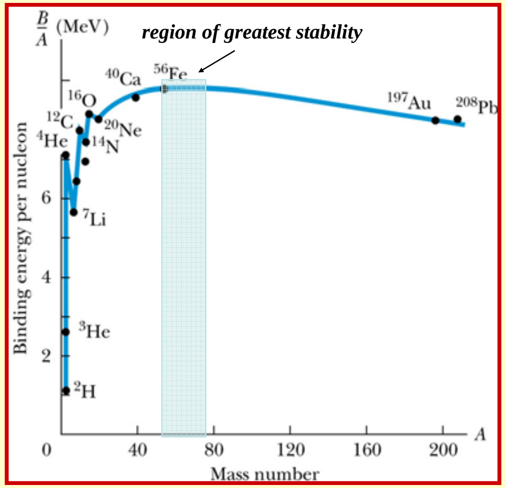
The nuclear force is

- independent of charge

- the range of action is extremely short (~fm)

Binding energy per nucleon

- The curve increases rapidly
- Sharp peaks for the even-even nuclides ^{4}He, ^{12}C, and ^{16}O
- Maximum is around A=56



Nuclear stability

The balance of proton and neutron number

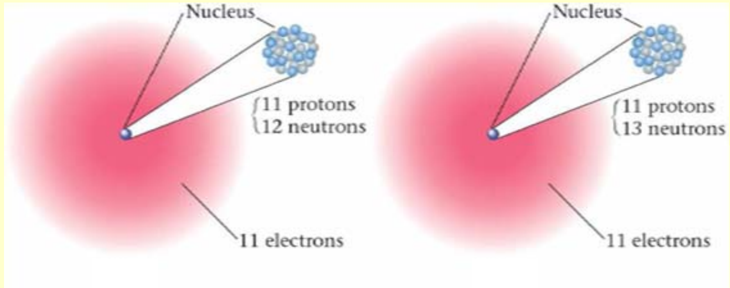
is extremely important for the stability of the nucleus

Isotopes

Greek ***isos topos* = *equal place***

Isotopes of an element have nuclei with
- the same number of protons
- different numbers of neutrons
- different mass number

Example of isotopes



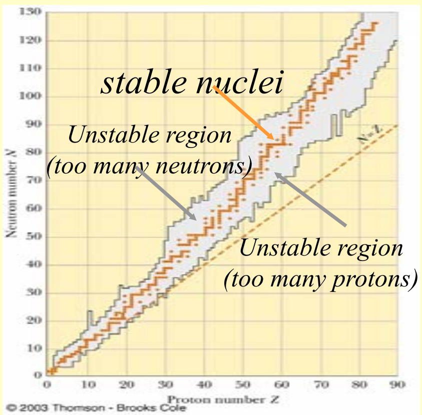
$^{23}_{11}Na$ $^{24}_{11}Na$

What is/are the stable combination(s)? 1:1 ?

Nuclear stability chart

- Light nuclei are most stable if N=Z
- Heavy nuclei are most stable when N > Z

  As the number of protons increase, the Coulomb force increases and so more nucleons are needed to keep the nucleus stable



- No nucleus is stable when Z > 83

Antoine Becquerel
1903 Nobel Prize in Physics for discovering radioactivity

Image of Becquerel's photographic plate which has been fogged by exposure to radiation from a uranium salt. The shadow of a metal Maltese Cross placed between the plate and the uranium salt is clearly visible. (1896)

Radioactive decay

- *Radioactivity* is the spontaneous release of energy in the form of radioactive particles or waves
- Experiments suggested that radioactivity was the result of the decay, or disintegration, of unstable nuclei
- Three types of radiation can be emitted
  - Alpha (α) particles
  - Beta (β) particles
  - Gamma (γ) rays (Rutherford 1896, see details later)

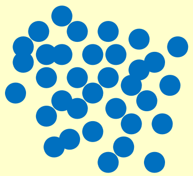
- The nuclides, as with most things in nature, want to be at their *lowest energy state* which is a stable nucleus.
- Radioactive decay occurs in nuclides where the *nucleus is unstable.*

•The nuclide reaches its stable state by undergoing radioactive decay.

Characteristics of radioactive decay

• it is *statistical process* – individual disintegrations occur ***randomly***

• it results in a decrease over time of the original amount of the radioactive nuclei



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decrease of the original amount of the radioactive nuclei over time

Activity : $$\Lambda = \left| \frac{dN}{dt} \right|$$

N: number of nuclei to be decayed

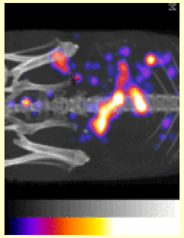
t: time

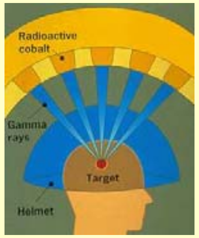
number of nuclei decayed in a unit time

measure: bequerel (Bq)
1Bq = 1 decay/sec

Typical activities in the practice

kBq,	***MBq,***	***GBq,***	***TBq***
natural background	*in vivo* diagnostics	laboratory practice	*therapy*





Radioactive decay law

Differential form $$\left|\frac{dN}{dt}\right| = -\lambda N$$

***λ* :** decay constant (measure: 1/s)

constant for a certain isotope

Activity depends both on the

- size of the population of radioactive atoms
- type of the isotope

$$\left|\frac{dN}{dt}\right| = -\lambda N$$

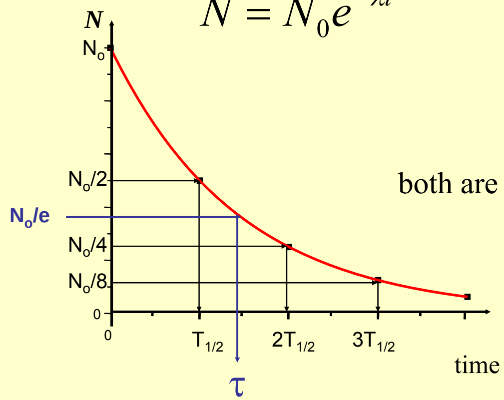
Solution of this equation yields

$$N = N_0 e^{-\lambda t}$$ **Integral form**

N_0 : number of radioactive nuclei at *t = 0,*
***N* :** the number radioactive of nuclei remaining after a period *t*

Graphical representation

$$N = N_0 e^{-\lambda t}$$

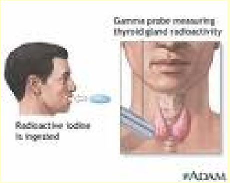


$T_{1/2}$: the half-life

τ : the mean life time

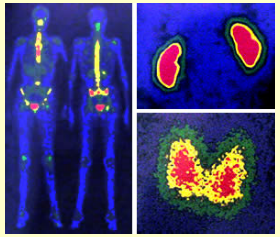
both are characteristic to the nuclei

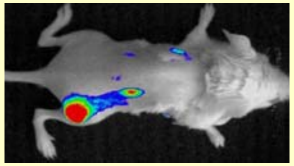
Half-lives in Medical Uses



Iodine - 131 (^{131}I) - ***$T_{1/2}$ = 8 days***
Thyroid treatment

Technetium-99m (^{99m}Tc) – ***$T_{1/2}$ = 6 hours***
Isotope diagnostics





Gold-198 (^{198}Au) - ***$T_{1/2}$ = 2.7 days***
Tumor therapy

Further considerations

$$N = N_0 e^{-\lambda t}$$

If t = $T_{1/2}$ $\Longrightarrow$ $N_0 / 2 = N_0 e^{-\lambda T_{1/2}}$

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{T_{1/2}}$$

If t = τ $\Longrightarrow$ $N_0 / e = N_0 e^{-\lambda \tau}$

$$\lambda = \frac{1}{\tau}$$

Definition of decay constant

Alteration of activity in time

$$N = N_0 e^{-\lambda t}$$

$$\Downarrow$$

$$\Lambda = \Lambda_0 e^{-\lambda t}$$

$$\Lambda = \lambda N$$

Specific activity : activity in a unit mass (Λ/m),

unit: Bq / kg

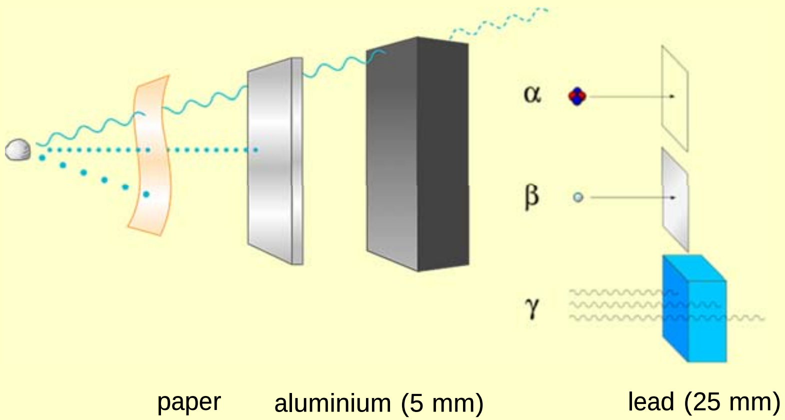
Characteristics of radioactive isotopes

Activity : depends both on the nucleus and the size of its population

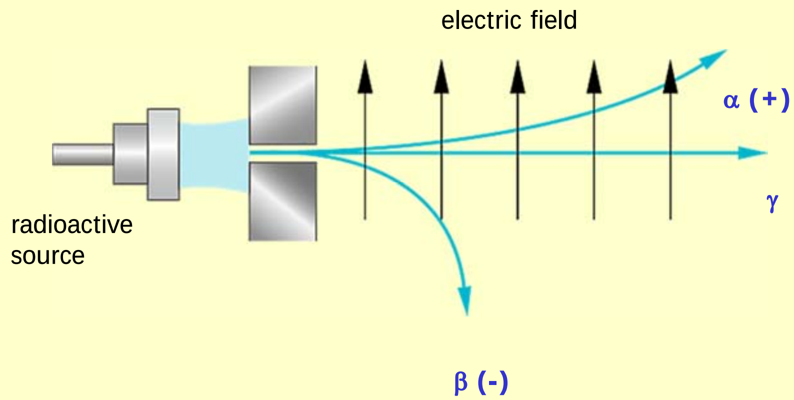
Half-life : physical parameter of each radioactive isotope

Type of radiation: physical characteristic of the nucleus

Types and nature of nuclear radiation

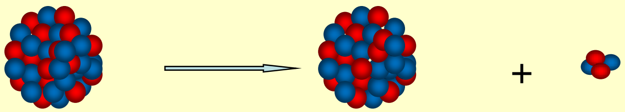


Deflection of radiation in electric field



Radiation from a radium source is split by an electric field.

α decay



parent nucleus *daughter nucleus* *α particle*

$$^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}\alpha$$

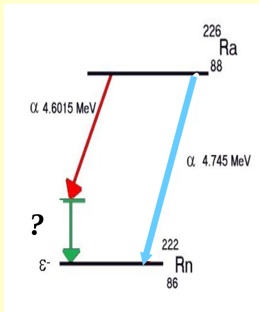
α particle is a nucleus of helium containing two neutrons and two protons

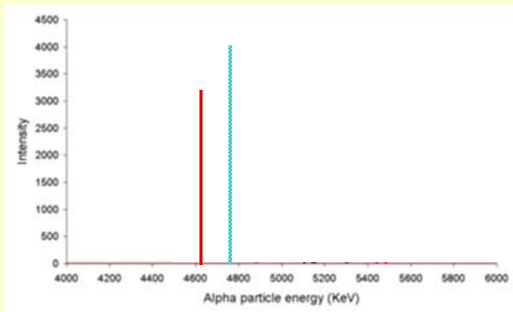
Heavy nuclei with mass numbers higher than 150 can disintegrate by emission of an α particle

example

$$^{226}_{88}Ra \longrightarrow {}^{222}_{86}Rn + {}^{4}_{2}\alpha$$

Energy spectrum of α radiation

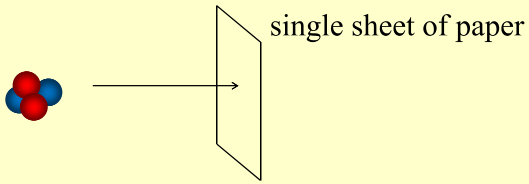




line spectrum

Energy is characteristic for the nucleus

Penetration depth of α particles

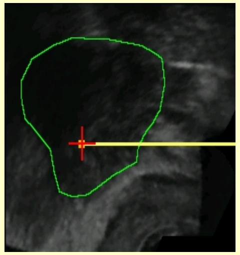


absorber	density	alpha range
air (STP)	1.2 mg/cm^3	3.7 cm
paper (20lb)	0.89 g/cm^3	53 µm
water (soft tissue)	1.0 g/cm^3	45 µm

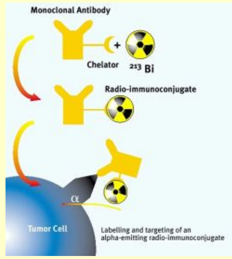
Medical application of α radiation

Diagnostics: none

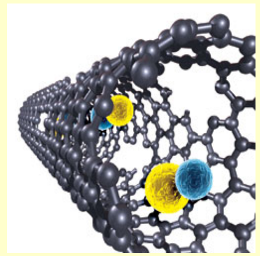
Targeted alpha **therapy** of cancer



Seed implantation by needle



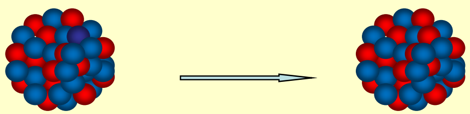
Monoclonal antibody



Carbon nano-tube

β decay

1. Neutron excess: β⁻ decay



+ • + $\bar{\nu}$

parent nucleus *daughter nucleus*

$$^{A}_{Z}X \longrightarrow \ ^{A}_{Z+1}Y + e^{-} + \bar{\nu}$$

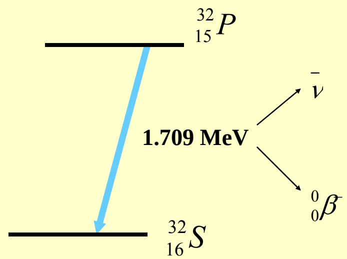
$$^{1}_{0}n \longrightarrow \ ^{1}_{1}p^{+} + \ ^{0}_{0}\beta^{-} + \bar{\nu}$$

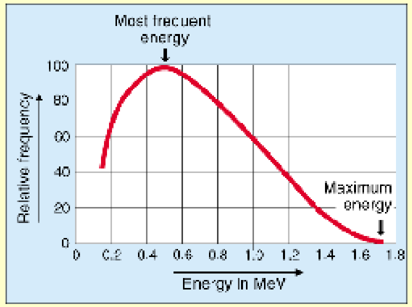
anti-neutrino

example

$$^{131}_{53}I \longrightarrow \ ^{131}_{54}Xe + \ ^{0}_{0}\beta^{-} + \bar{\nu}$$

Energy spectrum of β radiation

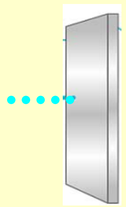




Energy distribution of the β- particles emitted during the β- decay of ^{32}P.

continuous spectrum
with maximum kinetic energy for the β particle

Penetration depth of β^- particles



5 mm aluminum

absorber	*density*	*maximum beta range* (2.3 MeV)	*maximum beta range* (1.1 MeV)
air	1.2 mg/cm^3	8.8 m	3.8 m
water (soft tissue)	1.0 g/cm^3	11 mm	4.6 mm
aluminum	2.7 g/cm^3	4.2 mm	2.0 mm
lead	11.3 g/cm^3	1.0 mm	0.4 mm

Medical application of β^- radiation

Diagnostics: none

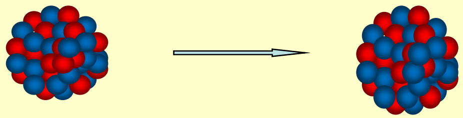
Targeted therapy: hyperthyroidism, thyroid and several other types of cancer

Brachytherapy: implants into the tumours

Endovascular irradiation

β decay

2. Proton excess: β^+ decay



$+ \; \cdot \; + \; \nu$

parent nucleus *daughter nucleus*

$${}^{A}_{Z}X \longrightarrow {}^{A}_{Z-1}Y + e^+ + \nu$$

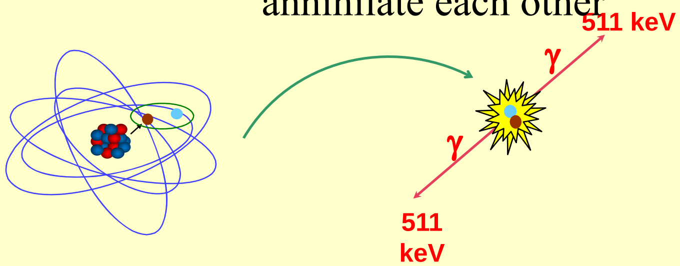
$${}^{1}_{1}p^+ \longrightarrow {}^{1}_{0}n + {}^{0}_{0}\beta^+ + \nu$$

neutrino

example

$${}^{11}_{6}C \longrightarrow {}^{11}_{5}B + e^+ + \nu$$

Annihilation - particle-antiparticle pairs can annihilate each other



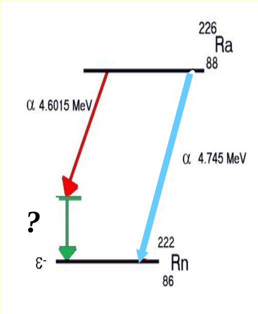
1. Conservation of momentum : two photons with opposite direction are produced

2. Energy balance:

$$m_e c^2 + m_p c^2 = 2\,hf$$

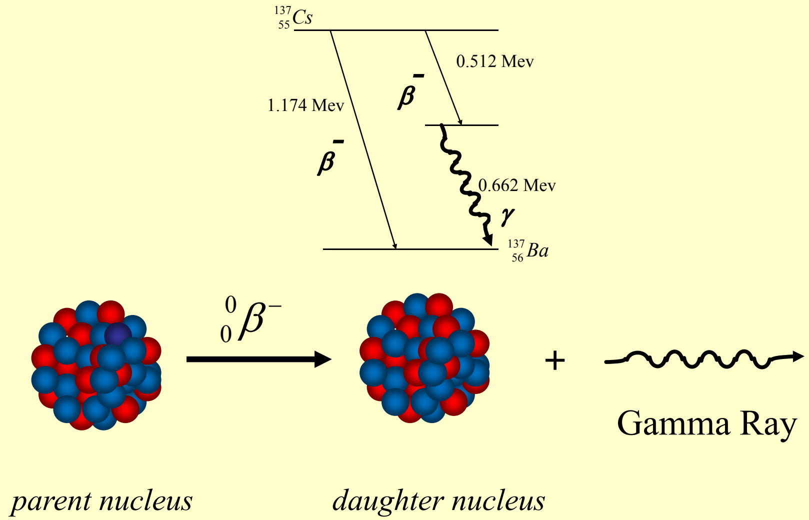
mass – energy equivalence

γ decay

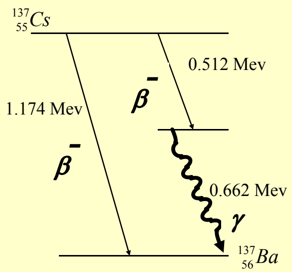


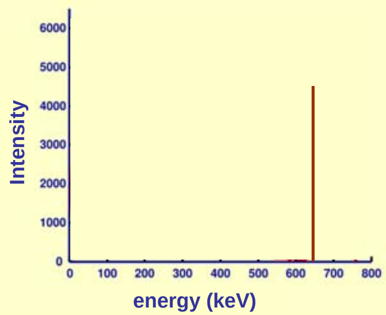
Sometimes the newly formed isotopes (after α or β decay) appear in the excited stat.

Excited nuclides have tendency to release the excess of energy by **electromagnetic radiation** - emission of gamma rays.



Energy spectrum of γ radiation

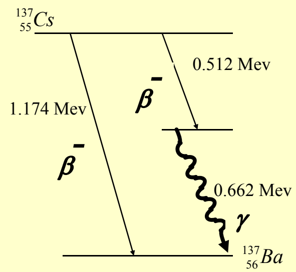


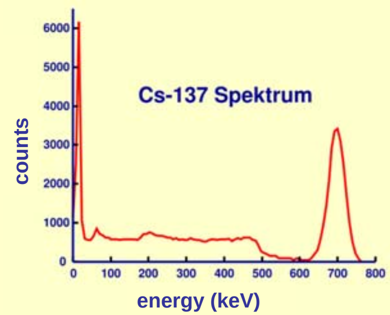


line spectrum

Energy is characteristic for the nucleus

Energy spectrum of γ radiation

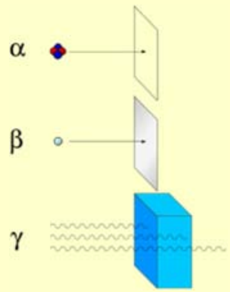




line spectrum

Energy is characteristic for the nucleus

Penetration depth of γ radiation



lead (25 mm)

Penetration distance is higher than that of α or β particles, but it is highly energy dependent.

Gamma rays can travel from 1 to 100s of meters in the air and can easily go right through people (~dm).

Timing of γ emission

Lifetime of the excited nucleus:

1.Prompt γ decay: ~ $\mathbf{10^{-13} - 10^{-18}}$ **s**

2. Isommeric transition: $\geq \mathbf{10^{-10}}$ **s**

Question of the week

What is the origin of nuclear force?

Isomeric transition

Some excited states may have a half-lifes ranging

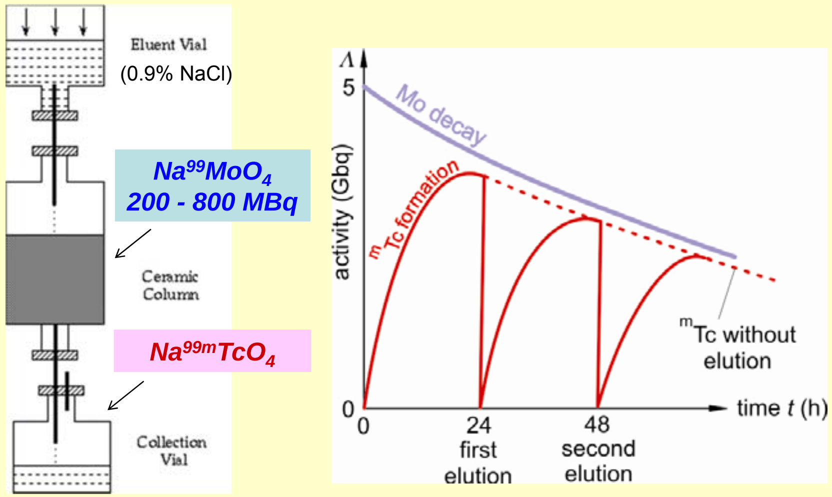
from hours up to more than 600 years

$$^{Am}_{Z}X \rightarrow {}^{A}_{Z}X + \gamma$$

$$^{99}_{42}Mo \rightarrow {}^{99m}_{43}Tc + \beta^{-} \rightarrow {}^{99}_{43}Tc + \gamma$$

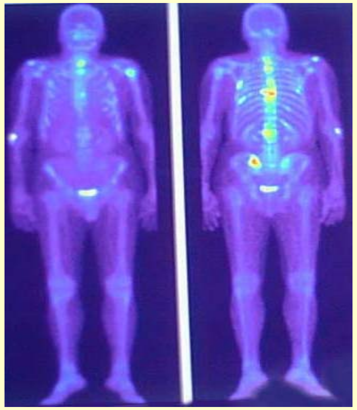
$T_{1/2}$=67 hours $\quad$ $T_{1/2}$=6 hours

Technetium-99m generator



Medical application

Diagnostics: ideal for isotope diagnostics



Bone scan using ^{99m}Technetium labeled phosphate compound

Radionuclide	Compound	Organ	Function
$^{99}Tc^m$	sodium pertechnetate	brain	blood flow
$^{99}Tc^m$	coagulated albumin	lung	blood flow
$^{99}Tc^m$	colloidal suspension	liver	liver function
$^{99}Tc^m$	complex phosphate	bone	bone metabolisms
$^{99}Tc^m$	red blood cells	heart	blood circulation
^{123}I	iodide	thyroid	metabolisms
^{123}I	hippuran	kidneys	renal function
^{133}X	gas	lungs	ventilation

Damjanovich, Fidy, Szöllősi: Medical Biophysics

I. 1.5
- 1.5.1
- 1.5.2
- 1.5.4

II.3.2
- 3.2.1
- 3.2.2
- 3.2.3
- 3.2.4