



Medical signal processing



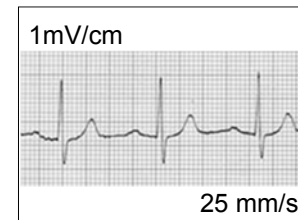
KAD 2016.12.06

A **signal** is any kind of physical quantity that conveys/transmits/stores information

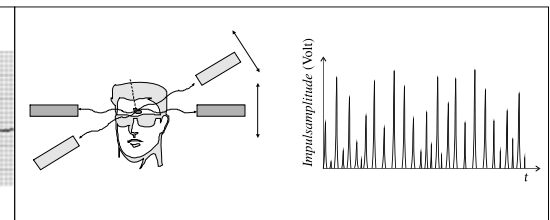
e.g. (1)
electrical voltage, that can be measured on the surface of the skin/head as a result of the heart-/muscle-/brain activities (ECG/EMG/EEG)

e.g. (2)
gamma photon detection in radioisotope diagnostics

(1)



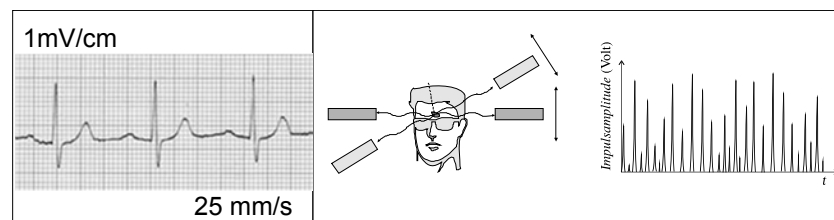
(2)



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Classification of signals

static	—	time-dependent
periodic	—	non-periodic
random	—	deterministic
pulsed	—	continuous
electric	—	non-electric
analog	—	digital



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in a very special role

electric signals

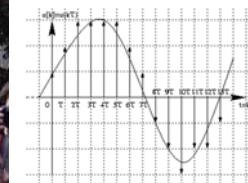
non-electric signals are transferred to electric ones

advantages of **electric** signals:
they are easy to transform, amplify, transmit

digital signals

analog signals are transferred to digital ones

advantages of **digital** signals:
they are easy to store, the noise can be engineered and influence can be reduced



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quantity that compares the magnitudes of two signals:

Signal level or Bel-number (or Decibel-number): n

(named after A. Bell)

unit of n : Bel (B) or decibel (dB)

$$n = \lg \frac{P_2}{P_1} \text{ B} = \lg \frac{J_2}{J_1} \text{ B} = \lg \frac{E_2}{E_1} \text{ B}$$

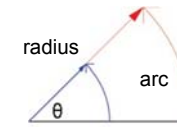
decimal logarithm of ratio of two powers (intensities, energies)

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cf. **radian**

$$\Theta = \frac{\text{arc}}{\text{radius}}$$

$$[\Theta] = \frac{\text{m}}{\text{m}} = \text{rad} = 1$$



cf. **pH** (power of Hydrogen)

$$\text{pH} = -\lg \frac{[\text{H}^+]}{1\text{M}}$$

$$\text{e.g.: } [\text{H}^+] = 10^{-7}\text{M}$$

$$\Rightarrow \text{pH} = -\lg 10^{-7} = -1 \cdot (-7) = 7$$

instead of Bel number we are using **decibel-number**

$$n = 10 \cdot \lg \frac{P_2}{P_1} \text{ dB}$$

$$(10\text{d} = 1)$$

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the **characteristic** unit: **power** (or intensity/energy),
the **practical** unit: (electric) **voltage**

the relation between power and voltage:

$$P = U \cdot I = \frac{U^2}{R} \quad (\text{Ohm: } U = R \cdot I)$$

signal level with voltages:

$$n = 10 \cdot \lg \frac{P_2}{P_1} \text{ dB} = 10 \cdot \lg \frac{\frac{U_2^2}{R_2}}{\frac{U_1^2}{R_1}} \text{ dB} =$$

$$= 10 \cdot \lg \frac{U_2^2}{U_1^2} \text{ dB} = 20 \cdot \lg \frac{U_2}{U_1} \text{ dB}$$

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$$\frac{P_2}{P_1} = 2 \Leftrightarrow 10 \lg 2 \text{ dB} =$$

$$= 10 \cdot 0,3 \text{ dB} = 3 \text{ dB}$$

$$\frac{P_2}{P_1} = \frac{1}{2} \Leftrightarrow -3 \text{ dB}$$

cf. half life,
half value thickness

$$\frac{P_2}{P_1} = 10 \Leftrightarrow 10 \cdot \lg 10 \text{ dB} =$$

$$= 10 \cdot 1 \text{ dB} = 10 \text{ dB}$$

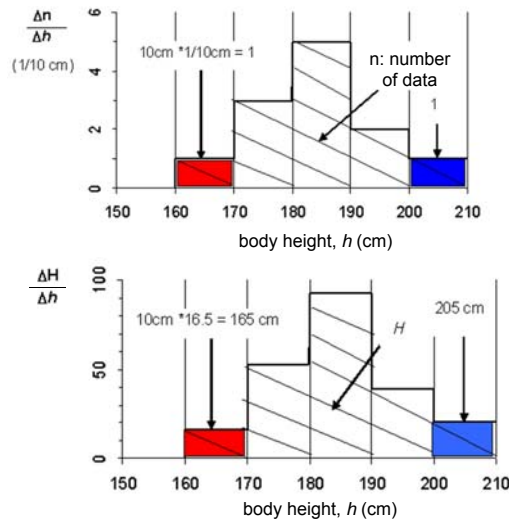
$$\frac{P_2}{P_1} = 100 \Leftrightarrow 10 \lg 100 \text{ dB} =$$

$$= 10 \cdot 2 \text{ dB} = 20 \text{ dB}$$

U_2/U_1	P_2/P_1	dB
1,414	2	3
2	4	6
	8	9
3,16	10	10
	20	13
10	100	20
	$1000=10^3$	30
$100=10^2$	$10000=10^4$	40
$1000=10^3$	10^6	60

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empirical density function



H

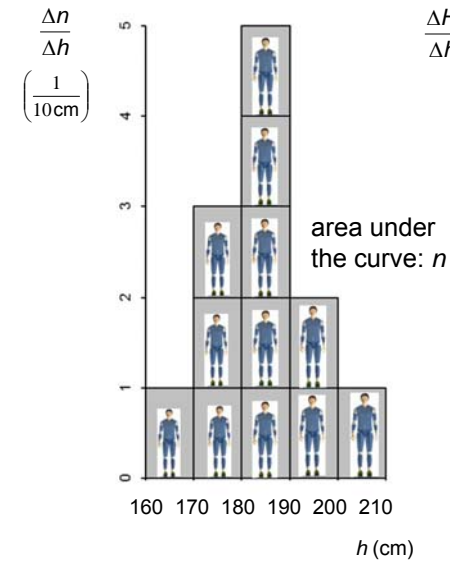


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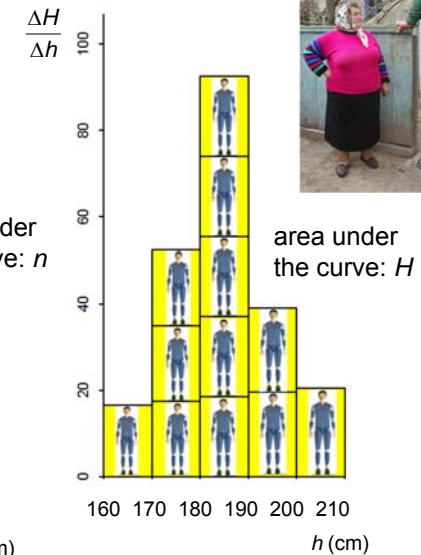
spectrum, as a special density function

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Density function



Spectrum



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Fourier's theorem for periodic functions (signals)

all (usual) periodic functions can be expressed as a sum of sine (and cosine) functions from the fundamental frequency and the overtones

periodic function:
there is a period, T



$\frac{1}{T} = f$, where f is the frequency T

the sine function, which has the same frequency as the periodic function:

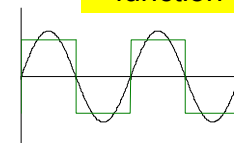
fundamental frequency

$2f, 3f, 4f, \dots$: **overtones**

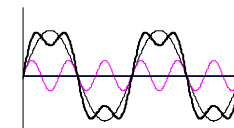
(line spectrum)

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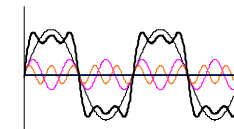
function



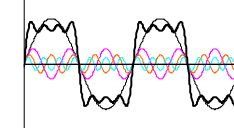
square pulse train
fundamental
fr(equency)



fundamental fr.+
3rd overtone

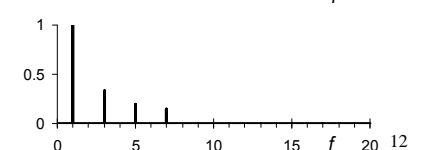
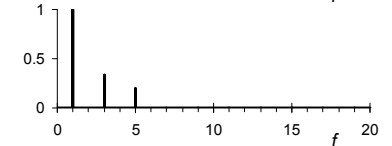
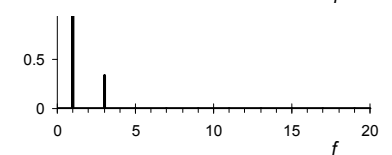
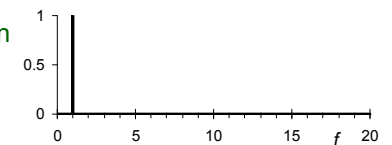


fundamental fr.+
3rd overtone +
5th overtone



fundamental fr.+
3rd overtone +
5th overtone +
7th overtone

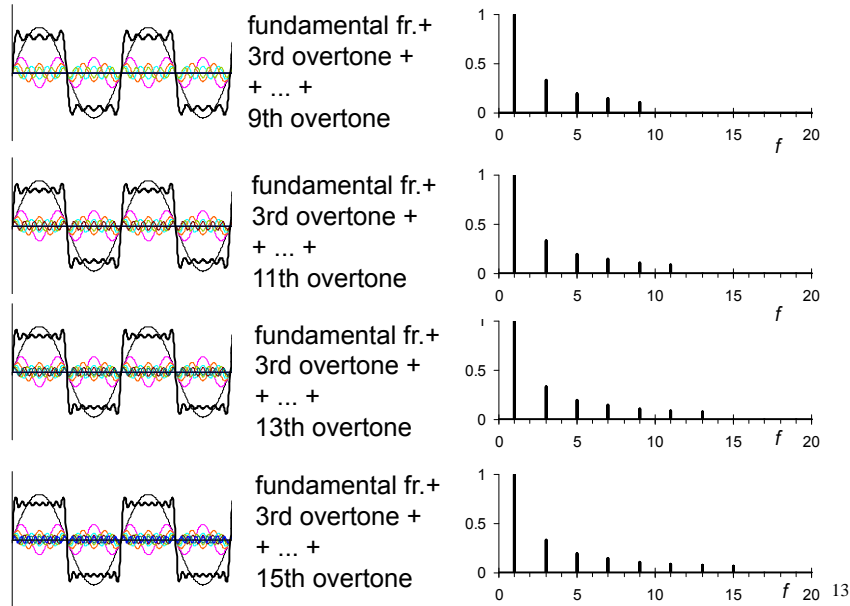
spectrum



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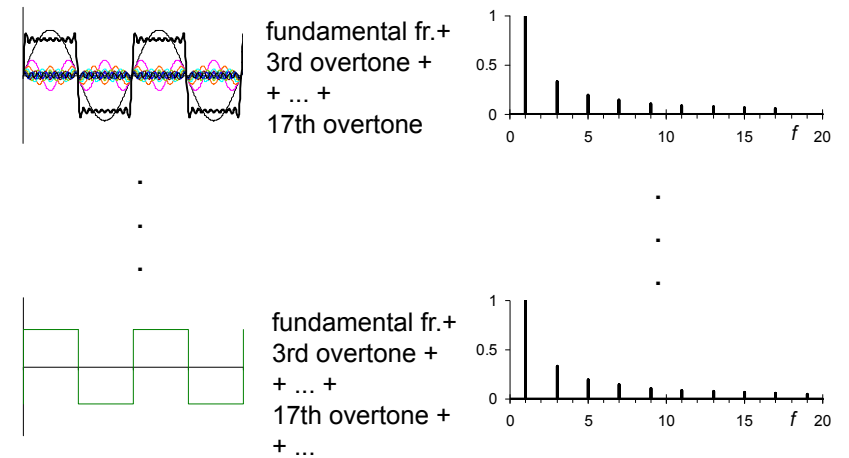
function

spectrum



function

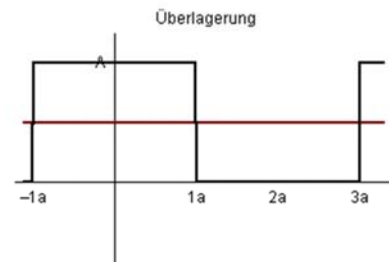
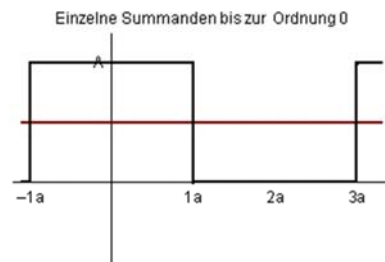
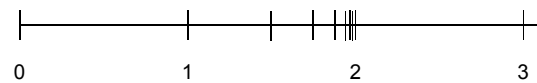
spectrum



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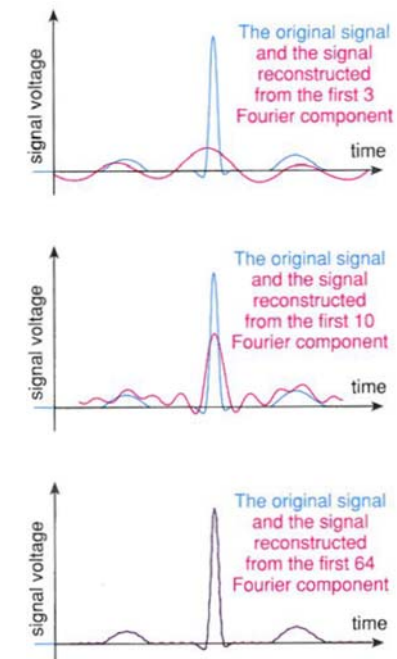
cf. infinite **series**

$$\sum_{k=0}^{\infty} \frac{1}{2^k} = \frac{1}{2^0} + \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \dots = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots = 2$$



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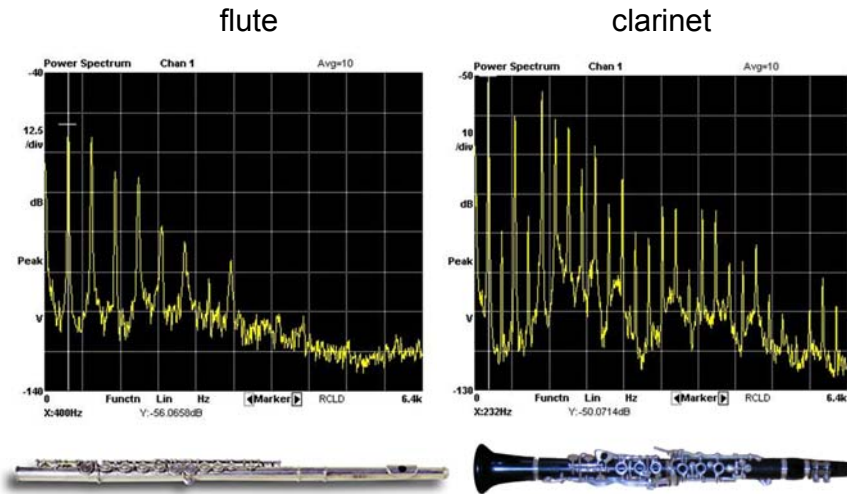
Creating an
ECG signal
from sine
functions



Textbook, Figure VII.3.

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Measured spectra



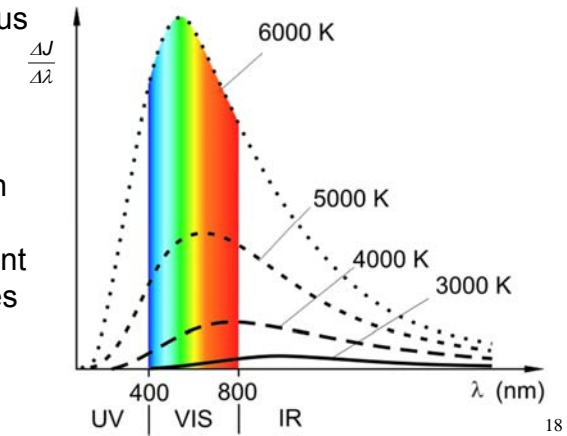
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Fourier's theorem for non-periodic functions (signals)

all (usual) functions can be expressed as a sum of sine (and cosine) functions

spectrum: continuous

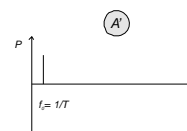
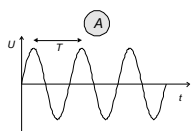
cf. emission spectra of incandescent light sources



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function

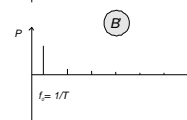
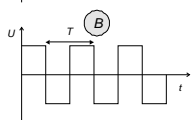
sine function



spectrum

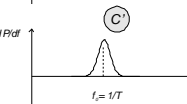
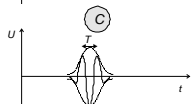
line spectrum (1 line)

periodic function



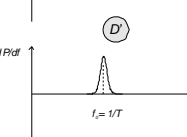
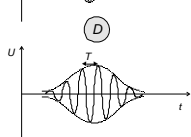
line spectrum

a few periods



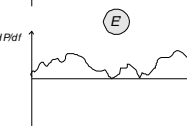
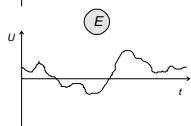
band spectrum

more periods



band spectrum

non-periodic function



continuous spectrum

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Inisheer

Music in time-frequency representation

Traditional

Penny Whistle



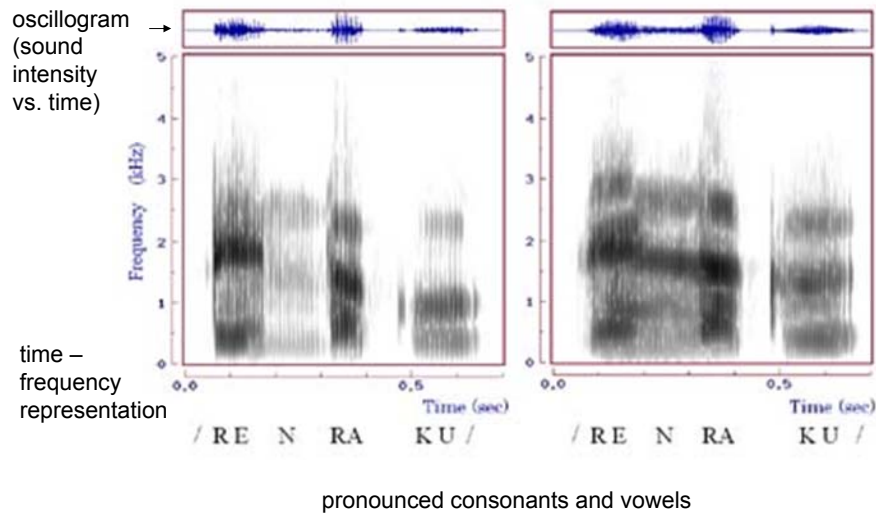
f_{sine}



t

20

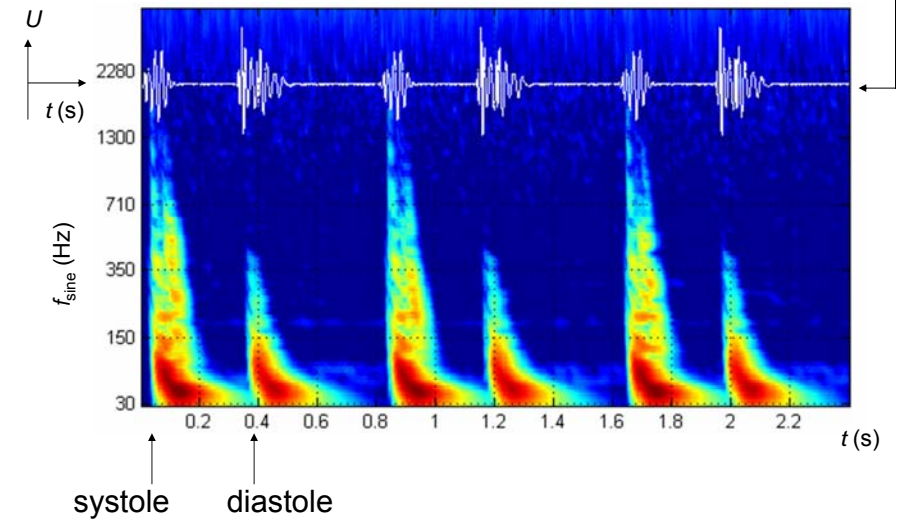
Voiceprint



<http://www.nrips.go.jp/org/fourth/info3/index-e.html>

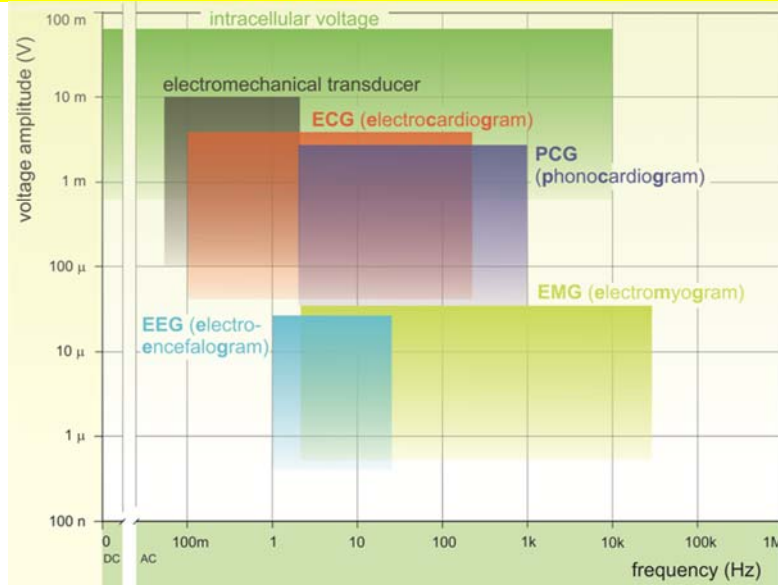
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Heart beats in time-frequency representation (+ oscillogram)



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Frequency and amplitude ranges of biological signals



Practical manual, titel page of meas. 17

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Frequency dependent unit: Electronic amplifier

- (1) $P_{in} < P_{out}$
- (2) P_{in} and P_{out} : same functions

same: „fundamentalist“ requirement
similar: realistic requirement

$$(1) + (2) \quad A_p \cdot P_{in}(t) \equiv P_{out}(t), \text{ where } A_p > 1$$

$$A_p = \frac{P_{out}}{P_{in}},$$

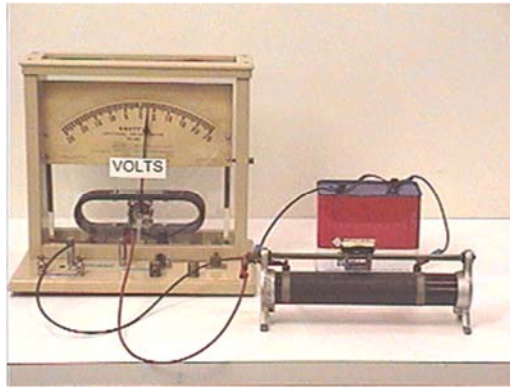
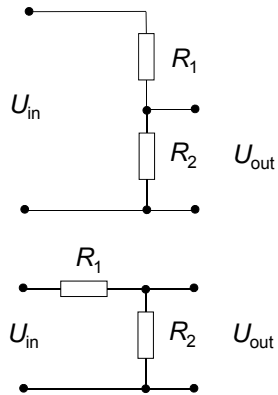
power gain (amplification)

$$A_U = \frac{U_{out}}{U_{in}},$$

voltage gain (amplification)

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(frequency independent) voltage-divider

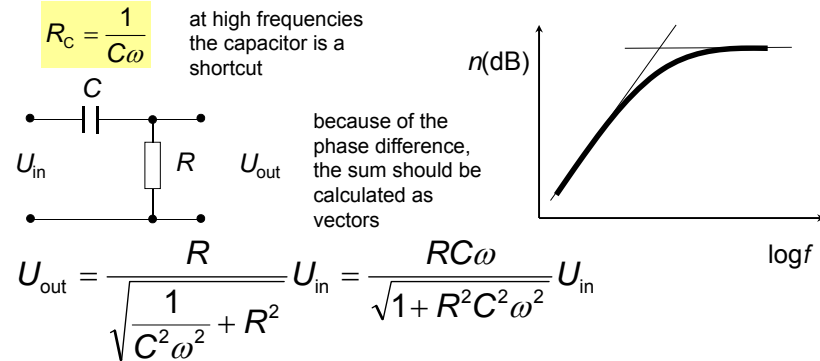


$$U_{\text{out}} = \frac{R_2}{R_1 + R_2} U_{\text{in}}$$

frequency dependent voltage-divider: with capacitor

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High-pass/low-cut filter



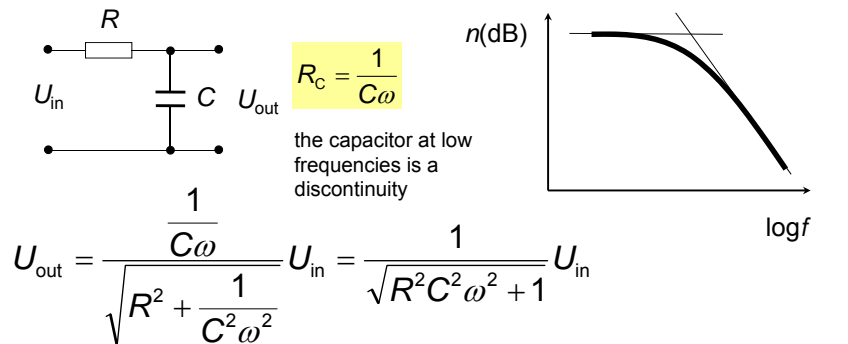
at very low frequencies: if $\omega \ll \omega_0$ ($\omega \approx 0$), $U_{\text{out}} = 0$

at low frequencies: if $\omega \ll \omega_0$, $U_{\text{out}} = RC\omega U_{\text{in}} \leftrightarrow 6 \text{ dB/octave}$

at high frequencies: if $\omega \approx \infty$, $U_{\text{out}} = U_{\text{in}}$

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Low-pass/high-cut filter



at low frequencies: if $\omega \ll \omega_0$ ($\omega \approx 0$), $U_{\text{out}} = U_{\text{in}}$

at high frequencies: if $\omega \gg \omega_0$, $U_{\text{out}} = \frac{1}{RC\omega} U_{\text{in}} \leftrightarrow -6 \text{ dB/octave}$

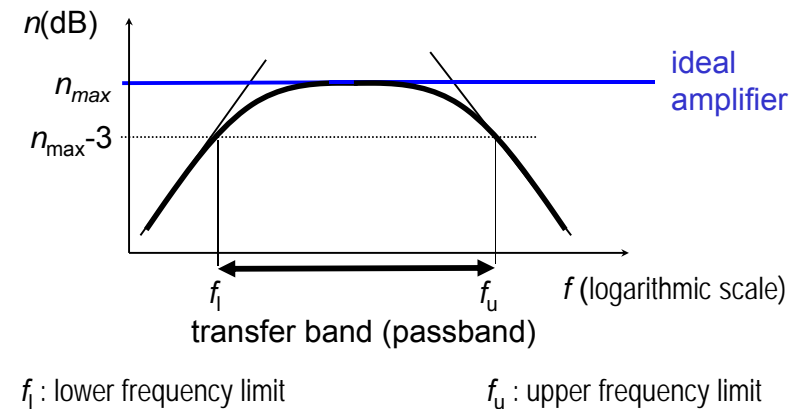
at very high frequencies: if $\omega \gg \omega_0$ ($\omega \approx \infty$), $U_{\text{out}} = 0$

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for (1): $A_P > 1$,

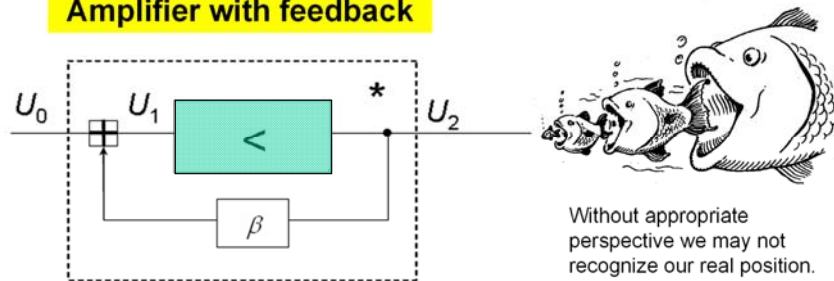
$$n = 10 \lg A_P = 20 \lg A_U > 0 \text{ dB}$$

for (2): **frequency characteristics**



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Amplifier with feedback



$$(a) U_1 = U_0 + \beta U_2 \quad (b) A_U = \frac{U_2}{U_1}$$

$$(c) A_U^* = \frac{U_2}{U_0} = \frac{U_1 A_U}{U_0} = \frac{(U_0 + \beta U_2) A_U}{U_0} = A_U + \beta \frac{U_2}{U_0} A_U = A_U + \beta A_U^* A_U$$

$$A_U^* - \beta A_U^* A_U = A_U \quad A_U^* = \frac{A_U}{1 - \beta A_U}$$

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$$A_U^* = \frac{A_U}{1 - \beta A_U}$$

A_U^* : voltage gain with feedback

A_U : voltage gain without feedback

$\beta > 0$, **positiv feedback** (same phase), $A_U^* > A_U$ (advantage)

$\beta < 0$, **negativ feedback** (in opposite phase), $A_U^* < A_U$ (disadv.)

positiv feedback:

(a) $\beta A_U = 1$, amplification: „infinite“

– sine wave oscillator

e.g.: ultrasound generator,
heat therapy

(b) $\beta A_U \leq 1$, amplification: very big

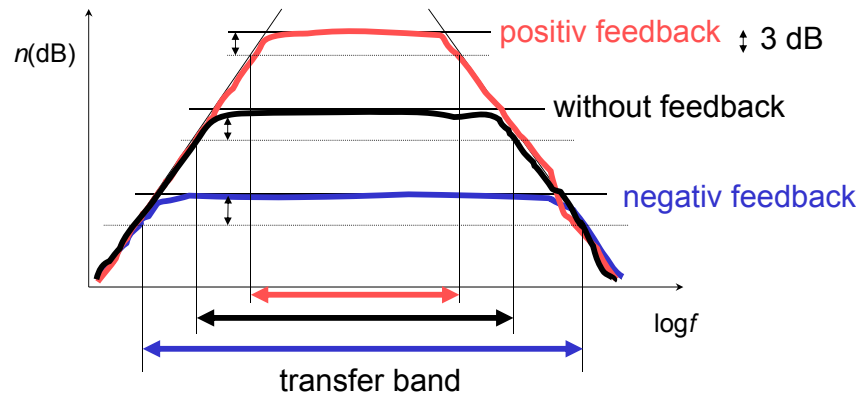
– regenerative amplifier

e.g.: hearing, outer haircells



negativ feedback: „all“ amplifier

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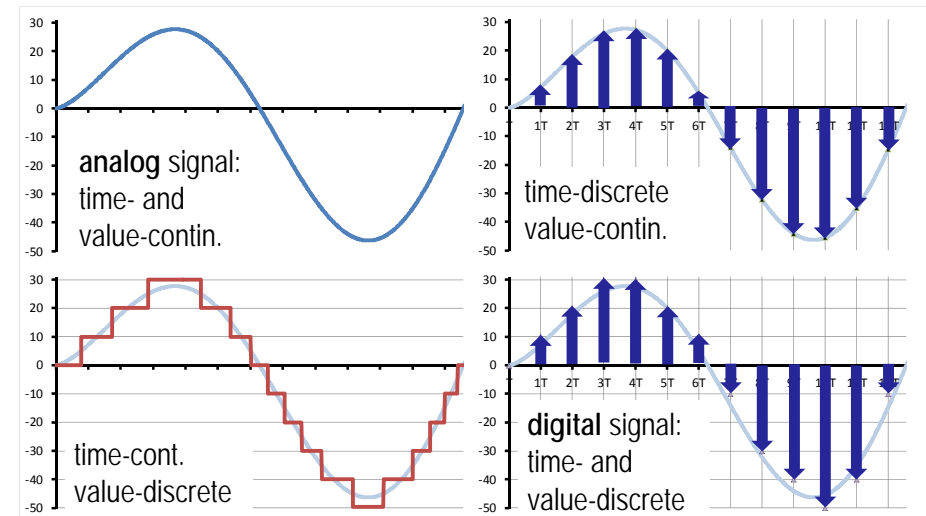


positiv feedback: transfer band – narrower (big disadvantage)
higher gain (advantage)

negativ feedback: transfer band – broader (advantage)
less gain (small disadvantage)

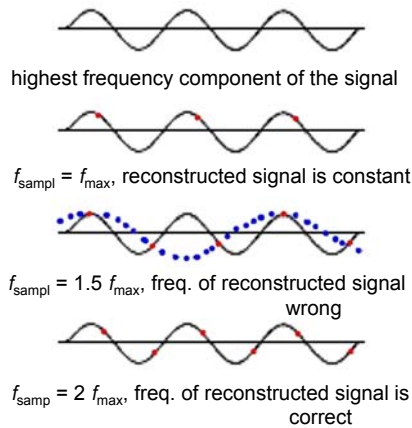
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Analog signal – digital signal



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time-discrete: the value of the signal is not known for all moments in time



Nyquist-Shannon sampling theorem:

for complete reconstruction the minimum sampling frequency should be twice the frequency of the highest overtone of the signal

e.g.: hifi, $f_{\text{max}} = 20 \text{ kHz}$

$f_{\text{samp}} = 44.1 \text{ kHz} > 2 \cdot 20 \text{ kHz}$

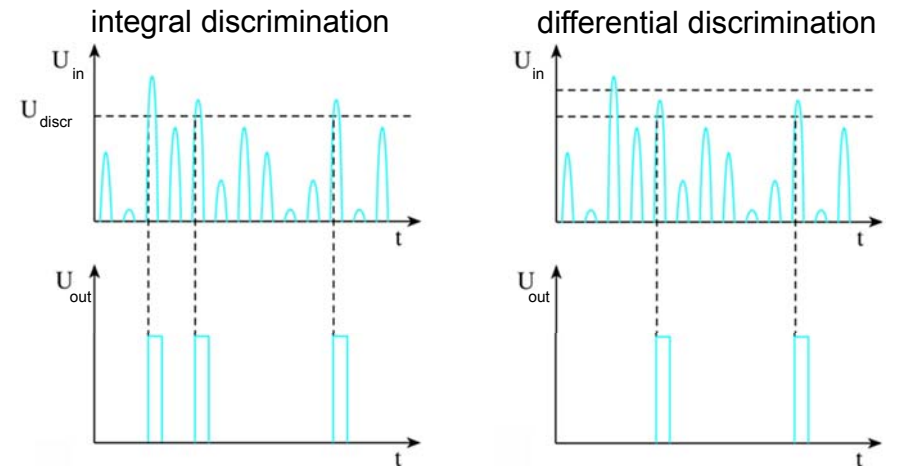
value discrete: the value of the signal can not be arbitrary

e.g.: hifi, 16 bit = $2^{16} = 65\,536$ (CD standard)

24 bit = $2^{24} = 16\,777\,216$ ("best" audio card)

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Pulse processing



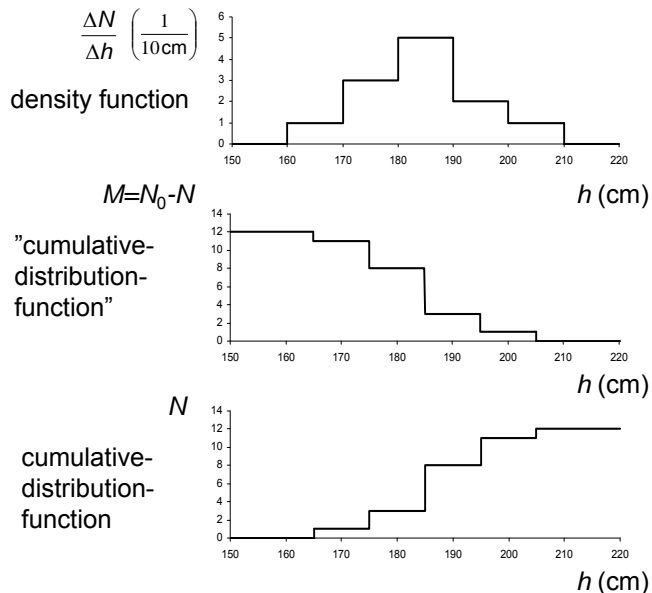
to select only those pulses that are larger than a preset amplitude

to select only those pulses whose amplitudes lie within a preset window

Textbook, Figure VII.32.

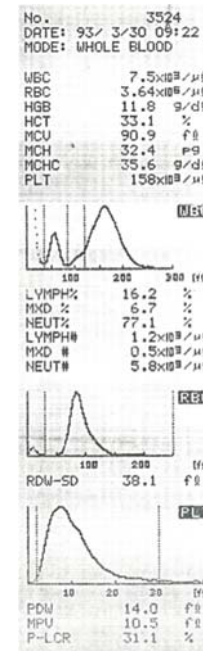
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Distribution functions and ID/DD "spectra"



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Concentration of white blood cells



Coulter counter

