

Medical Biophysics II.

4th lecture: Diffusion, Brownian motion, Osmosis

23 February 2017.

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Diffusion?

Why?

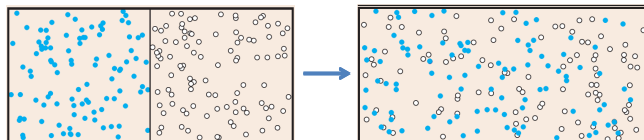
- physiology: cell function – ion diffusion...
- disorders: fibrosis, oedema, vasculitis, ascites...
- diagnostics: DWI MRI...
- therapy: dialysis, physiological saline....
- drug delivery: transdermal (liposomal), inhaled...

.....

Diffusion?

The change in the spatial distribution of particles because of **random thermal motion**.

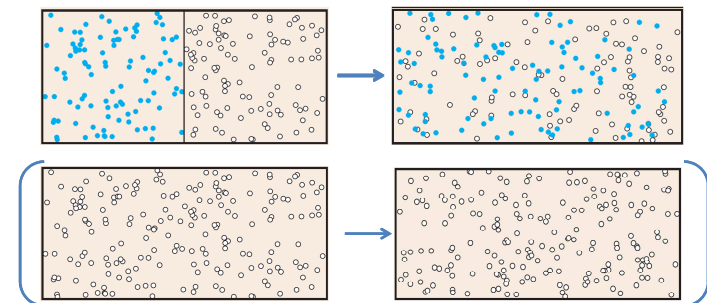
In microscopic level with **net matter transport**.



Diffusion?

The change in the spatial distribution of particles because of **random thermal motion**.

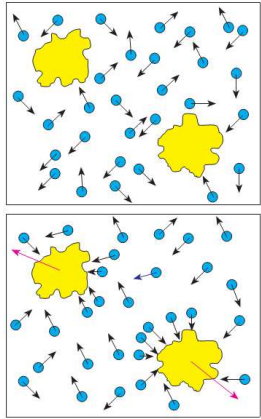
In microscopic level with **net matter transport**.



Relevant: NET transport of substance „A” in „B”.

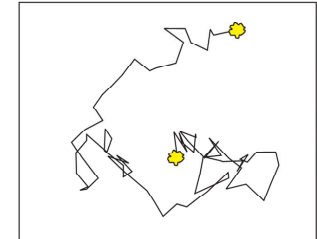
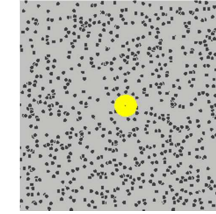
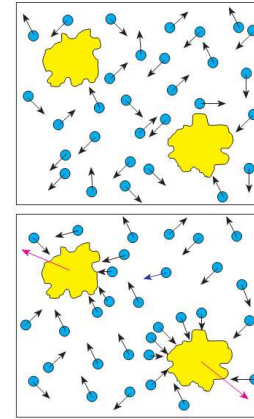
Brownian motion

The „random walk” of a particles resulting from their collision with other particles.



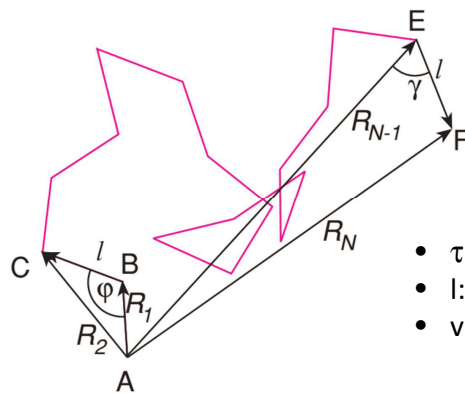
Brownian motion

The „random walk” of a particles resulting from their collision with other particles.



- τ : mean time between collisions
- l : mean free path
- v : mean speed of particles

How far reaches a particle?

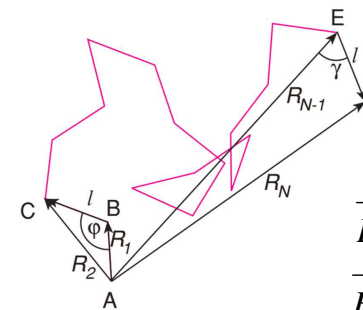


- τ : mean time between collisions
- l : mean free path
- v : mean speed of particles

One particle: $R_2^2 = R_1^2 + l^2 - 2 \cdot R_1 \cdot l \cdot \cos \varphi$

A „mean” particle:
(mean of n particles): $\overline{R_2^2} = \frac{1}{n} \cdot \sum_{i=1}^n (R_1^2 + l^2 - 2 \cdot R_1 \cdot l \cdot \cos \varphi_i)$

How far reaches a particle?



$$\overline{R_2^2} = \frac{1}{n} \cdot \sum_{i=1}^n (R_1^2 + l^2 - 2 \cdot R_1 \cdot l \cdot \cos \varphi_i)$$

$$\overline{R_2^2} = \frac{1}{n} \cdot n \cdot (R_1^2 + l^2) - 2 \cdot R_1 \cdot l \cdot \sum_{i=1}^n (\cos \varphi_i)$$

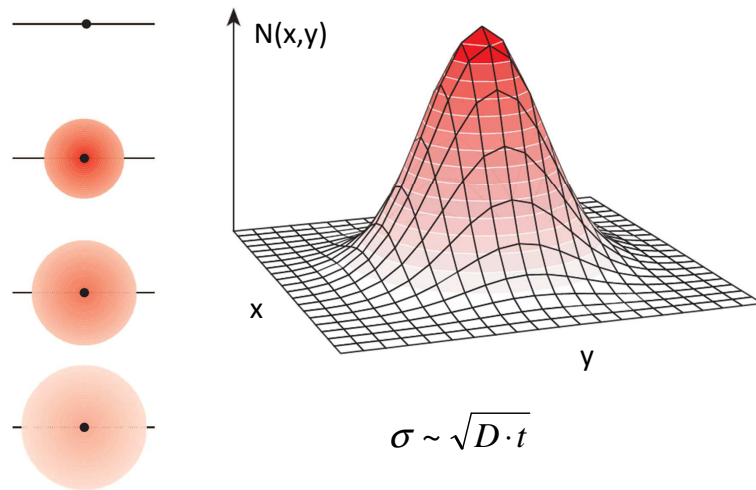
$$\overline{R_2^2} = R_1^2 + l^2 = l^2 + l^2 = 2 \cdot l^2$$

$$\overline{R_N^2} = N \cdot l^2$$

$$\overline{R_t} = \sqrt{N \cdot l^2} = \sqrt{\frac{t}{\tau} \cdot l \cdot l} = \sqrt{t \cdot v \cdot l} = \sqrt{3 \cdot D \cdot t}$$

$$\frac{v \cdot l}{3} = D$$

Experiment – 2D distribution



Matter transport - flow

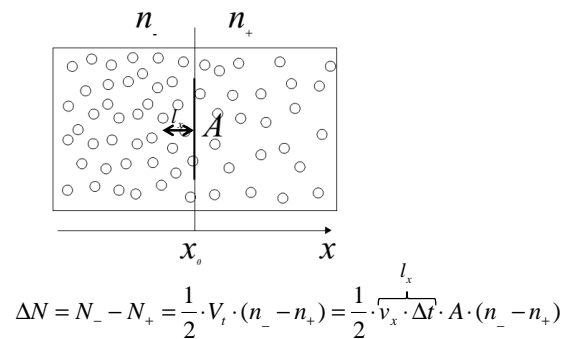
Particle flow rate: $I_N = \frac{\Delta N}{\Delta t}; \left[\frac{1}{s} \right]$

Particle flow density (flux): $J_N = \frac{\Delta I_N}{\Delta A}; \left[\frac{1}{m^2 \cdot s} \right]$

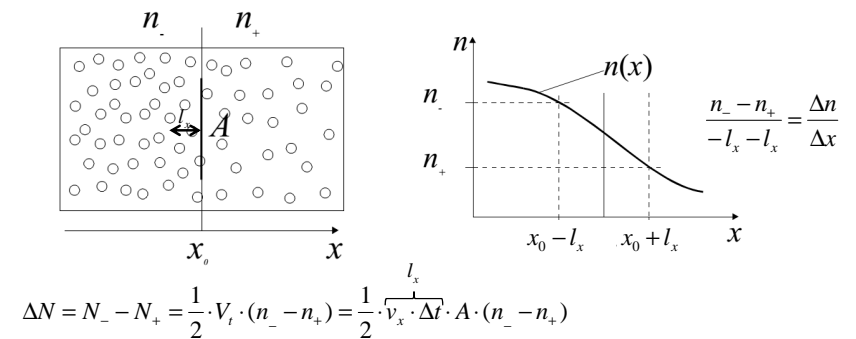
Matter flow rate: $I_v = \frac{\Delta v}{\Delta t}; \left[\frac{mol}{s} \right]$

Matter flow density (flux): $J_v = \frac{\Delta I_v}{\Delta A}; \left[\frac{mol}{m^2 \cdot s} \right]$

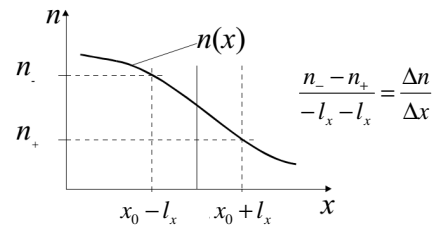
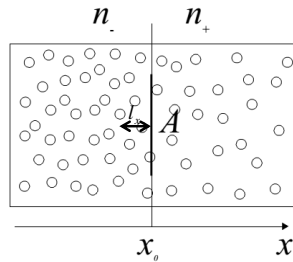
Fick's first law



Fick's first law



Fick's first law



$$\Delta N = N_- - N_+ = \frac{1}{2} \cdot V_l \cdot (n_- - n_+) = \frac{1}{2} \cdot \sqrt{v_x \cdot \Delta t} \cdot A \cdot (n_- - n_+)$$

$$\Delta N = \frac{1}{2} \cdot v_x \cdot \Delta t \cdot A \cdot 2 \cdot l_x \cdot \frac{\Delta n}{\Delta x}$$

$$v_x \cdot l = D$$

$$J_{Nx} = \frac{1}{2} \cdot v_x \cdot 2 \cdot l_x \cdot \frac{\Delta n}{\Delta x} = -D \cdot \frac{\Delta n}{\Delta x}$$

$$J_v = -D \cdot \frac{\Delta c}{\Delta x}$$

BUT!: Δc is Not the „driving force“!

Diffusion coefficient

D gives the amount of matter diffused across a unit area in a unit time in a case of unit concentration gradient.

$$D = \frac{v \cdot l}{3} \left[\frac{m^2}{s} \right]$$

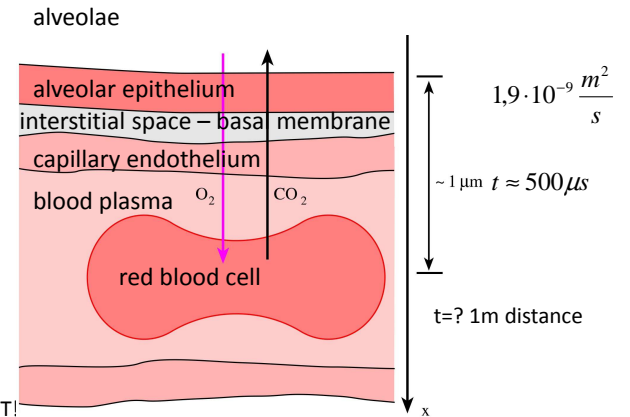
$$D = u \cdot k \cdot T$$

Einstein-Stokes
(spheres)

$$D = \frac{k \cdot T}{6 \cdot \pi \cdot \eta \cdot r}$$

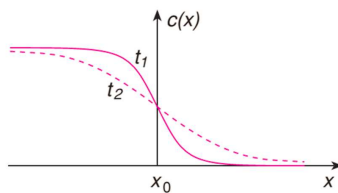
BUT!

Not directly proportional with T !

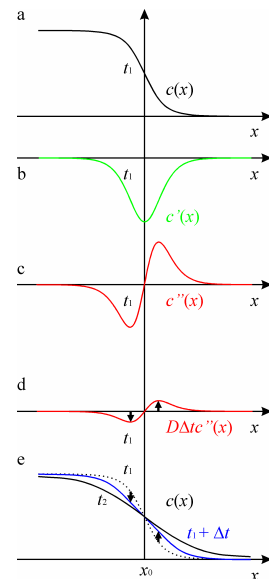


Fick's second law

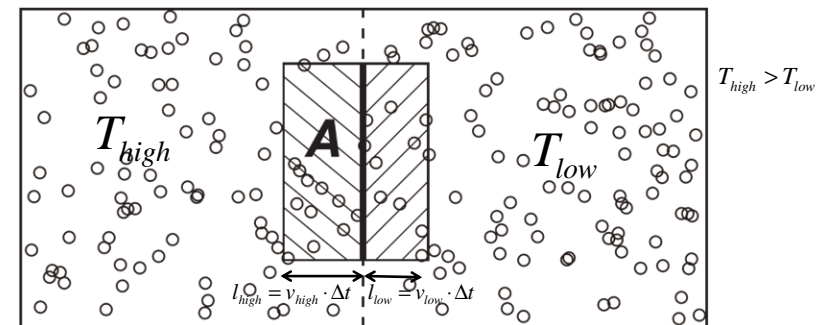
Fick II: change of the concentration gradient in time



$$c(t + \Delta t) = c(t) + D \cdot \Delta t \cdot \frac{\Delta \left(\frac{\Delta c}{\Delta x} \right)}{\Delta x}$$

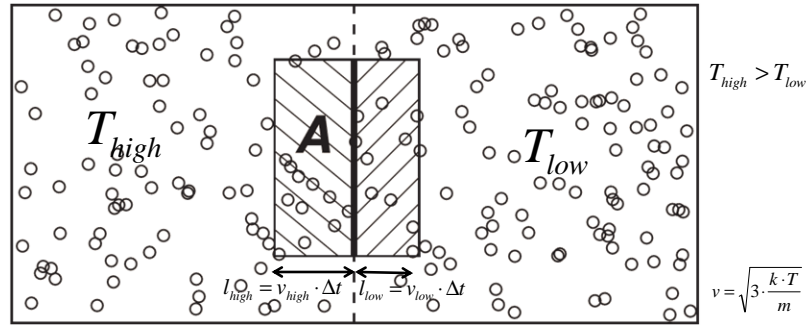


Thermodiffusion



$$\Delta N = N_{high} - N_{low} = \frac{1}{2} \cdot n \cdot \Delta t \cdot A \cdot (v_{high} - v_{low}) = \frac{1}{2} \cdot n \cdot \Delta t \cdot A \cdot (v_{high} - v_{low}) \cdot \frac{(v_{high} + v_{low})}{(v_{high} + v_{low})}$$

Thermodiffusion

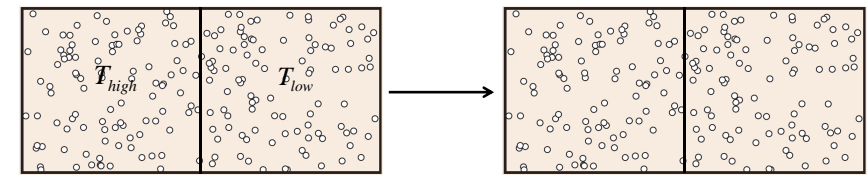


$$\frac{1}{2} \cdot n \cdot \Delta t \cdot A \cdot \frac{(v_{high}^2 - v_{low}^2)}{(v_{high} + v_{low})} = \frac{1}{2} \cdot n \cdot \Delta t \cdot A \cdot \frac{(v_{high}^2 - v_{low}^2)}{(v_{high} + v_{low})} = \frac{1}{2} \cdot n \cdot \Delta t \cdot A \cdot \frac{(3 \cdot k \cdot T_{high} - 3 \cdot k \cdot T_{low})}{2 \cdot v_{mean}}$$

$$\frac{1}{2} \cdot \left(\frac{n \cdot \Delta t \cdot A \cdot 3 \cdot k}{m \cdot 2 \cdot v_{mean}} \right) \cdot (T_{high} - T_{low})$$

$$J_v = -L_T \cdot \frac{\Delta T}{\Delta x} \quad (\text{Ludwig-Soret effect})$$

Heat conduction



$$T_{high} > T_{low} \quad \Delta N = N_{high} - N_{low} = 0$$

$$N_{high} = N_{low} \quad \bar{\epsilon} = \frac{3}{2} \cdot k \cdot T$$

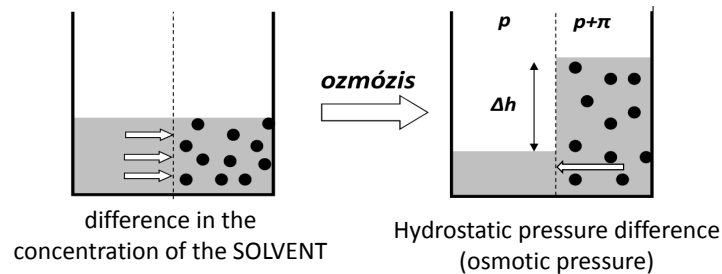
Energy flow density

$$J_v = \frac{\Delta E}{A \cdot \Delta t} = \frac{N_{high} \cdot \frac{3}{2} \cdot k \cdot (T_{high} - T_{low})}{A \cdot \Delta t} = -\lambda \cdot \frac{\Delta T}{\Delta x}$$

(Fourier law for heat conduction)

Osmosis

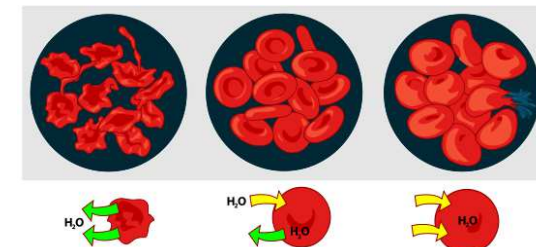
One-way diffusion of the SOLVENT.



$$p_{osm} = \pi = c_{solute} \cdot R \cdot T$$

Osmotic concentration (equivalent osmotic pressure, „ozmolarity”, „ozmolality”):
The concentration of a solution that keeps balance with a heterogeneous solution.
Derived units: *mOsm/(L)*, mmol/L (mmol/kg),

Medical practice



Osmotic concentration of the blood: about 300mOsm/L

Physiological („isotonic”) solutions:

Physiological/Normal/Isotonic saline: 0,9% (w/v) NaCl – 58,44g/mol...

d5W: 5% (w/v) glucose – 278mOsm/L

Ringer, Ringer's lactate