Mathematical and Physical Basis of Medical Biophysics

Lecture 2
Kinematics – Physics of Motion

11th September 2017
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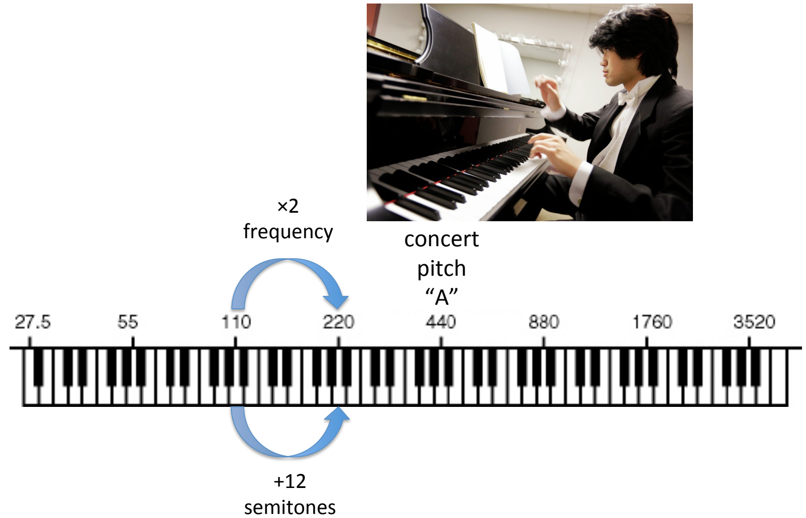
Logarithmic Function: Example



Logarithmic Function

INTEGRAL FORM

$$y = b \cdot \log_a(x)$$

PRACTICAL CONSIDERATIONS:
- base is 10 (sometimes *e* or 2)
- if the base is fixed this will modify the factor paramater according to the following identity:

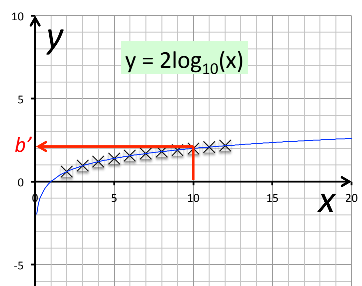
$$b \cdot \log_a(x) = b/\log_{10}(a) \cdot \log_{10}(x) = b' \cdot \log_{10}(x)$$

VARIABLES: dependent variable, independent variable

$$y = b' \cdot \log_{10}(x)$$

PARAMETERS: factor parameter

if $x = 10$
then $y = b'$



„DIFFERENTIAL" FORM

$$\Delta y \sim \Delta x / x$$

The **absolute change** of the dependent variable is proportional to the **relative change** of the independent variable

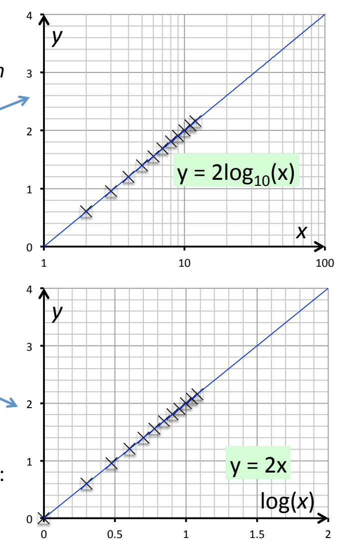
Logarithmic Function: Linearization

graphical linearization
plot *y* on lin and x on log scales:
*the relationship **looks** linear but it **is** still a log function*

INTEGRAL FORM

$$y = b' \cdot \log_{10}(x)$$

arithmetical linearization
plot *y* as a function of log(*x*):
*the relationship **is** linear*



Logarithmic Function: Some Examples

from the Biophysics Formula Collection ...and elsewhere

#1: The statistical definition of entropy (III.72)

$S = k \ln \Omega$

$S = k \cdot \log_e(\Omega)$

$y = b \cdot \log_a(x)$

#2: The decibel (dB) scale (VII.10)

$n = 10 \log A_p$

$n = 10 \cdot \log_{10}(A_p)$

$y = b \cdot \log_a(x)$

#3: The definition of absorbance (VI.34)

$A = \lg(J_0/J)$

$A = 1 \cdot \log_{10}\underbrace{(J_0/J)}$

$y = b \cdot \log_a(x)$

#4: The pH scale

$\mathrm{pH} = -\log[\mathrm{H}^+]$

$\mathrm{pH} = -1 \cdot \log_{10}\underbrace{([\mathrm{H}^+]/(1\ \mathrm{M}))}$

$y = b \cdot \log_a(x)$

Derivative and Integral: Example #1

Δ Δ

x	y = x^2	y' = Δy/Δx	y'' = Δ(Δy/Δx)/Δx
0	0		
1	1	1	
2	4	3	2
3	9	5	2
4	16	7	2
5	25	9	2
6	36	11	2
7	49	13	2
8	64	15	2
9	81	17	2
10	100	19	2

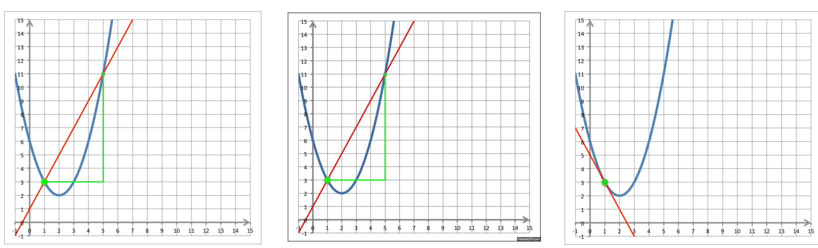
Σ Σ

Derivative: slope of tangent line

difference quotient:
$\Delta y/\Delta x$
slope of **secant** line

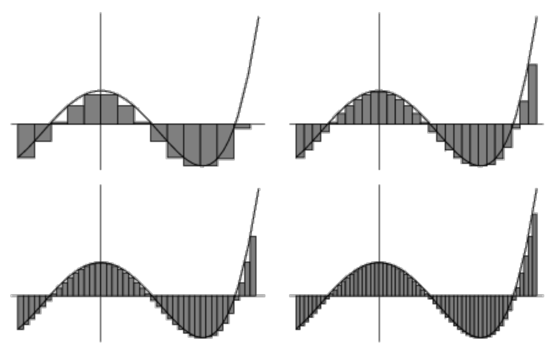
$\Delta \longrightarrow \mathrm{d}$

derivative:
dy/dx
slope of **tangent** line



Integral: Area Under the Curve (AUC)

$\Sigma \longrightarrow \int$



Rectilinear Motions

Quantities, Units, and Equations

displacement: $\Delta s = s_2 - s_1$ $[\Delta s]$ = m
velocity: $v = \Delta s/\Delta t$ $[v]$ = m/s
acceleration: $a = \Delta v/\Delta t$ $[a]$ = m/s²

Uniform Rectilinear Motion

$s_t = s_0 + v \cdot t$
v = constant
$a = 0$

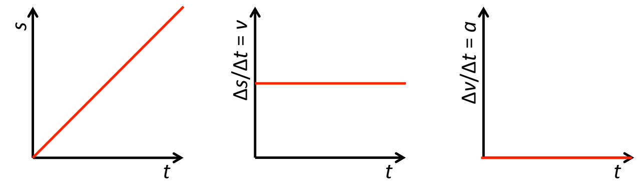
Rectilinear Motion with Uniform Acceleration

$s_t = s_0 + v_0 \cdot t + a/2 \cdot t^2$
$v_t = v_0 + a \cdot t$
a = constant

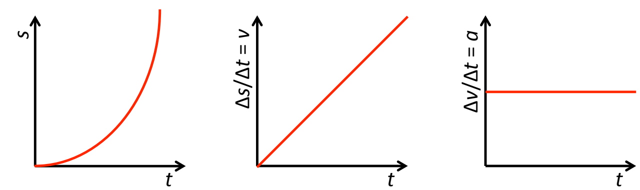
Derivative and Integral: Application

Rectilinear Motion

Uniform Rectilinear Motion:



Rectilinear Motion with Uniform Acceleration:

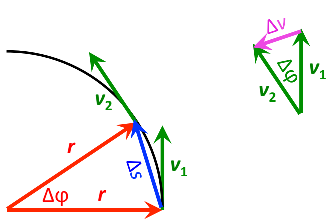


Circular Motion

Quantities, Units, and Equations

angular displacement: $\Delta\varphi = \varphi_2 - \varphi_1$ $[\Delta\varphi]$ = rad
angular velocity, angular frequency: $\omega = \Delta\varphi/\Delta t$ $[\omega]$ = rad/s
tangential speed: $v = r \cdot \Delta\varphi/\Delta t = r \cdot \omega$ $[v]$ = m/s

centripetal acceleration: $a_{cp} = v^2/r = r \cdot \omega^2$ $[a]$ = m/s²



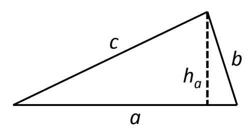
(1) approximation for small angles:
displacement = arc length = $v \cdot \Delta t \approx \Delta s$

(2) due to properties of similar triangles:
$\Delta v/v = \Delta s/r$

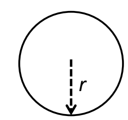
(1) + (2):
$\Delta v/v = v \cdot \Delta t/r$

$a_{cp} = v^2/r$

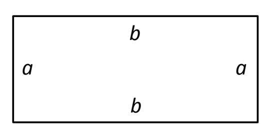
Perimeter & Area



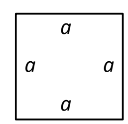
TRIANGLE
perimeter: $a+b+c$
area: $a*h_a/2$



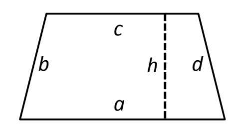
CIRCLE
perimeter: $2r\pi$
area: $r^2\pi$



RECTANGLE
perimeter: $2*(a+b)$
area: $a*b$

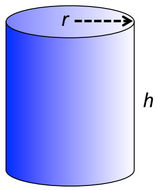


SQUARE
perimeter: $4a$
area: $a*a=a^2$

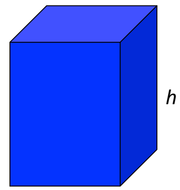


TRAPEZOID
perimeter: $a+b+c+d$
area: $(a+c)/2*h$

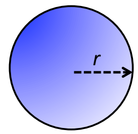
Surface & Volume



CYLINDER (open)
surface (wall only): $2r\pi*h$
volume: $r^2\pi*h$



PRISM (open)
surface (wall only):
(perimeter of base)*h
volume: (area of base)*h



SPHERE
surface: $4r^2\pi$
volume: $4r^3\pi/3$

Units – SI Base & Derived Units

physical quantity	symbol	unit	symbol
length	*l, x, s, d*	meter	m
mass	*m*	kilogram	kg
time	*t*	second	s
temperature	*T*	kelvin	K
electric current	*I*	ampere	A
amount of substance	*n, N*, ν [nu]	mole	mol
luminous intensity	I_v	candela	cd

The SI base units

physical quantity	symbol	unit	symbol	derivation
speed	*v, c*	–	–	$m\cdot s^{-1}$
acceleration	*a*	–	–	$m\cdot s^{-2}$
force	*F*	newton	N	$kg\cdot m\cdot s^{-2}$
energy	*E*	joule	J	$kg\cdot m^2\cdot s^{-2}$
power	*P*	watt	W	$kg\cdot m^2\cdot s^{-3}$
intensity	*I*	–	–	$kg\cdot s^{-3}$
pressure	*p*	pascal	Pa	$kg\cdot m^{-1}\cdot s^{-2}$

Some SI derived units

Units – SI Prefixes

prefix	symbol	meaning	etymology
exa	E	$\times10^{18} = \times1000^6$	Greek 6 (ἕξ = hex)
peta	P	$\times10^{15} = \times1000^5$	Greek 5 (πέντε = pente)
tera	T	$\times10^{12} = \times1000^4$	Greek 4 (τέτταρες = tettares), originally: monster (τέρας = teras)
giga	G	$\times10^9 = \times1000^3$	Greek giant (γίγας = gigas)
mega	M	$\times10^6 = \times1000^2$	Greek great (μέγας = megas)
kilo	k	$\times10^3 = \times1000^1$	Greek 1000 (χίλιοι = khilioi)
hekto	h	$\times10^2$	Greek 100 (ἑκατόν = hekaton)
deca	da (dk)	$\times10^1$	Greek 10 (δέκα = deka)
deci	d	$\times10^{-1}$	Latin 10 (decem)
centi	c	$\times10^{-2}$	Latin 100 (centum)
milli	m	$\times10^{-3} = \times1000^{-1}$	Latin 1000 (mille, *pl.* milia)
micro	µ	$\times10^{-6} = \times1000^{-2}$	Greek small (μικρός = mikros)
nano	n	$\times10^{-9} = \times1000^{-3}$	Greek dwarf (νᾶνος = nanos)
pico	p	$\times10^{-12} = \times1000^{-4}$	Spanish small, bit (pico)
femto	f	$\times10^{-15} = \times1000^{-5}$	Danish 15 (femten)
atto	a	$\times10^{-18} = \times1000^{-6}$	Danish 18 (atten)

Units – Conversion

from "with prefix" to "no prefix":
15 km = 15 · 10^3 m
15 cg = 15 · 10^{-2} g

from "no prefix" to "with prefix":
15 m = 15 / 10^3 km
15 g = 15 / 10^{-2} cg

from "with prefix" to "with prefix":
15 km = 15 · 10^3 m = 15 · 10^3 / 10^{-2} cm

when the unit has an exponent:
15 km^3 = 15 · $(10^3\ m)^3$ = 15 · $(10^3)^3$ m^3
15 m^3 = 15 / $(10^3)^3$ km^3

liters to and from cubic meters:
1 m^3 = 10 hL = 1000 L
1 dm^3 = 1 L
1 cm^3 = 1 mL
1 mm^3 = 1 µL

time to seconds:
2 days 3 h 12 min 30 s = ((2·24+3)·60+12)·60+30 s

degrees, minutes of arc, seconds of arc:
45° 40′ 30″ = $(45+40/60+30/60^2)°$

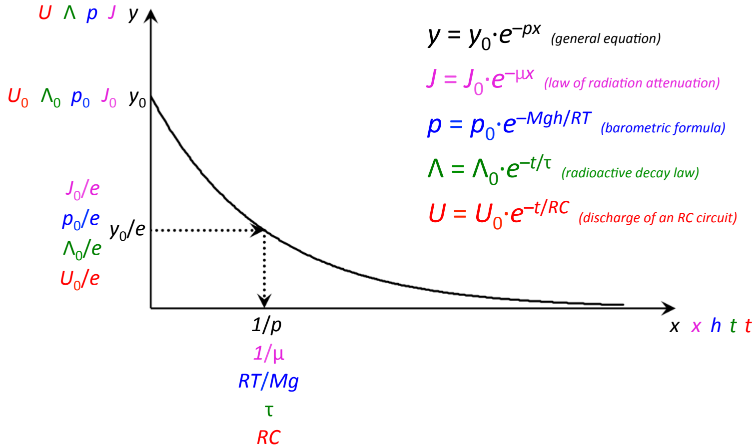
degrees to and from radians:
1 rad = (360/2π)°
1° = (2π/360) rad

compound units:
15 kg/m^3 = 15 · 10^3 / $(1/(10^{-2})^3)$ g/cm^3
45 km/h = 45 · 10^3 / 3600 m/s

degrees Celsius to and from kelvins:
T = 15 °C = (15+273) K
T = 15 K = (15–273) °C
ΔT = 15 °C = 15 K
ΔT = 15 K = 15 °C

Addendum: Graph of Exponential Functions

from the Biophysics Formula Collection



$y = y_0 \cdot e^{-px}$ *(general equation)*

$J = J_0 \cdot e^{-\mu x}$ *(law of radiation attenuation)*

$p = p_0 \cdot e^{-Mgh/RT}$ *(barometric formula)*

$\Lambda = \Lambda_0 \cdot e^{-t/\tau}$ *(radioactive decay law)*

$U = U_0 \cdot e^{-t/RC}$ *(discharge of an RC circuit)*