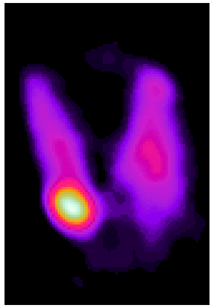
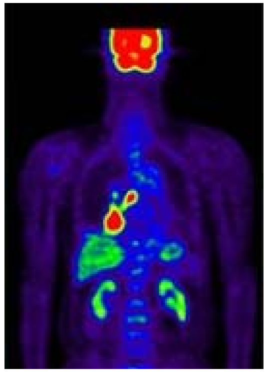
The atomic nucleus. Radioactivity. Nuclear radiations

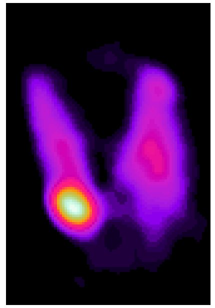
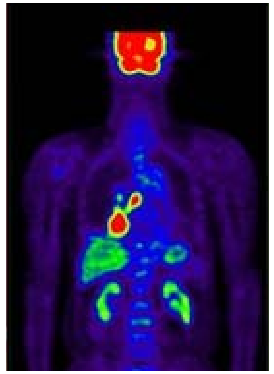
László Smeller

Why?

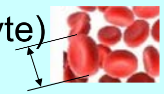
Medical applications of the nuclear radiation:

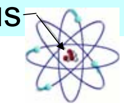
- Nuclear imaging
- Radiotherapy

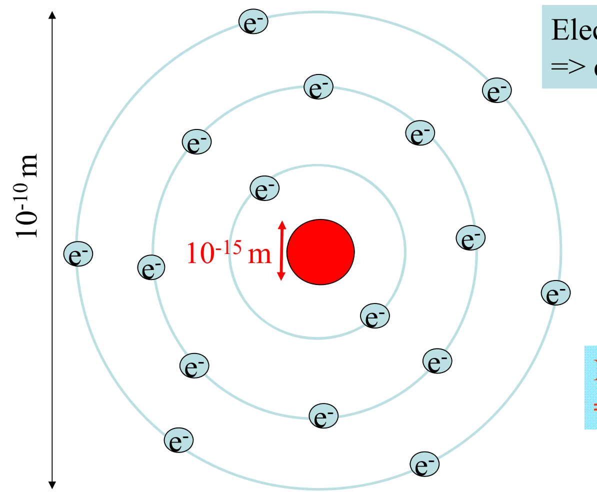
Length scale of the nature

m		
10^{0}	meter	men
10^{-3}	millimeter	size of letters you can read
10^{-6}	micrometer	size of a cell (e.g. erytrocyte) ∅ 7μm
10^{-9}	nanometer	protein
10^{-10}	– angstrom	diameter of an atom, bond length H atom ∅ ≈ 1 angstrom (Å)
10^{-12}	picometer	wavelength of the X-ray
10^{-15}	femtometer	size of the nucleus

The electrons and the nucleus

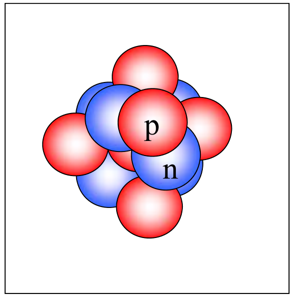


Electrons:
=> chemical properties

Nucleus:
=> radioactivity

Structure of the nucleus

Elementary charge=$1.6 \cdot 10^{-19}$ C



	charge	mass
proton	**+1 e**	**1 atomic mass unit**
neutron	**0**	**1 atomic mass unit**

A (mass number) = number of protons + number of neutrons

Z (atomic number) = number of protons

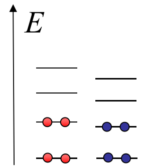
$^{99}_{43}\mathbf{Tc}$

99 nucleon, 43 proton and 56 neutron

Stability of the nucleus

- Coulomb force: destabilization (electrostatic repulsion between the protons)
- Nuclear force: very strong attractive force, acts only on short range (~fm), independent on the charge

- Quantized energy levels for the nucleus.



- Typical binding energy is in the MeV range
  eV=$1.6 \cdot 10^{-19}$ J

Isotopes

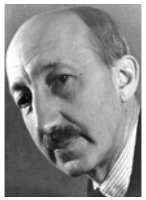
Variants of an element, where the

- number of protons are the same

⇒ the chemical properties are identical.

- number of neutrons are different

=> the atomic numbers are different

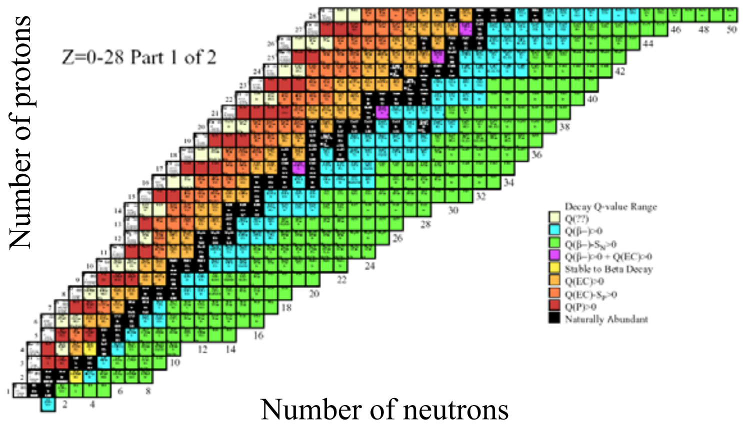


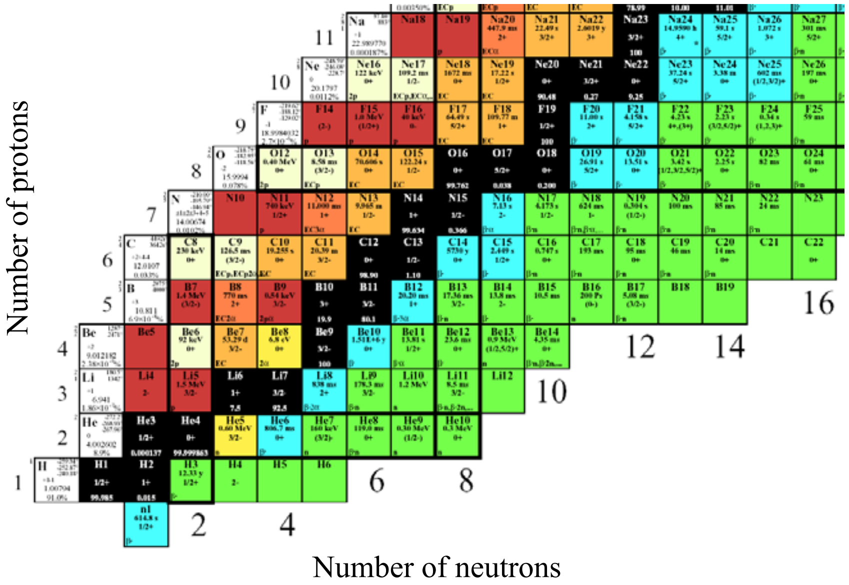
György Hevesi

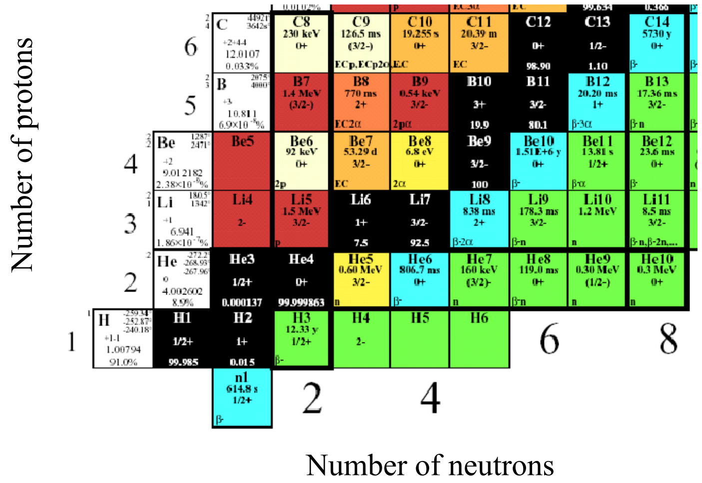
E.g.: $^{18}_{9}$F unstable (radioactive); $^{19}_{9}$F stable; $^{20}_{9}$F unstable (radioactive)

isotope <-> radioactive isotope

Table of isotopes







Radioactive decays and particles

α - decay	α - particle	= $^{4}_{2}He$ nucleus
β - decay : β^-	β^- particle	= electron
β^+	β^+ particle	= positron
Isomeric transition	γ-ray	
K-electron capture	characteristic x-ray photon	

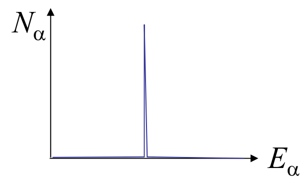
α - decay

α - decay: an α particle (^{4}He nucleus) will be emitted typical for the heavy atoms

$$^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}\alpha$$

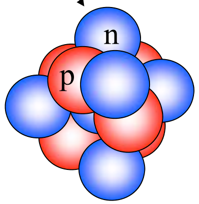
$$\text{e.g.} \quad ^{226}_{88}\text{Ra} \longrightarrow {}^{222}_{86}\text{Rn} + {}^{4}_{2}\alpha$$

Line spectrum
$E_\alpha \sim$ MeV



β^- - decay

neutron surplus



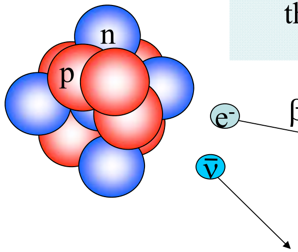
$${}^{A}_{Z}X \longrightarrow {}^{A}_{Z+1}Y + {}^{0}_{-1}\beta + \bar{\nu}$$

$${}^{1}_{0}n \rightarrow {}^{1}_{1}p + {}^{0}_{-1}\beta + \bar{\nu}$$

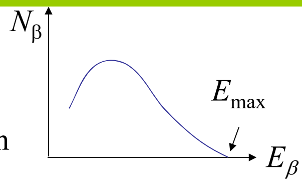
remains in the nucleus (${}^{1}_{1}p$) — leave the nucleus (${}^{0}_{-1}\beta$, $\bar{\nu}$)

e.g.: ${}^{20}_{9}F$, ${}^{32}_{15}P$, ${}^{59}_{26}Fe$, ${}^{131}_{53}I$

e.g. : ${}^{32}_{15}\mathrm{P} \longrightarrow {}^{32}_{16}\mathrm{S} + {}^{0}_{-1}\beta + \bar{\nu}$



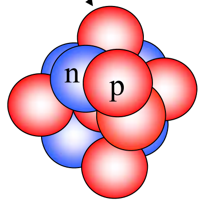
continous energy spectrum



$\beta^- = {}^{0}_{-1}\beta = e^-$

β^+ - decay

proton surplus



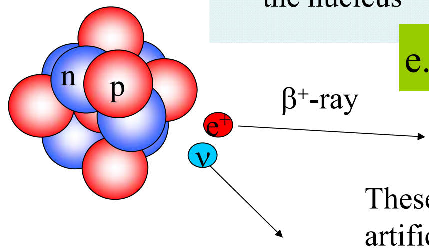
$${}^{A}_{Z}X \longrightarrow {}^{A}_{Z-1}Y + {}^{0}_{+1}\beta + \nu$$

$${}^{1}_{1}p \rightarrow {}^{1}_{0}n + {}^{0}_{+1}\beta + \nu$$

remains in the nucleus (${}^{1}_{0}n$) — leave the nucleus (${}^{0}_{+1}\beta$, ν)

e.g.: ${}^{11}_{6}C$, ${}^{15}_{8}O$, ${}^{18}_{9}F$, ${}^{52}_{26}Fe$

e.g. : ${}^{30}_{15}\mathrm{P} \longrightarrow {}^{30}_{14}\mathrm{Si} + {}^{0}_{+1}\beta + \nu$



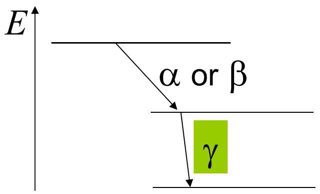
continous energy spectrum

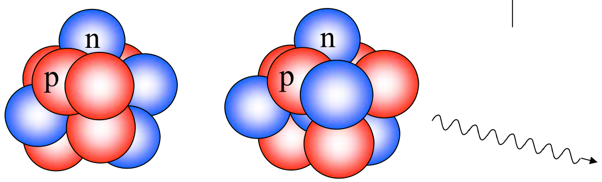
These isotopes must be produced artificially (e.g. in cyclotron)

Prompt γ-radiation

The daughter nucleus has an energetically unfavoured arrangement of nucleons.

The surplus energy will normally be emitted immediately (<ps) in form of the γ radiation





Atomic number, mass number are unchanged.

Isomeric transition

If the excited state of the daughter nucleus is metastable, the γ-radiation will be emitted later.

The parent and daughter atoms can be separated: the daughter atom emits **only γ-radiation!**

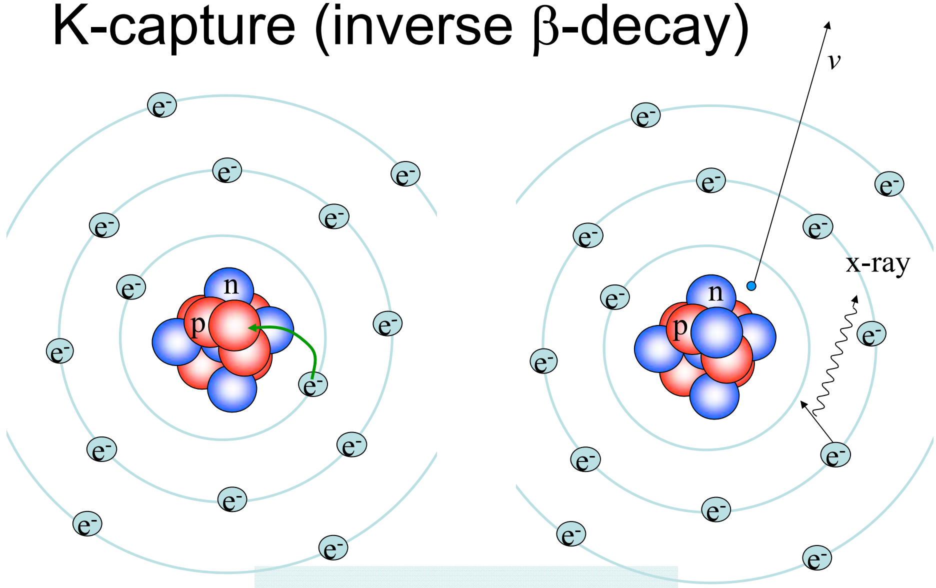
=> **Isotope diagnostics (nuclear imaging)**

E.g.: ${}^{99m}\mathrm{Tc}$

$${}^{99}_{42}Mo \xrightarrow{\beta^-} {}^{99m}_{43}Tc \xrightarrow{\gamma} {}^{99}_{43}Tc$$

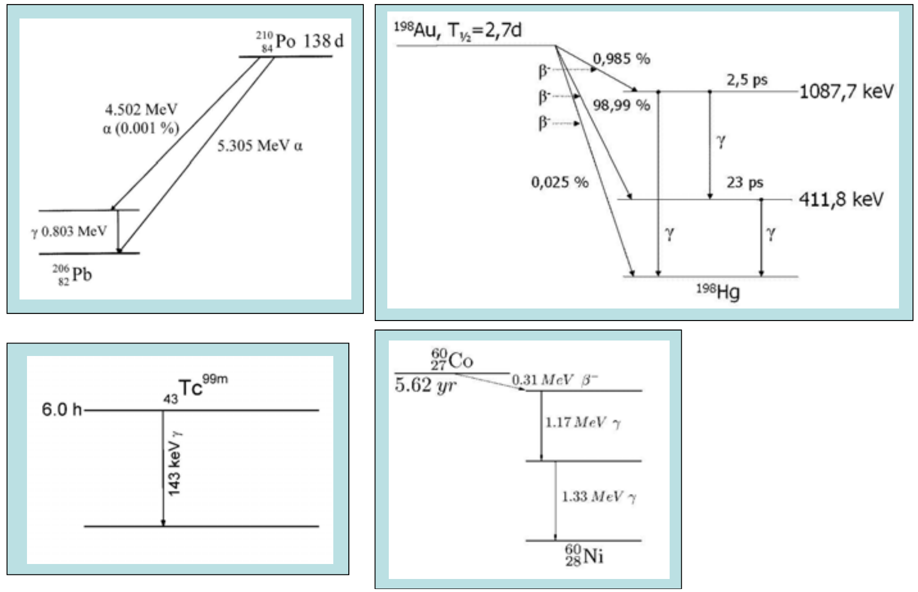
66 h — 6 h

K-capture (inverse β-decay)



$$^{1}_{1}p + ^{0}_{-1}\beta \rightarrow ^{1}_{0}n + \nu$$

Some examples of the decay paths



Characteristics of radioactive decays in general

activity	characterizes the source
half life time	characterizes the speed of the decay*
particle energy	characterizes the radiation*

*depends on the type of the isotope

Activity (Λ)

$$\Lambda = \left|\frac{dN}{dt}\right| \quad \left(= \left|\frac{\Delta N}{\Delta t}\right| \right)$$

N = Number of undecayed atoms
t = time
ΔN = Number of decays during Δt time

Activity= number of decays in a unit time

unit: becquerel Bq
1 Bq= 1 decay/sec

kBq, MBq, GBq, TBq

immeasurably small

level of natural activity

in vivo diagn.

careful work with it!

activity applied in radiotherapy

Law of radioactive decay

$\Delta N = -\lambda N \Delta t$ N: Number of undecayed nuclei

$\frac{dN}{dt} = -\lambda N$ λ: decay constant (probability of the decay [1/s])

$1/\lambda = \tau$ average lifetime

Differential equation

solution:

$$N(t) = N_0 e^{-\lambda t}$$

Exponential decrease

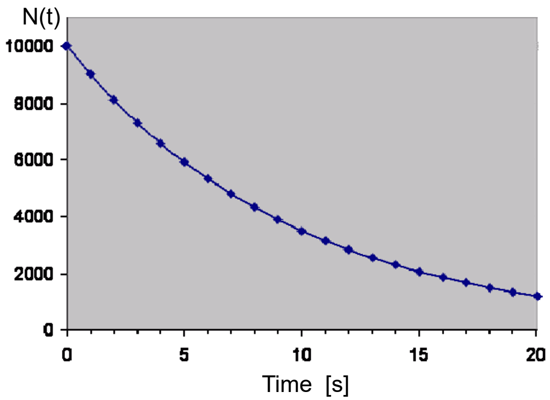
number of undecayed atoms at t=0

An example

- At start: N_0=10000 (number of atoms
  λ=0.1 $^1/_s$ (decay probability)
- after 1 sec: 9000 (10000x0.1=1000 decayed)
- after 2 sec: 8100 (9000x0.1=900 decayed)
- after 3 sec: 7290 (8100x0.1=810 decayed)
- after 4 sec: 6561 (7290x0.1=729 decayed)
- ….

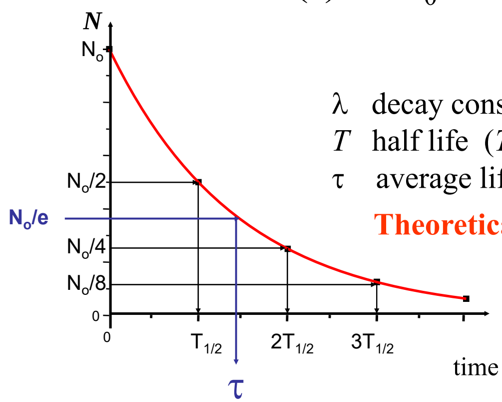
An example

- At start: N_0=10000 (number of atoms
  λ=0.1 $^1/_s$ (decay probability)
- after 1 s: 9000
- after 2 s: 8100
- after 3 s: 7290
- after 4 s: 6561
- ….



Law of radioactive decay

$$N(t) = N_0 e^{-\lambda t} = N_0 2^{-\frac{t}{T}}$$



λ decay constant (probability of the decay)

T half life ($T_{1/2}$)

τ average lifetime

Theoretically never decreases to zero !

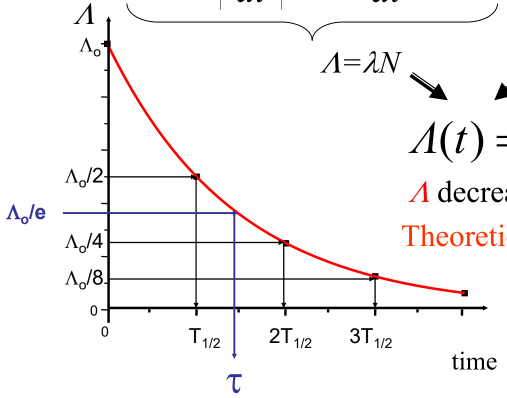
$$\lambda = \frac{\ln 2}{T} = \frac{0{,}693}{T}$$

Decrease of the activity as a function of time

$$\Lambda = \left|\frac{dN}{dt}\right| \qquad \frac{dN}{dt} = -\lambda N \qquad N(t) = N_0 e^{-\lambda t}$$

$\Lambda = \lambda N$

$$\Lambda(t) = \Lambda_0 e^{-\lambda t} = \Lambda_0 2^{-\frac{t}{T}}$$



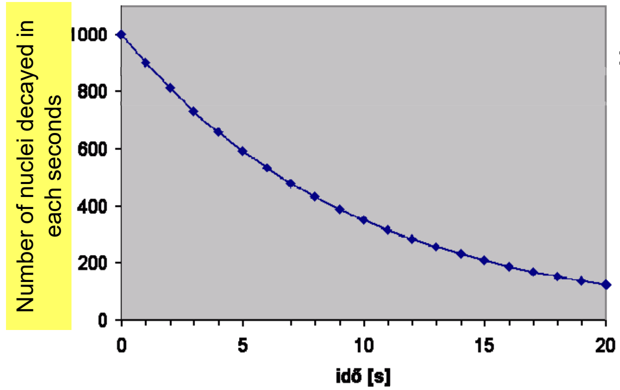
Λ decreases on the same mode as N!

Theoretically never decreases to zero!

During about 10 T the activity decreases to its 1/1000 (e.g GBq->MBq)

An example

- At start: N_0=10000 (number of atoms)
  λ=0.1 $^1/_s$ (decay probability)



=1000 decayed)

=900 decayed)

=810 decayed)

=729 decayed)

Few examples for half life

^{232}Th	$1.4\cdot10^{10}$ y
^{238}U	$4.5\cdot10^{9}$ y
^{40}K	$1.3\cdot10^{9}$ y
^{14}C	5736 y
^{137}Cs	30 y
^{3}H	12.3 y

^{60}Co	5.3 y
^{59}Fe	1.5 m
^{56}Cr	1 m (28 d)
^{131}I	8 d
^{99m}Tc	6 h
^{18}F	110 min
^{11}C	20 min
^{15}O	2 min
^{222}Th	2.8 ms

Don't learn these numbers!

Typical energy levels in the microworld

Excitation of the outer electrons	Electron transition between inner electrons	Transformation of the nucleus (decay)
eV (aJ)	keV (fJ)	MeV (pJ)
light	X-ray	Nuclear radiation α, β, γ

Absorption of the nuclear radiation

α, β⁻, β⁺ — particles with electric charge — direct ionisation

different types of interactions with matter!

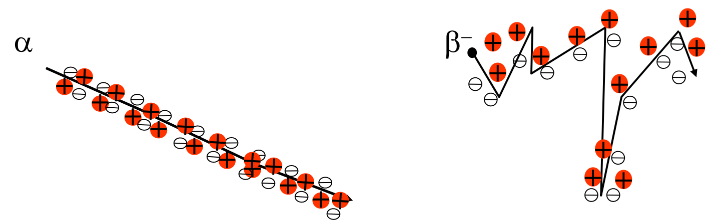
γ, (x-ray) — uncharged particles (photons) — indirect ionisation

not a nuclear radiation, but its absorption is similar to that of the γ radiation

Absorption of the charged particles

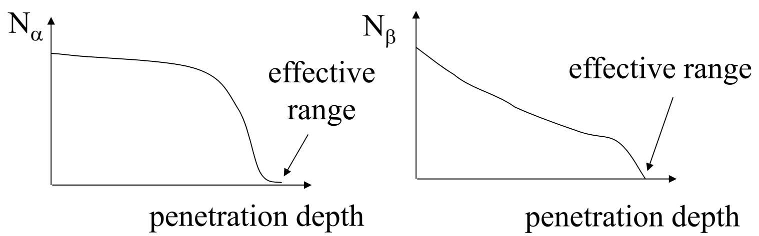
Ionizing during the path => continuous decrease of the particle energy
The energy after a given path length decreases to the thermal value

effective range



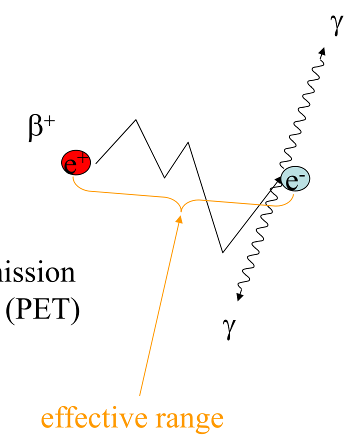
Effective range

	α-particle	β⁻-particle
in air	**few cm**	**~ m**
in tissue	**0,01-0,1 mm**	**~ cm**



β⁺-radiation

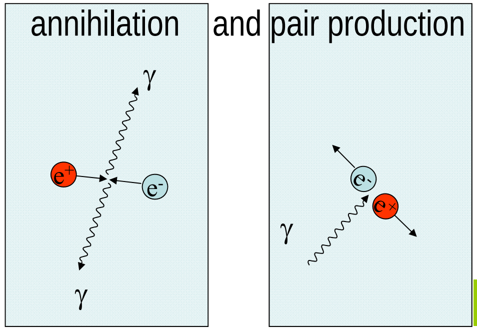
annihilation



see: Positron Emission Tomography (PET)

Electron and positron

- particle - antiparticle
- same mass,
- charge: same value, but differen[redacted]



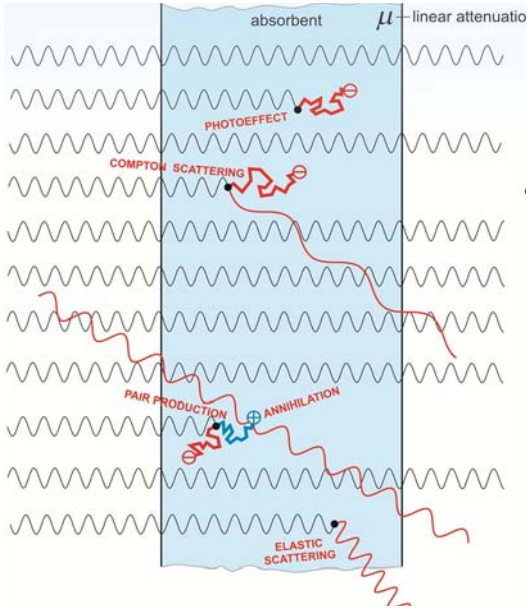
Einstein:

mass-energy ekvivalence

$$E=mc^2$$

m_ec^2=511 keV≈0.5 MeV

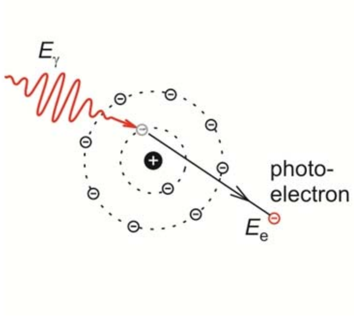
Absorption of the γ-radiation (and x-ray)



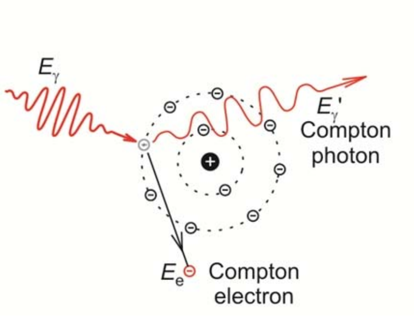
Absorption pricesses happen accidentally :

Photoeffect,
Compton-effect,
Pair production,
(elastic scattering)

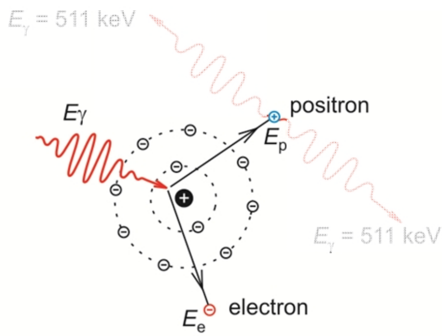
Photoeffect



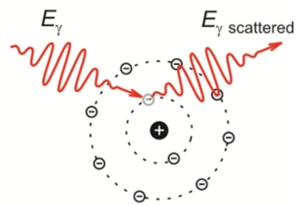
Compton effect
Compton-scattering



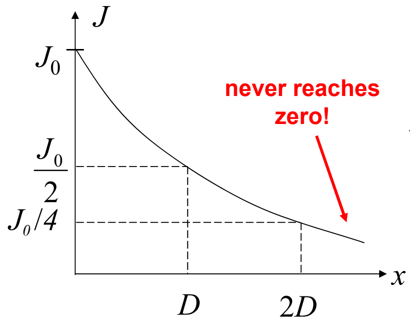
Pair production

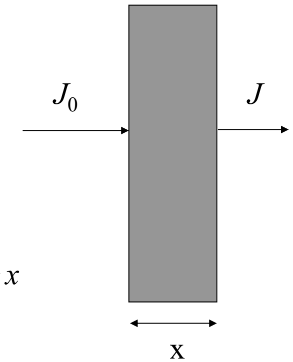


elastic scattering



Attenuation
of the γ–radiation and x-ray

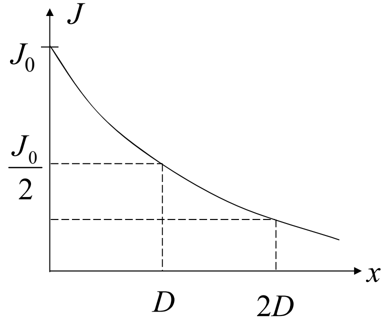




$$J = J_0 e^{-\mu x}$$

no finite effective range!

few rules of thumb: $x_{1/10}$=3,33 D $x_{1/1000}$=10 D



$$J = J_0 e^{-\mu x}$$

μ: (linear) attenuation coefficient

its units are: 1/m, 1/cm

$$\delta = \frac{1}{\mu}$$

„penetration depth"

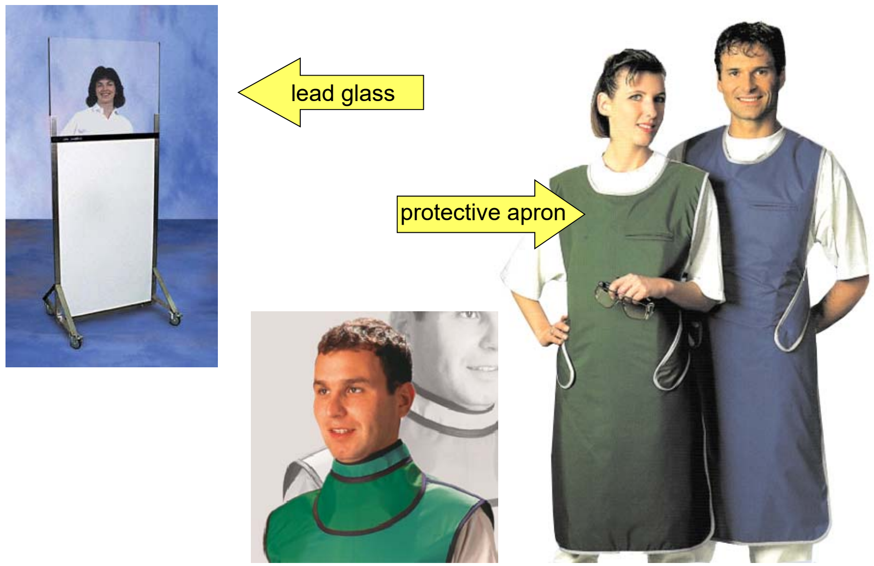
Intensity decreases to the *e*-th part (c.a. 37%)

μ(material, number of absorbing centers, energy of the radiation)
$=\mu(\text{material}, \rho, E_{photon}) \sim \rho$

$$\mu_m = \frac{\mu}{\rho}$$

mass attenuation coefficient

Applications (attenuation)



Applications: isotopes and nuclear radiation