Nuclear radiation, radioactive isotopes

Constituents of atoms

Particle	Symbol	Rest Energy (MeV)	Relative Charge*	Mass (kg)	Relative Mass (AMU)**
electron	e	0.51100	1-	9.11×10^{-31}	5.4858×10^{-4}
proton	p	938.272	0	1.6726×10^{-27}	1.0072765
neutron	n	939.566	1+	1.6749×10^{-27}	1.0086649

* electrons have an electric charge of -1.602×10^{-19} C
**The atomic mass unit is defined as 1/12 of the carbon (^{12}C)atom

Nuclear notation

Mass number
$A = Z + N$

Chemical symbol for the element

$^{24}_{12}N$

Atomic number = Number of protons

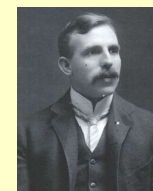N = **number of neutrons**

Nuclear stability

- There are very large *repulsive electrostatic forces* between protons

  These forces should cause the nucleus to fly apart

- $\Delta M = [Zm_p + (A-Z)m_n] - M(A,Z)$ (?)

  It must be that a different type of force exists within the nucleus

  1911 Rutherford, Geiger and Marsden

Nuclear stability

$$\Delta M = [Zm_p + (A-Z)m_n] - M(A,Z)$$

Instead of two separate conservation laws, a single conservation law states that the sum of mass and energy is conserved. Mass does not magically appear and disappear at random. A decrease in mass will be accompanied by a corresponding increase in energy and vice versa.

$$\Delta E = \Delta Mc^2$$

Another, short-range force is present, called the *nuclear force* (Rutherford, 1911)

Nuclear stability

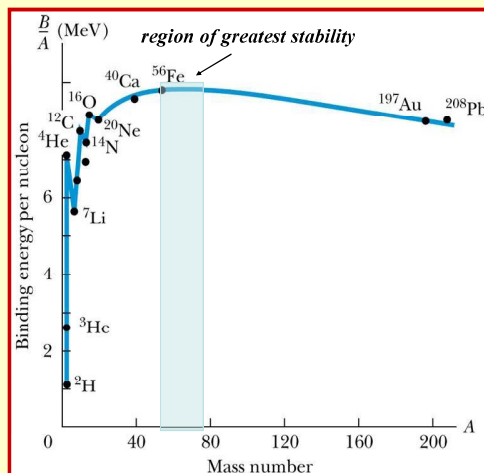- Another, short-range force is present, called the *nuclear force* (Rutherford, 1911)

This is an attractive force that acts between all nuclear particles

The nuclear attractive force is stronger than the Coulomb repulsive force at the short ranges within the nucleus

$$\Delta E = \Delta Mc^2$$

The nuclear force is
- independent of charge
- the range of action is extremely short (~fm)

Binding energy per nucleon

- The curve increases rapidly

- Sharp peaks for the even-even nuclides ^{4}He, ^{12}C, and ^{16}O

- Maximum is around A=56

Nuclear stability

The balance of proton and neutron number

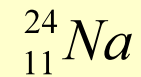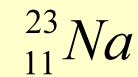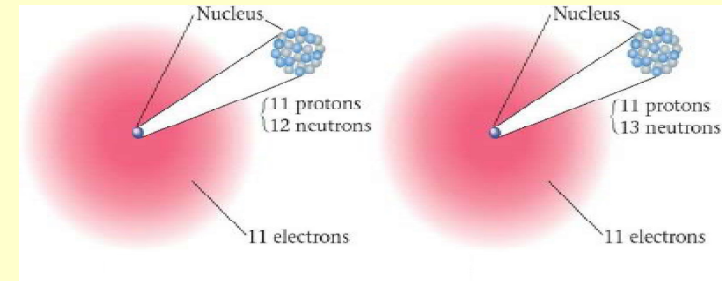*is extremely important for the stability of the nucleus*

Isotopes

Greek *isos topos = equal place*

Isotopes of an element have nuclei with

- the same number of protons

- different numbers of neutrons
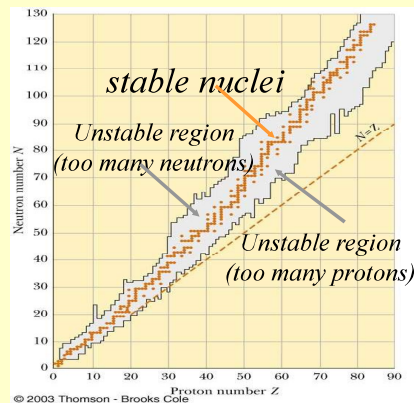
- different mass number

Example of isotopes

$$^{23}_{11}Na \qquad ^{24}_{11}Na$$

What is/are the stable combination(s)? 1:1 ?
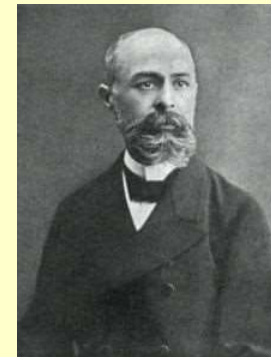
Nuclear stability chart

- Light nuclei are most stable if
    N=Z

- Heavy nuclei are most stable when
    N > Z

    As the number of protons increase,
    the Coulomb force increases and
    so more nucleons are needed to
    keep the nucleus stable

- No nucleus is stable when   Z > 83

Antoine Becquerel
1903 Nobel Prize in Physics
for discovering radioactivity

Image of Becquerel's
photographic plate which has
been fogged by exposure to
radiation from a uranium salt.
The shadow of a metal
Maltese Cross placed between
the plate and the uranium salt
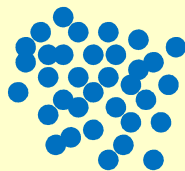is clearly visible. (1896)

Radioactive decay

- *Radioactivity* is the spontaneous release of energy in the form of radioactive particles or waves

- Experiments suggested that radioactivity was the result of the decay, or disintegration, of unstable nuclei

- Three types of radiation can be emitted
  - Alpha (α) particles
  - Beta (β) particles
  - Gamma (γ) rays (Rutherford 1896, see details later)

- The nuclides, as with most things in nature, want to be at their *lowest energy state* which is a stable nucleus.

- Radioactive decay occurs in nuclides where the *nucleus is unstable*.

- **The nuclide reaches its stable state by undergoing**

  radioactive decay.

Characteristics of radioactive decay

- it is *statistical process* ☐ individual

disintegrations occur *randomly*

- it results in a decrease over time of the original

amount of the radioactive nuclei

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decrease of the original amount of the radioactive nuclei over time

$$Activity : \quad \Lambda = \left| \frac{dN}{dt} \right| \quad \begin{array}{l} \textit{N: number of nuclei} \\ \quad \textit{to be decayed} \end{array}$$

t: time

number of nuclei decayed in a unit time

measure: bequerel (Bq)
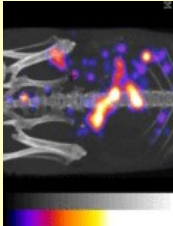*1Bq = 1 decay/sec*

Typical activities in the practice

kBq,   **MBq,**   **GBq,**   **TBq**

natural background   *in vivo* diagnostics   laboratory practice   *therapy*

Radioactive decay law

Differential form $\left|\dfrac{dN}{dt}\right| = -\lambda N$

λ : decay constant (measure: 1/s)

constant for a certain isotope

Activity depends both on the

- size of the population of radioactive atoms
- type of the isotope

$$\left|\frac{dN}{dt}\right| = -\lambda N$$
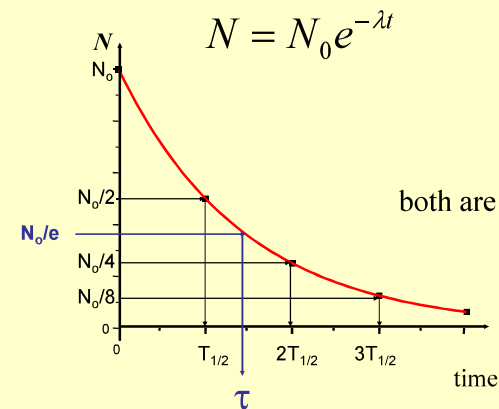
Solution of this equation yields

$$N = N_0 e^{-\lambda t} \qquad \textbf{Integral form}$$

N_0 : number of radioactive nuclei at $t = 0$,

N : the number radioactive of nuclei remaining after a period t

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Graphical representation

$$N = N_0 e^{-\lambda t}$$
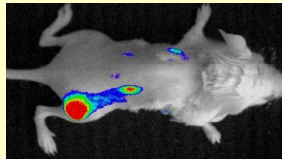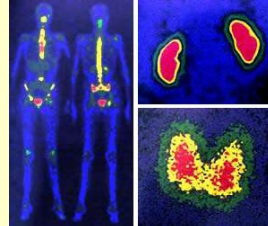

$T_{1/2}$: the half-life

τ : the mean life time

both are characteristic to the nuclei

Half-lives in Medical Uses

Iodine - 131 (^{131}I) - $T_{1/2}$ = *8 days*
Thyroid treatment

Technetium-99m (^{99m}Tc) – $T_{1/2}$ = *6 hours*
Isotope diagnostics

Gold-198 (^{198}Au) - $T_{1/2}$ = *2.7 days*
Tumor therapy

Further considerations

$$N = N_0 e^{-\lambda t}$$

If $t = T_{1/2}$ $\Longrightarrow$ $N_0 / 2 = N_0 e^{-\lambda T_{1/2}}$

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{T_{1/2}}$$

If $t = \tau$ $\Longrightarrow$ $N_0 / e = N_0 e^{-\lambda \tau}$

$$\lambda = \frac{1}{\tau}$$

Definition of decay constant

Alteration of activity in time

$$N = N_0 e^{-\lambda t}$$

$$\Lambda = \lambda N$$

$$\Lambda = \Lambda_0 e^{-\lambda t}$$

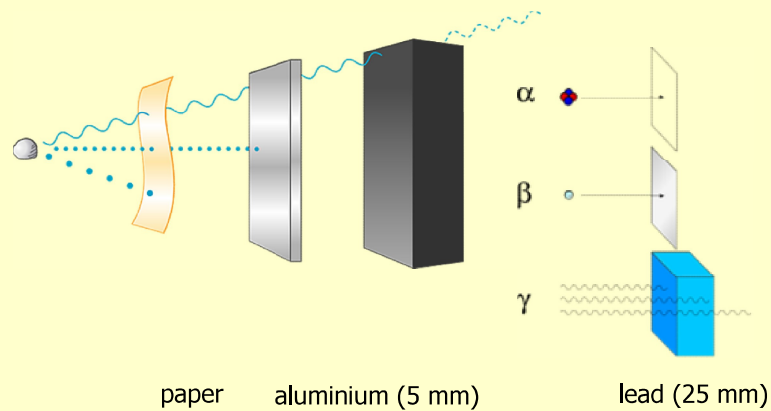**Specific activity** : activity in a unit mass (Λ/m),

unit: Bq / kg

Characteristics of radioactive isotopes

Activity : depends both on the nucleus and the size of its population

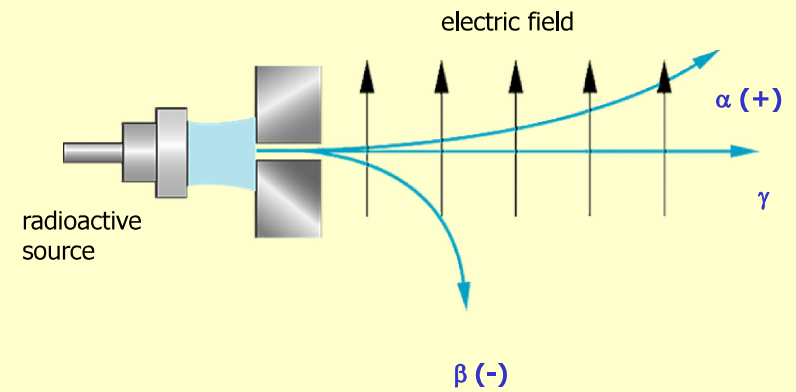*Half-life* : physical parameter of each radioactive isotope

Type of radiation: physical characteristic of the nucleus
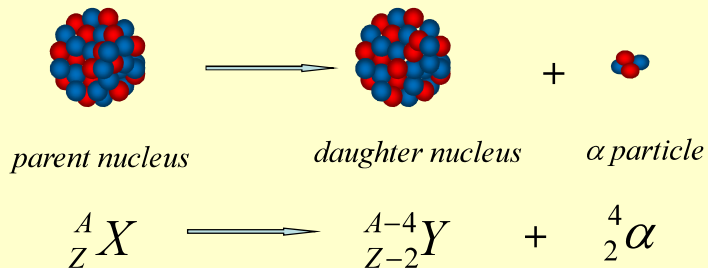
Types and nature of nuclear radiation

paper   aluminium (5 mm)   lead (25 mm)

Deflection of radiation in electric field

electric field

α (+)

γ

radioactive source

β (-)

Radiation from a radium source is split by an electric field.

α decay

parent nucleus   *daughter nucleus*   *α particle*

$$_{Z}^{A}X \longrightarrow \ _{Z-2}^{A-4}Y \ + \ _{2}^{4}\alpha$$
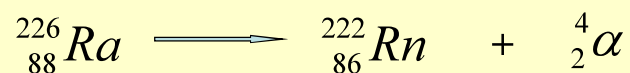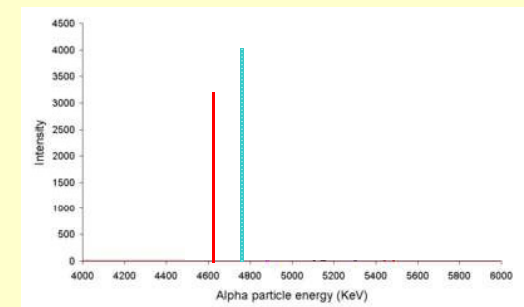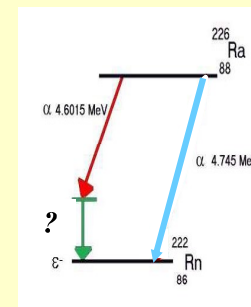
α particle is a nucleus of helium containing two neutrons and two protons

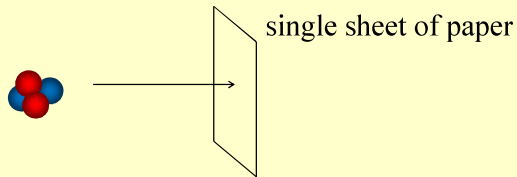Heavy nuclei with mass numbers higher than 150 can disintegrate by emission of an α particle

example

$$_{88}^{226}Ra \longrightarrow \ _{86}^{222}Rn \ + \ _{2}^{4}\alpha$$

Energy spectrum of α radiation

line spectrum

Energy is characteristic for the nucleus

Penetration depth of α particles

single sheet of paper

absorber	density	alpha range
air (STP)	1.2 mg/cm^3	3.7 cm
paper (20lb)	0.89 g/cm^3	53 μm
water (soft tissue)	1.0 g/cm^3	45 μm

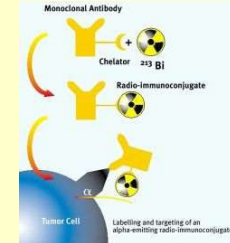# Medical application of α radiation

Diagnostics: none
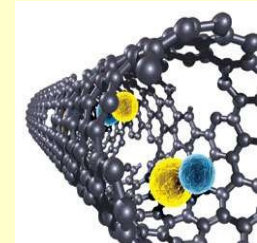
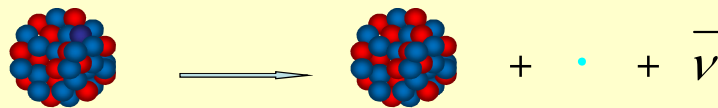Targeted alpha **therapy** of cancer

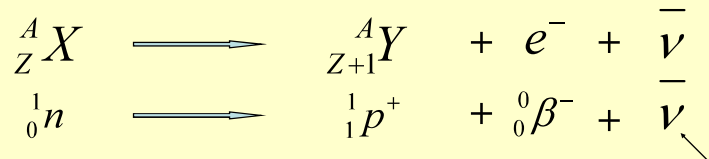Seed implantation by needle   Monoclonal antibody   Carbon nano-tube
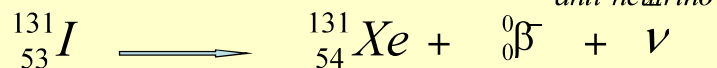
β decay

1. Neutron excess: β⁻ decay

parent nucleus   *daughter nucleus*

$$_{Z}^{A}X \longrightarrow \,_{Z+1}^{A}Y \; + \; e^- \; + \; \bar{\nu}$$

$$_{0}^{1}n \longrightarrow \,_{1}^{1}p^+ \; + \; _{0}^{0}\beta^- \; + \; \bar{\nu}$$

anti-neutrino

example

$$_{53}^{131}I \longrightarrow \,_{54}^{131}Xe \; + \; _{0}^{0}\beta^- \; + \; \nu$$

Energy spectrum of β radiation

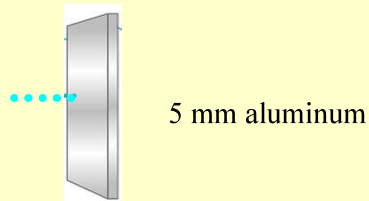$_{15}^{32}P$

1.709 MeV

$\bar{\nu}$

$_{0}^{0}\beta^-$

$_{16}^{32}S$

Energy distribution of the ß- particles emitted during the ß- decay of ^{32}P.

continuous spectrum
with maximum kinetic energy for the
β particle

Penetration depth of β⁻ particles

5 mm aluminum

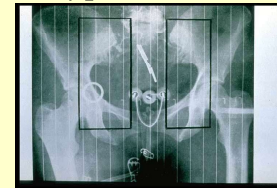| absorber	density	maximum beta range	
		(2.3 MeV)	*(1.1 MeV)*
air	1.2 mg/cm^3	8.8 m	3.8 m
water (soft tissue)	1.0 g/cm^3	11 mm	4.6 mm
aluminum	2.7 g/cm^3	4.2 mm	2.0 mm
lead	11.3 g/cm^3	1.0 mm	0.4 mm

Medical application of β⁻ radiation

Diagnostics: none

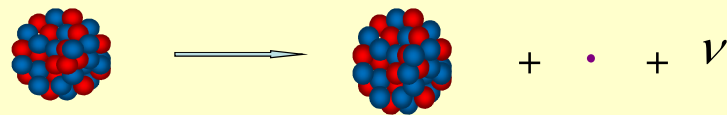Targeted therapy: hyperthyroidism, thyroid and several other types of cancer

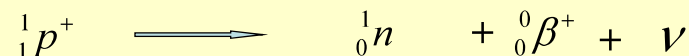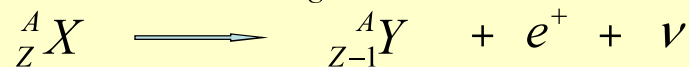Brachytherapy: implants into the tumours

Endovascular irradiation

β decay

2. Proton excess: β⁺ decay

$$+ \; \cdot \; + \; \nu$$

parent nucleus   *daughter nucleus*

$${}^{A}_{Z}X \longrightarrow {}^{A}_{Z-1}Y + e^{+} + \nu$$

$${}^{1}_{1}p^{+} \longrightarrow {}^{1}_{0}n + {}^{0}_{0}\beta^{+} + \nu$$

neutrino

example

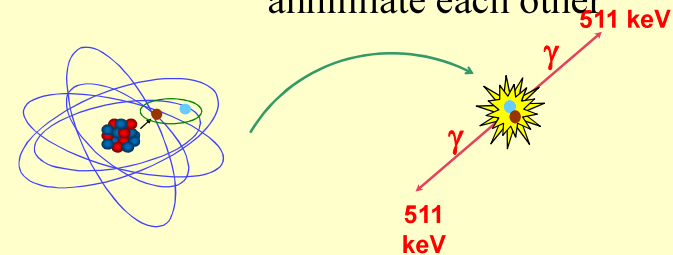$${}^{11}_{6}C \longrightarrow {}^{11}_{5}B + e^{+} + \nu$$

Annihilation - particle-antiparticle pairs can annihilate each other
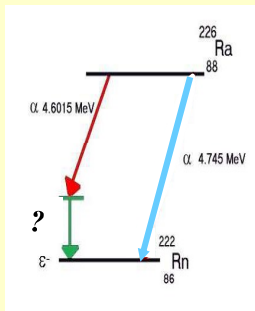

511 keV

γ

γ

511 keV

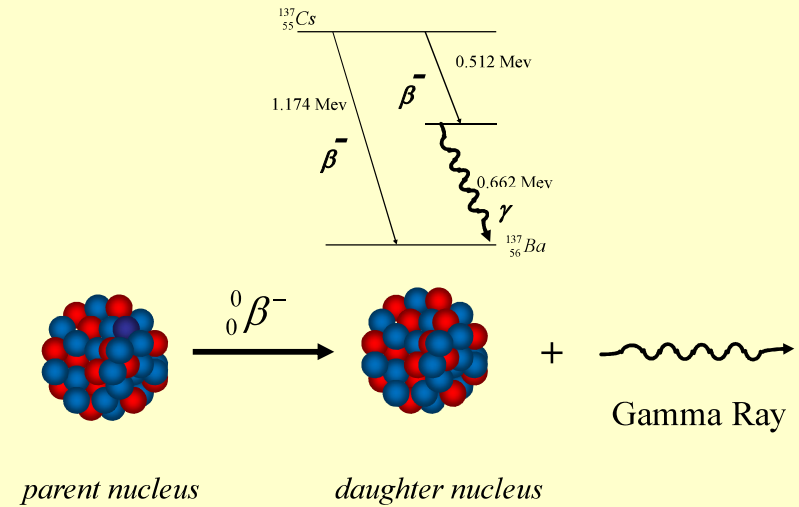1. Conservation of momentum : two photons with opposite direction are produced

2. Energy balance:

$$m_e c^2 + m_p c^2 = 2\,hf$$
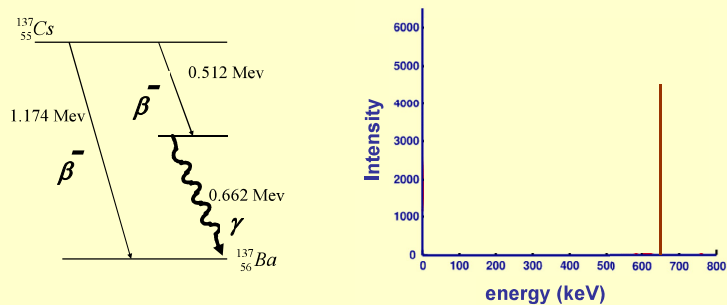
mass − energy equivalence

γ decay

Sometimes the newly formed isotopes (after α or β decay) appear in the excited stat.

Excited nuclides have tendency to release the excess of energy by **electromagnetic radiation** - emission of gamma rays.
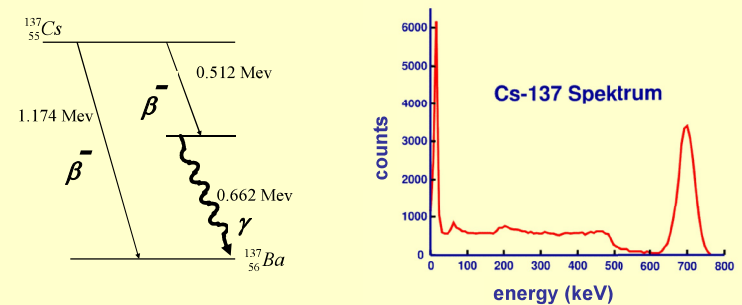
$^{226}_{88}$Ra
α 4.6015 MeV
α 4.745 MeV
?
ε⁻
$^{222}_{86}$Rn

$^{137}_{55}Cs$
1.174 Mev
0.512 Mev
β^-
β^-
0.662 Mev
γ
$^{137}_{56}Ba$

$^{0}_{0}\beta^-$

parent nucleus   *daughter nucleus*   + Gamma Ray

Energy spectrum of γ radiation

$^{137}_{55}Cs$
1.174 Mev
0.512 Mev
β^-
β^-
0.662 Mev
γ
$^{137}_{56}Ba$

Intensity / energy (keV)

line spectrum

Energy is characteristic for the nucleus

Energy spectrum of γ radiation

$^{137}_{55}Cs$
1.174 Mev
0.512 Mev
β^-
β^-
0.662 Mev
γ
$^{137}_{56}Ba$

Cs-137 Spektrum
counts / energy (keV)

line spectrum

Energy is characteristic for the nucleus

Penetration depth of γ radiation

lead (25 mm)

Penetration distance is higher than that of α or β particles, but it is highly energy dependent.

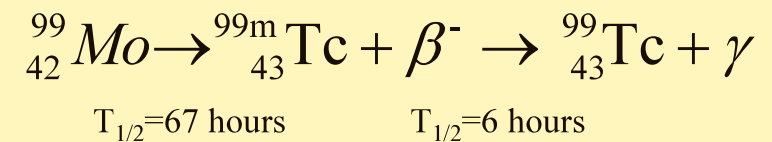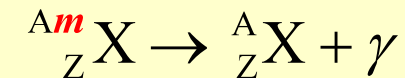Gamma rays can travel from 1 to 100s of meters in the air and can easily go right through people (~dm).

Timing of γ emission

Lifetime of the excited nucleus:

1. Prompt γ decay: $\sim 10^{-13} - 10^{-18}$ s

2. Isommeric transition: $\geq 10^{-10}$ s

Question of the week

What is the origin of nuclear force?

Isomeric transition

Some excited states may have a half-lifes ranging from hours up to more than 600 years

$$^{Am}_{Z}X \rightarrow\ ^{A}_{Z}X + \gamma$$

$$^{99}_{42}Mo \rightarrow\ ^{99m}_{43}Tc + \beta^{-} \rightarrow\ ^{99}_{43}Tc + \gamma$$

$T_{1/2}$=67 hours $T_{1/2}$=6 hours

Technetium-99m generator

Eluent Vial (0.9% NaCl)

$Na^{99}MoO_4$ 200 - 800 MBq

Ceramic Column

$Na^{99m}TcO_4$

Collection Vial

^{m}Tc formation

Mo decay

^{m}Tc without elution

activity (Gbq)

time t (h)

0   24 first elution   48 second elution

Medical application

Diagnostics: ideal for isotope diagnostics

Bone scan using ^{99m}Technetium labeled phosphate compound

Radionuclide	Compound	Organ	Function
$^{99}Tc^m$	sodium pertechnetate	brain	blood flow
$^{99}Tc^m$	coagulated albumin	lung	blood flow
$^{99}Tc^m$	colloidal suspension	liver	liver function
$^{99}Tc^m$	complex phosphate	bone	bone metabolisms
$^{99}Tc^m$	red blood cells	heart	blood circulation
^{123}I	iodide	thyroid	metabolisms
^{123}I	hippuran	kidneys	renal function
^{133}X	gas	lungs	ventilation

Damjanovich, Fidy, Szöllősi: Medical Biophysics

I. 1.5
    1.5.1
    1.5.2
    1.5.4

II.3.2
    3.2.1
    3.2.2
    3.2.3
    3.2.4