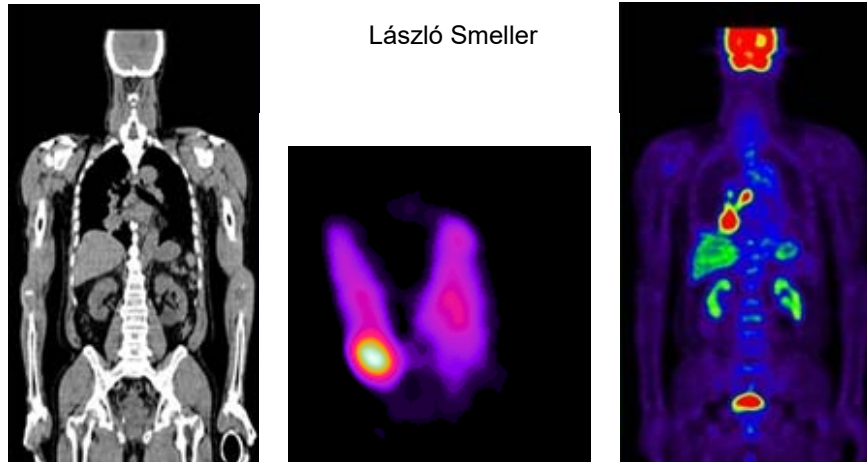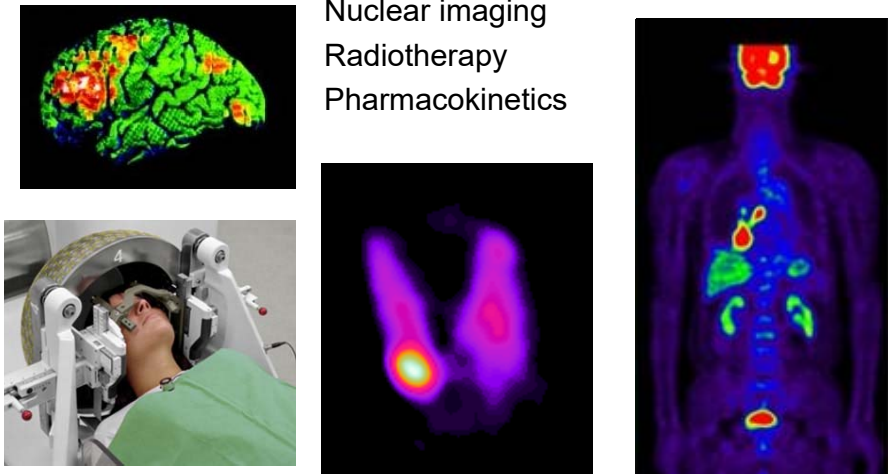# The atomic nucleus.
Radioactivity. Nuclear radiations

László Smeller
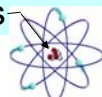
Why?

Medical/pharmaceutical applications of nuclear radiation:

Nuclear imaging
Radiotherapy
Pharmacokinetics

Length scale of the nature
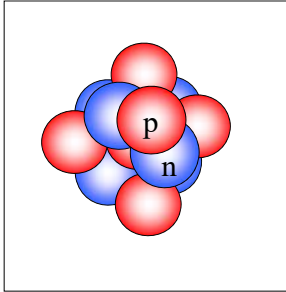
m		
10^0	meter	men
10^{-3}	millimeter	letters you can read
10^{-6}	micrometer	size of a cell (e.g. erytrocyte)
		$\varnothing$ 7μm
10^{-9}	nanometer	protein
10^{-10}	– angstrom	diameter of an atom, bond length
		H atom $\varnothing \approx$ 1 angstrom (Å)
10^{-12}	picometer	wavelength of the X-ray
10^{-15}	femtometer	size of the nucleus

The electrons and the nucleus

Changes in the electron shell:
=>chemical processes

10^{-10} m

10^{-15} m

**Changes of the nucleus:
=> radioactivity**
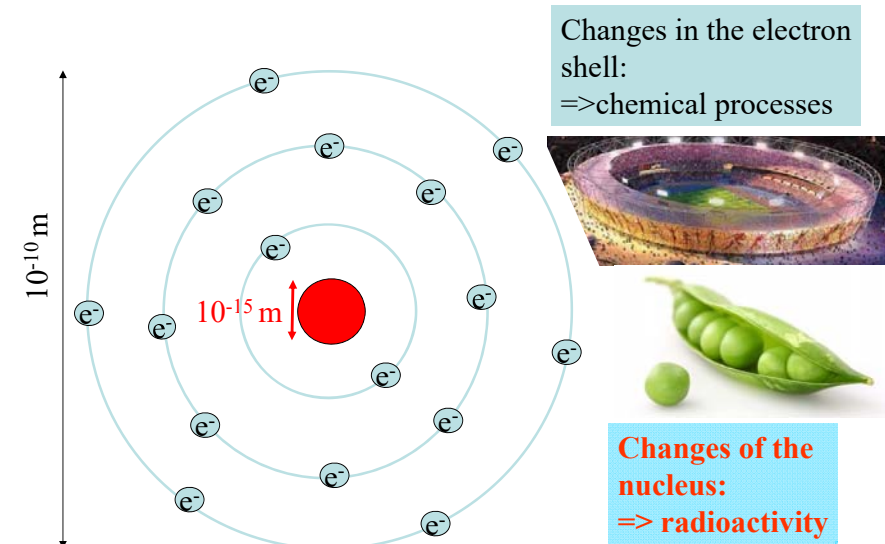
Structure of the nucleus

Elementary charge=$1,6\cdot10^{-19}$ C

	charge	mass
proton	+1 e	1 atomic mass unit
neutron	0	1 atomic mass unit

A (mass number) = number of protons + number of neutrons

Z (atomic number) = number of protons

$$^{99}_{43}\text{Tc}$$

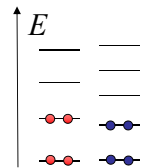99 nucleon, 43 proton and 56 neutron

Stability of the nucleus

- Coulomb force: destabilization
  (electrostatic repulsion between the protons)

- Nuclear force: very strong
  attractive force
  acts only on short range (~fm)
  independent on the charge

- Quantized energy levels for the nucleus.

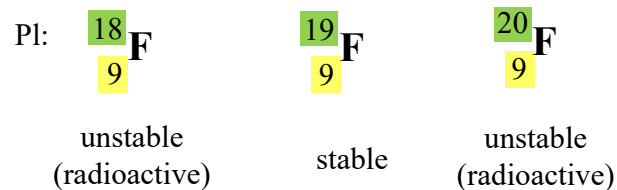- Typical binding energy is in the MeV range
  eV=$1,6\cdot10^{-19}$ J

E

Isotopes

Number of protons is the same

Number of neutrons is different

Variants of the same element

$\Rightarrow$ the chemical properties are identical.

Pl: $^{18}_{9}\text{F}$ $^{19}_{9}\text{F}$ $^{20}_{9}\text{F}$

unstable (radioactive)   stable   unstable (radioactive)

isotope <-> radioactive isotope

Table of isotopes

Number of protons

Number of neutrons

Z=0-28 Part 1 of 2

Decay Q-value Range
Q(??)
Q(β-)>0
Q(β-)-S$_n$>0
Q(β-)>0 + Q(EC)>0
Stable to Beta Decay
Q(EC)>0
Q(EC)-S$_p$>0
Q(P)>0
Naturally Abundant

Number of protons (vertical axis, top-left chart)

Number of neutrons (horizontal axis, top-left chart)

Number of protons (vertical axis, top-right chart)

Number of neutrons (horizontal axis, top-right chart)
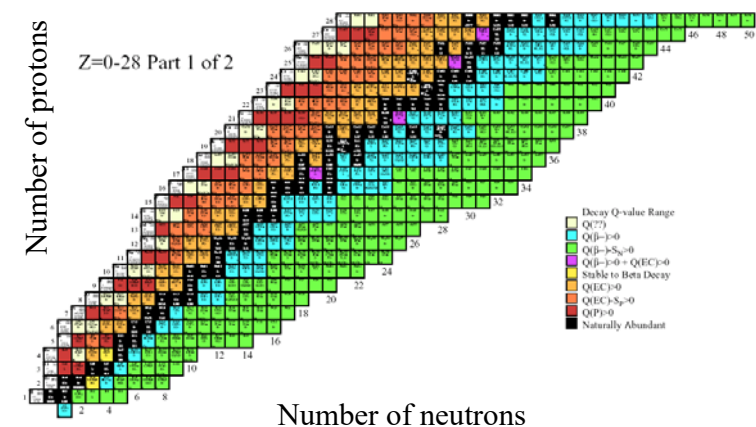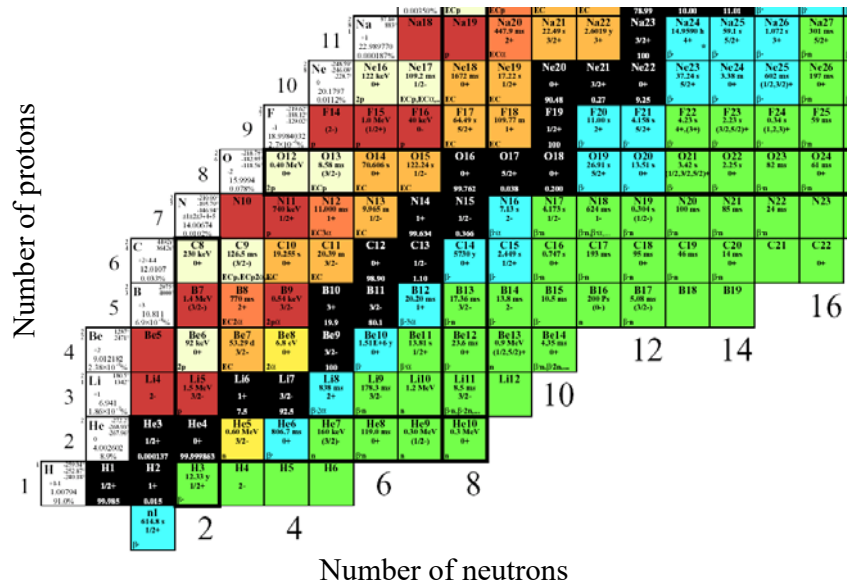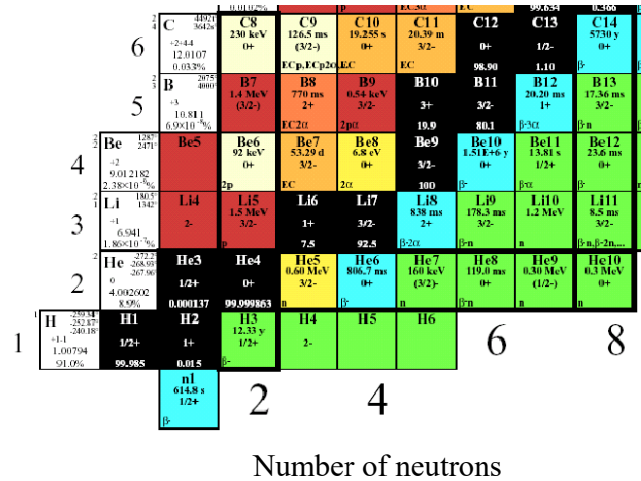
Radioactive decays and particles

α - decay α - particle $= {}^{4}_{2}\text{He}$ nucleus
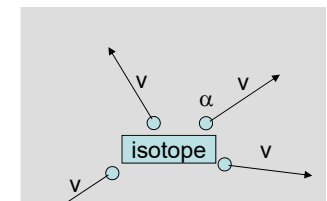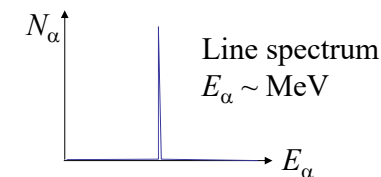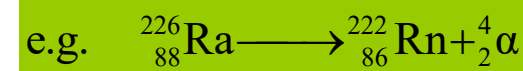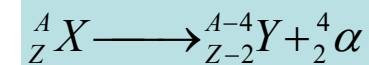
β - decay : β^{-} β^{-} particle $=$ electron
          β^{+} β^{+} particle $=$ positron

Isomeric transition γ-ray

K-electron capture   characteristic x-ray photon

α - decay

α - decay: an α particle (^{4}He nucleus) will be emitted
           typical for the heavy atoms

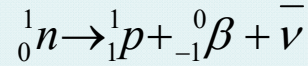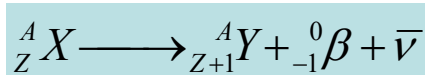$$ {}^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}\alpha $$

e.g. $${}^{226}_{88}\text{Ra} \longrightarrow {}^{222}_{86}\text{Rn} + {}^{4}_{2}\alpha$$

N_{α}

Line spectrum
$E_{\alpha} \sim \text{MeV}$

E_{α}

isotope

β⁻ - decay

neutron surplus

$$^A_Z X \longrightarrow\ ^A_{Z+1}Y +\ ^0_{-1}\beta + \overline{\nu}$$

$$^1_0 n \rightarrow\ ^1_1 p +\ ^0_{-1}\beta + \overline{\nu}$$

remains in the nucleus · · · leave the nucleus

e.g.:
$^{20}_{9}F$
$^{32}_{15}P$
$^{59}_{26}Fe$
$^{131}_{53}I$

e.g.: $^{32}_{15}P \longrightarrow\ ^{32}_{16}S +\ ^0_{-1}\beta + \overline{\nu}$

β⁻-ray → continuous energy spectrum

$\beta^- =\ ^0_{-1}\beta = e^-$

N_β

E_{max}

E_β

β⁺ - decay

proton surplus

$$^A_Z X \longrightarrow\ ^A_{Z-1}Y +\ ^0_{+1}\beta + \nu$$

$$^1_1 p \rightarrow\ ^1_0 n +\ ^0_{+1}\beta + \nu$$
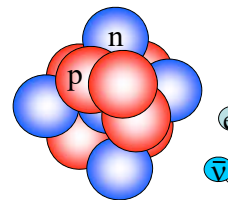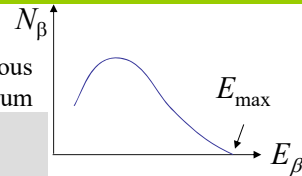
remains in the nucleus · · · leave the nucleus

e.g.: $^{11}_{6}C$
$^{15}_{8}O$
$^{18}_{9}F$
$^{52}_{26}Fe$

e.g.: $^{30}_{15}P \longrightarrow\ ^{30}_{14}Si +\ ^0_{+1}\beta + \nu$
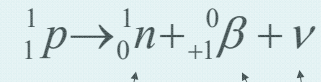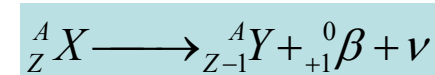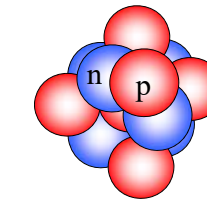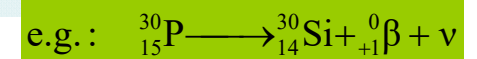
β⁺-ray → continuous energy spectrum

These isotopes must be produced artificially (e.g. in cyclotron)

Prompt γ-radiation

The daughter nucleus has an energetically unfavoured arrangement of nucleons.

The surplus energy will be immediately (ns) emitted in form of the γ radiation

α or β

γ

Atomic number, mass number are unchanged.

Isomeric transition

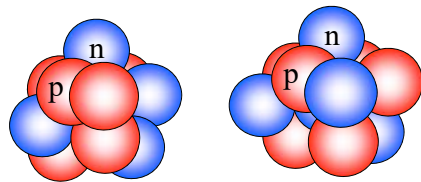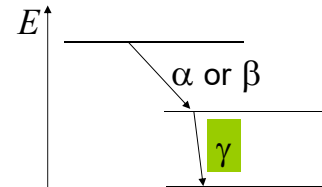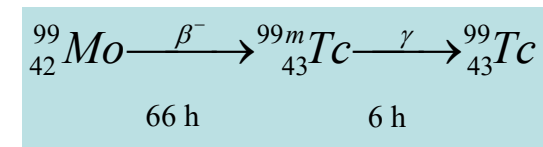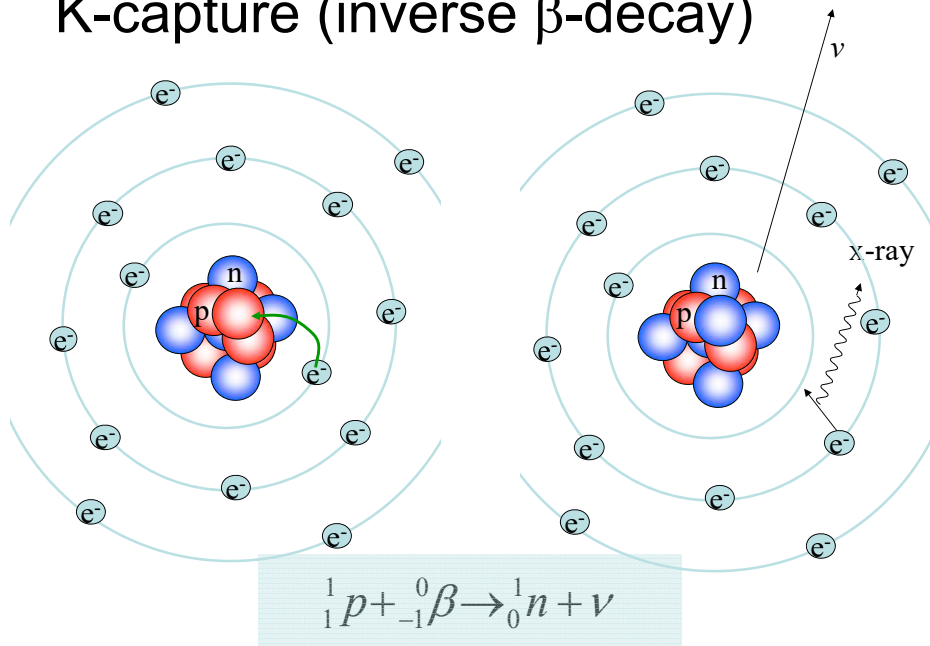If the excited state of the daughter nucleus is metastable, the γ-radiation will be emitted later.
The parent and daughter atoms can be separated: the daughter atom emits **only γ-radiation!**
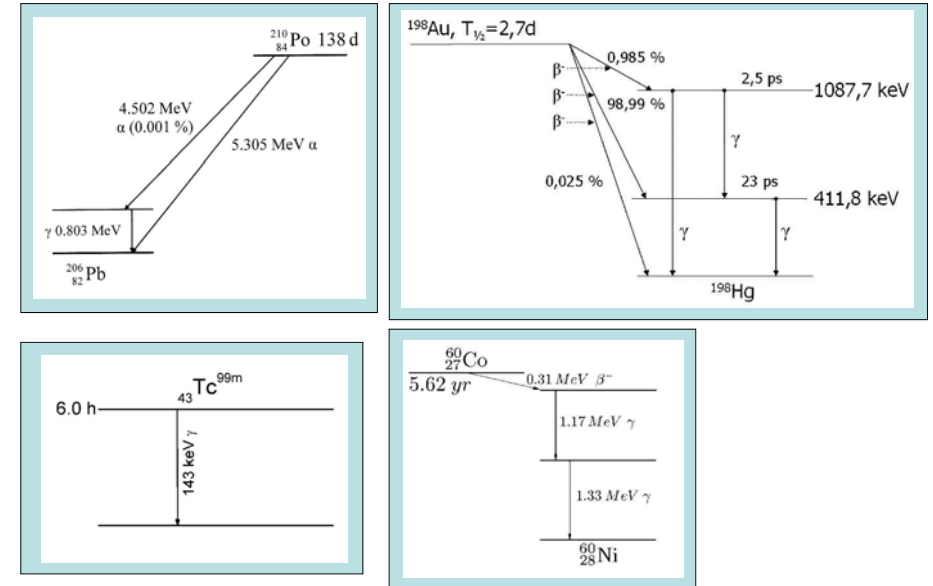
=> **Isotope diagnostics (nuclear imaging)**

E.g.: ^{99m}Tc

$$^{99}_{42}Mo \xrightarrow{\beta^-}\ ^{99m}_{43}Tc \xrightarrow{\gamma}\ ^{99}_{43}Tc$$

66 h · · · 6 h

K-capture (inverse β-decay)

v

x-ray

$$_1^1 p + _{-1}^0 \beta \rightarrow _0^1 n + \nu$$

Some examples of the decay paths

Characteristics of radioactive decays in general

activity	characterizes the source
half life time	characterizes the speed of the decay (depends on the type of the isotope)
particle energy	characterizes the radiation (depends on the type of the isotope)

Activity (Λ)

$$\Lambda = \left| \frac{dN}{dt} \right| \quad \left(= \left| \frac{\Delta N}{\Delta t} \right| \right)$$

N = Number of undecayed atoms
t = time
ΔN = Number of decays during Δt time

Activity = number of decays in a unit time

unit: becquerel Bq
1 Bq = 1 decay/sec

kBq, MBq, GBq, TBq

immeasurably small

level of natural activity

in vivo diagn.

careful work with it!

activity applied in radiotherapy
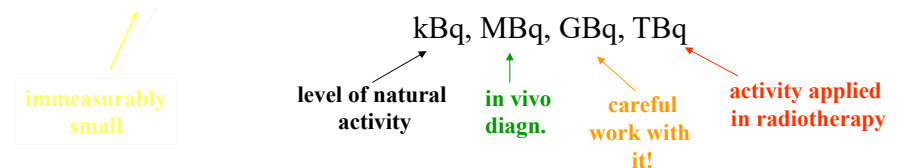
Law of radioactive decay

$$\Delta N = -\lambda N \Delta t$$

N: Number of undecayed nuclei

$$\frac{dN}{dt} = -\lambda N$$

λ: decay constant (probability of the decay [1/s])

$1/\lambda = \tau$ average lifetime

Differential equation

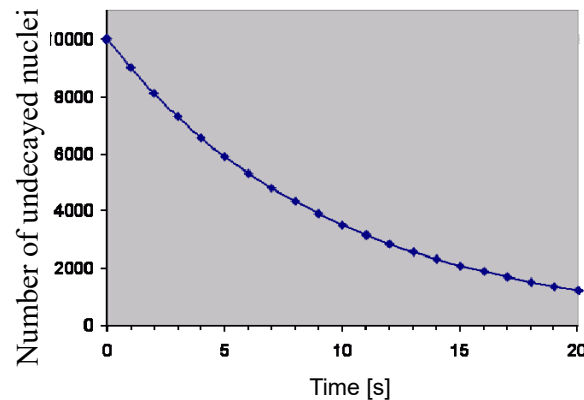solution:

$$N(t) = N_0 e^{-\lambda t}$$

Exponential decrease

number of undecayed atoms at $t=0$

An example

- $N_0 = 10000 \quad \lambda = 0.1 \ ^1/_s$
- After 1 sec : 9000   (10000x0.1=1000 decayed)
- After 2 sec : 8100   (9000x0.1=900 decayed)
- After 3 sec : 7290   (8100x0.1=810 decayed)
- After 4 sec : 6561   (7290x0.1=729 decayed)
- ….

An example

- $N_0 = 10000 \quad \lambda = 0,1 \ ^1/_s$
- 1 sec   9000
- 2 sec   8100
- 3 sec   7290
- 4 sec   6561
- ….

Law of radioactive decay

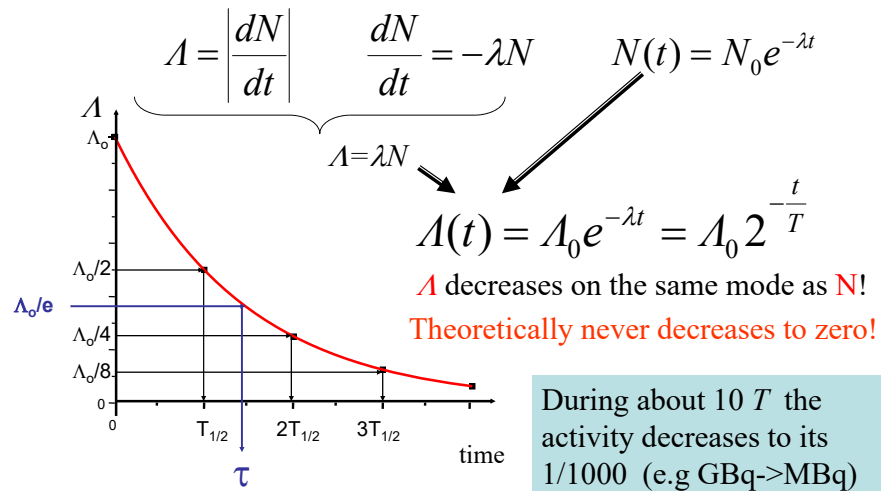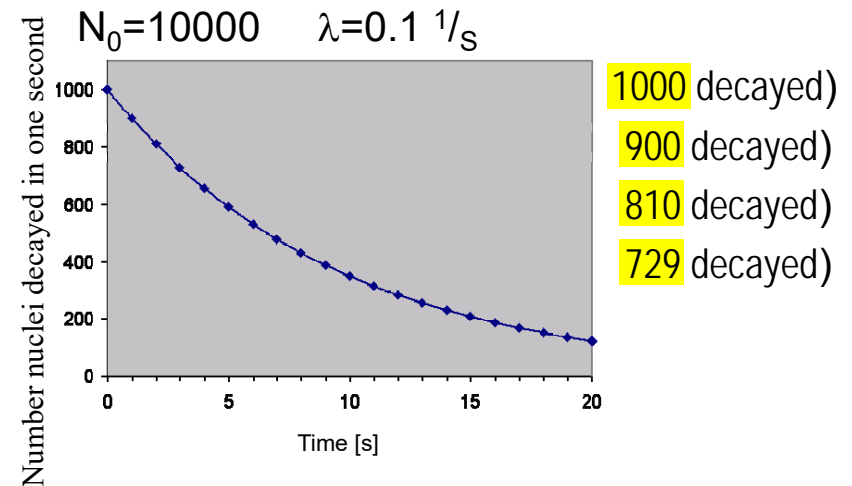$$N(t) = N_0 e^{-\lambda t} = N_0 2^{-\frac{t}{T}}$$

λ decay constant (probability of the decay)

T half life $(T_{1/2})$

τ average lifetime

Theoretically never decreases to zero !
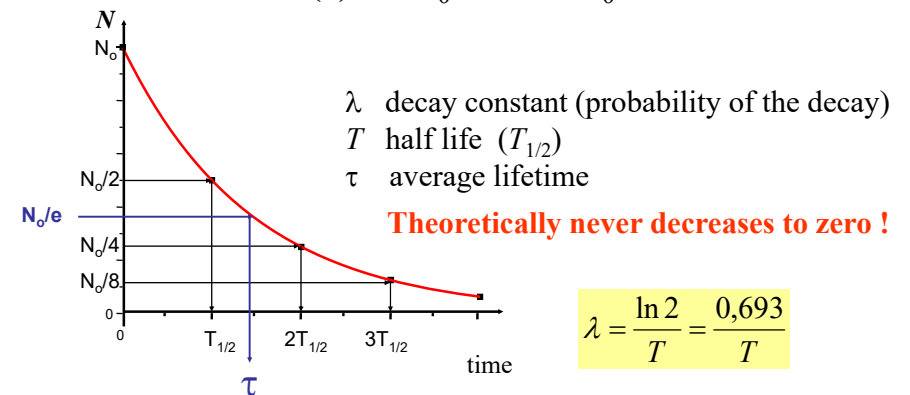
$$\lambda = \frac{\ln 2}{T} = \frac{0,693}{T}$$

Decrease of the activity as a function of time

$$\Lambda = \left| \frac{dN}{dt} \right| \qquad \frac{dN}{dt} = -\lambda N \qquad N(t) = N_0 e^{-\lambda t}$$

$$\Lambda = \lambda N$$

$$\Lambda(t) = \Lambda_0 e^{-\lambda t} = \Lambda_0 2^{-\frac{t}{T}}$$

Λ decreases on the same mode as N!

Theoretically never decreases to zero!

During about 10 T the activity decreases to its 1/1000 (e.g GBq->MBq)

Example

$N_0 = 10000 \qquad \lambda = 0.1 \; {}^{1}/_{s}$

1000 decayed)

900 decayed)

810 decayed)

729 decayed)

Number nuclei decayed in one second

Time [s]

Few examples for half life

²³²Th	1,4·10¹⁰ y		⁶⁰Co	5,3 y
²³⁸U	4,5 ·10⁹ y		⁵⁹Fe	1,5 m
⁴⁰K	1,3 ·10⁹ y		⁵⁶Cr	1 m (28 d)
¹⁴C	5736 y		¹³¹I	8 d
¹³⁷Cs	30 y		⁹⁹ᵐTc	6 h
³H	12,3 y		¹⁸F	110 min
			¹¹C	20 min
			¹⁵O	2 min
			²²²Th	2,8 ms

Don't learn these numbers!

Typical energy levels in the microworld

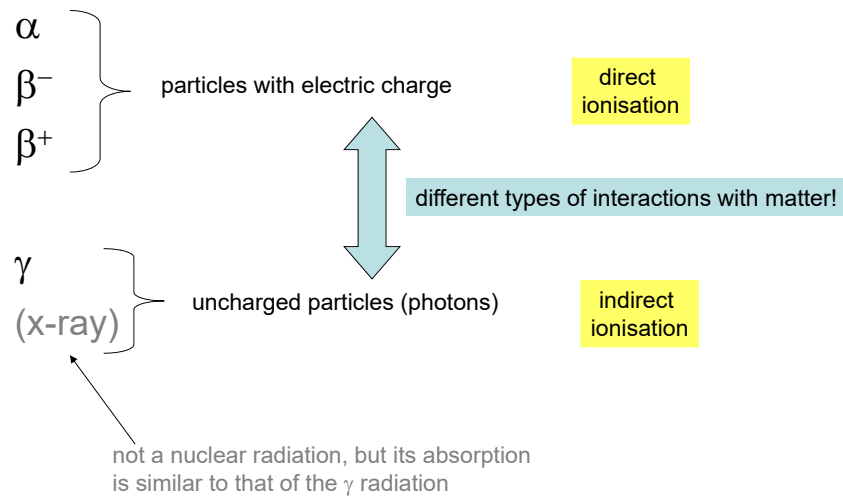| Excitation of the outer electrons	Electron transition between inner electrons	Transformation of the nucleus (decay)
eV (aJ)	keV (fJ)	MeV (pJ)
light	X-ray	Nuclear radiation α, β, γ
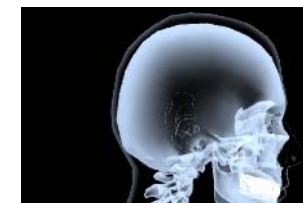
Absorption of the nuclear radiation

α
β^- particles with electric charge   direct ionisation
β^+

different types of interactions with matter!

γ
(x-ray)   uncharged particles (photons)   indirect ionisation
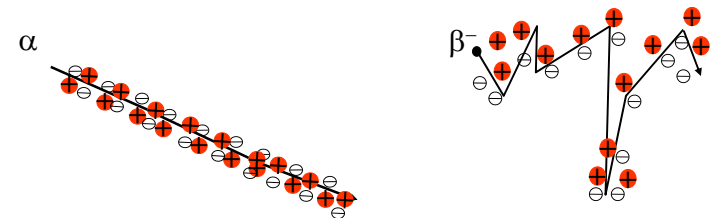
not a nuclear radiation, but its absorption
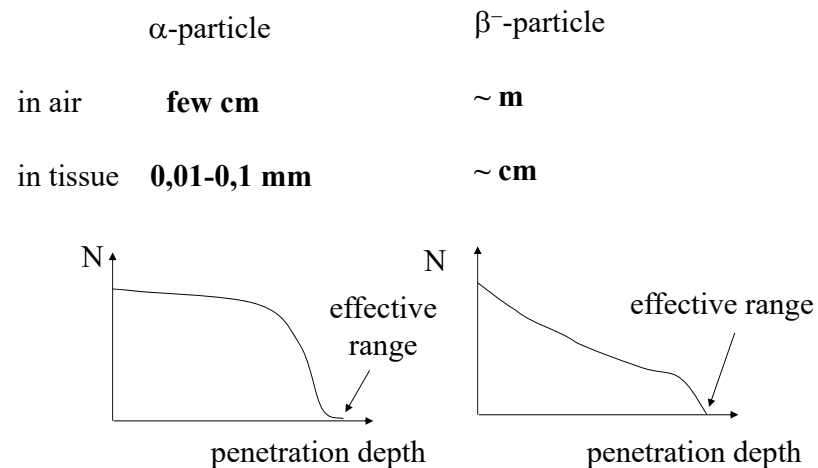is similar to that of the γ radiation

Absorption of the charged particles

Ionizing during the path => continuous decrease of the particle energy
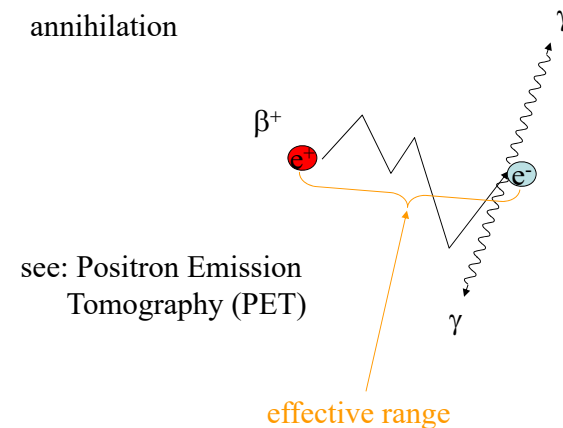The energy after a given path length decreases to the thermal value

effective range

α

β^-
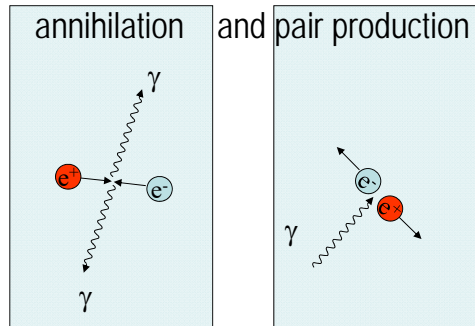
Effective range

	α-particle	β^--particle
in air	**few cm**	**~ m**
in tissue	**0,01-0,1 mm**	**~ cm**

N

effective range

penetration depth

N

effective range

penetration depth

β^+-radiation

annihilation

γ

β^+

e⁻

see: Positron Emission
Tomography (PET)

γ

effective range

Electron and positron

- particle - antiparticle
- same mass,
- charge: same value, but differen

annihilation and pair production

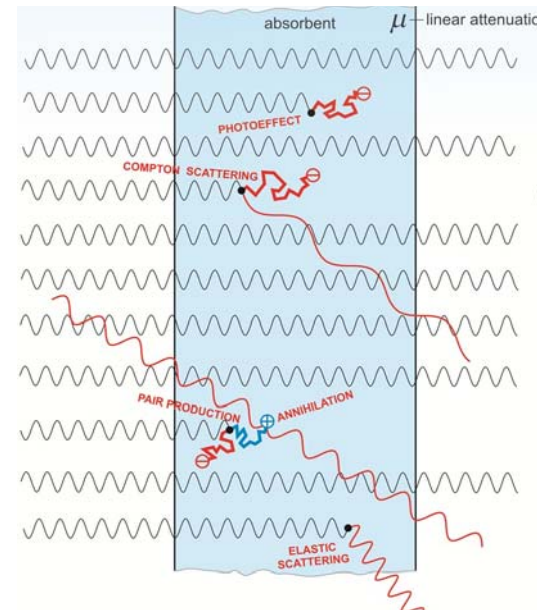
Einstein:

mass-energy ekvivalence

$$E=mc^2$$
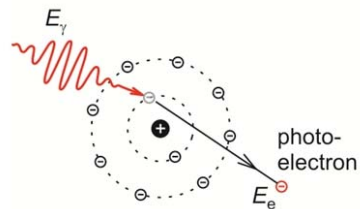
$m_e c^2 = 511 \text{ keV} \approx 0,5 \text{ MeV}$

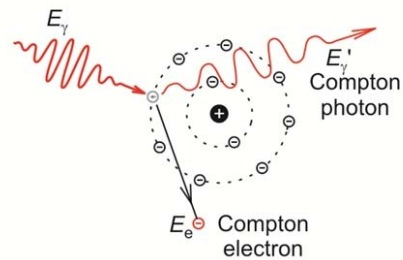# Absorption of the γ-radiation (and x-ray)

Absorption processes happen accidentally :

Photoeffect,
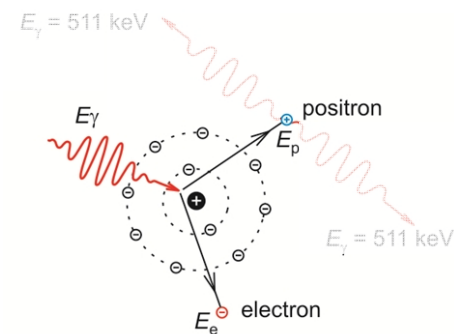Compton-effect,
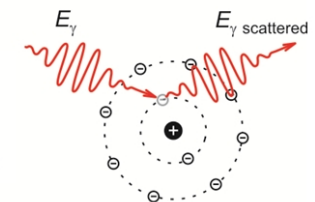Pair production,
(elastic scattering)

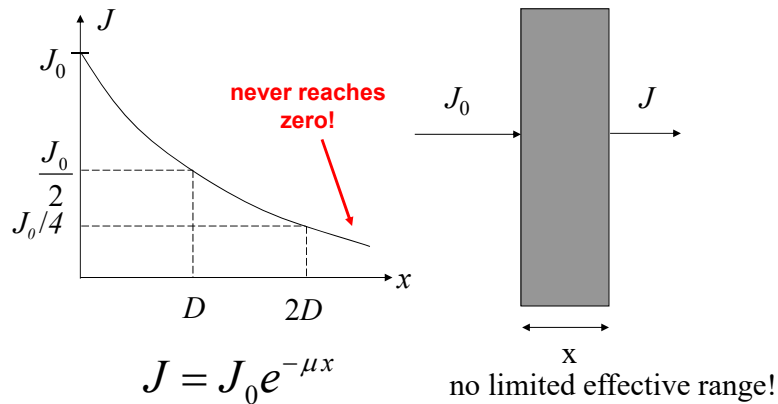## Photoeffect

Compton effect
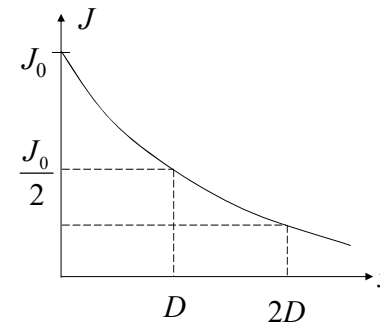## Compton-scattering

Pair production

elastic scattering

Attenuation
of the γ−radiation and x-ray

never reaches zero!

J_0 → J

$$J = J_0 e^{-\mu x}$$

x
no limited effective range!

few rules of thumb: $x_{1/10}=3,33\ D$ $x_{1/1000}=10\ D$

$$J = J_0 e^{-\mu x}$$

μ: (linear) attenuation coefficient
its units are: 1/m, 1/cm

$$\delta = \frac{1}{\mu}$$ „penetration depth"
Intensity decreases to
the e-th part (c.a. 37%)

μ(material, number of absorbing centers, energy of the radiation)
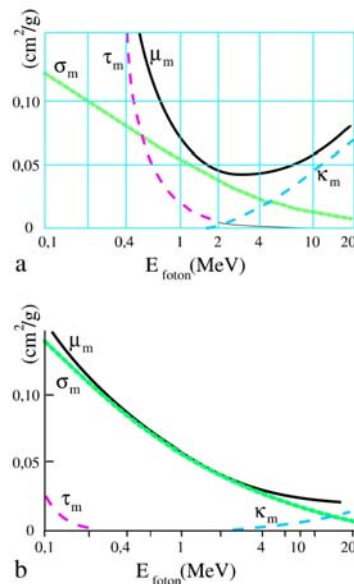$=\mu$(material,ρ,E_{photon}) $\sim \rho$

$$\mu_m = \frac{\mu}{\rho}$$ mass attenuation coefficient

---

mass attenuation coeff.

$$\mu_m = \frac{\mu}{\rho}$$

$$\mu_m = \tau_m + \sigma_m + \kappa_m$$

$$\tau_m = c\lambda^3 Z^3$$

Applications (attenuation)

lead glass

protective apron