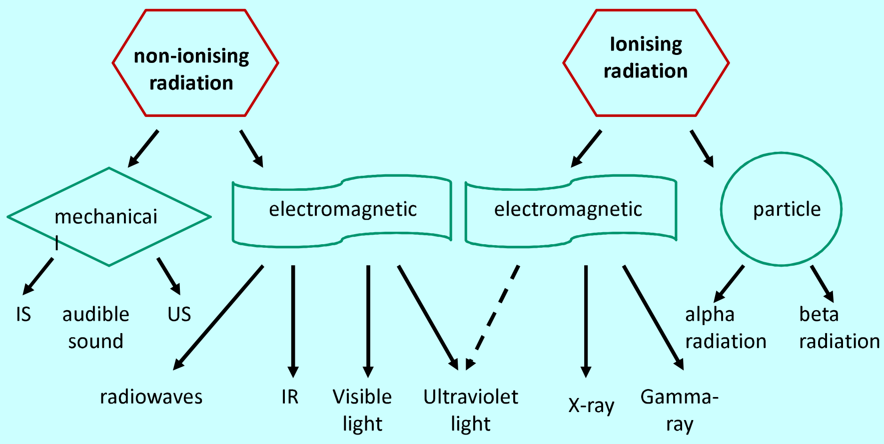
Radiation



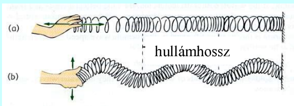
Sound

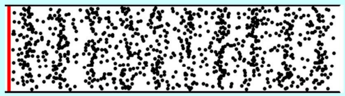
and

ultrasound

Physics of sound

Longitudinal vs. Transverse wave





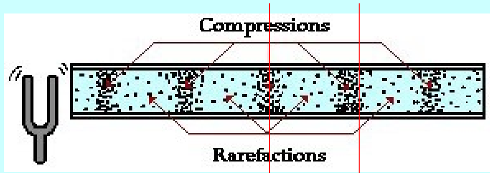
longitudinal wave
(in the interior of liquids and gases only this type)

transverse wave
can generated in solid materals and at liquid surfaces

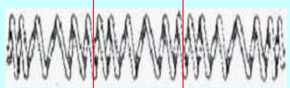
Physics of sound

Sound: mechanical wave (model)

whistle

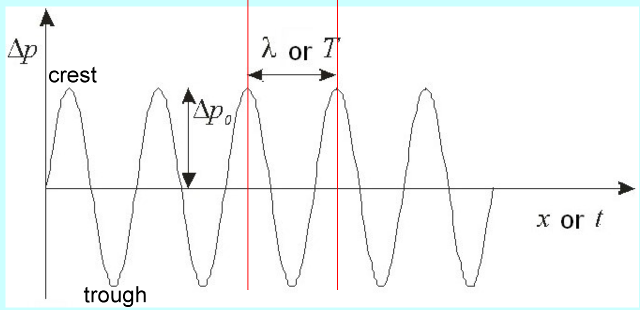


spring

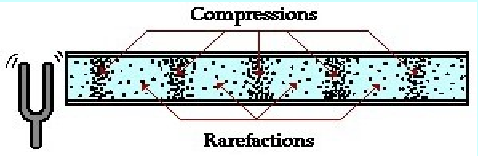


spatial and temporal periodicity

function



whistle



hydrostatic pressure

pressure change, sound pressure

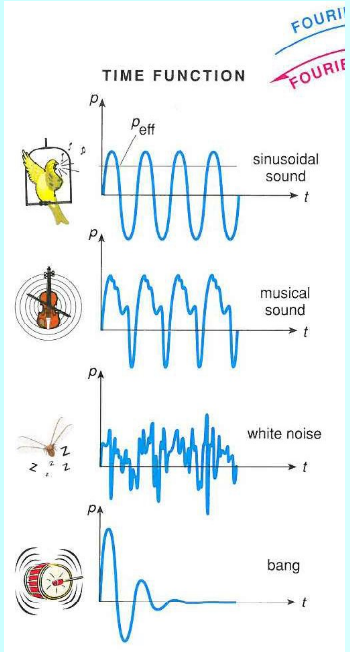
$p_{total} = p_{hydrostat} + \Delta p$

pressure DC + AC

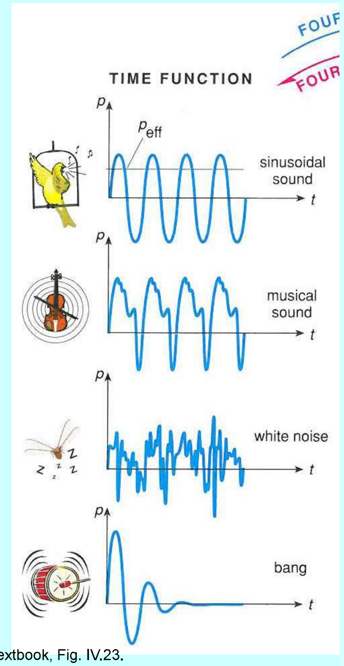
amplitude phase

$$\Delta p(t,x) = \Delta p_{max} \sin\left[2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)\right]$$

$c \cdot T = \lambda, \quad c = f \cdot \lambda$



$$p(t) = p_1 \sin(\omega t) + p_2 \sin(2\omega t) + p_3 \sin(3\omega t) + .$$



Textbook, Fig. IV.23.

pitch
frequency of the fundamental

hight

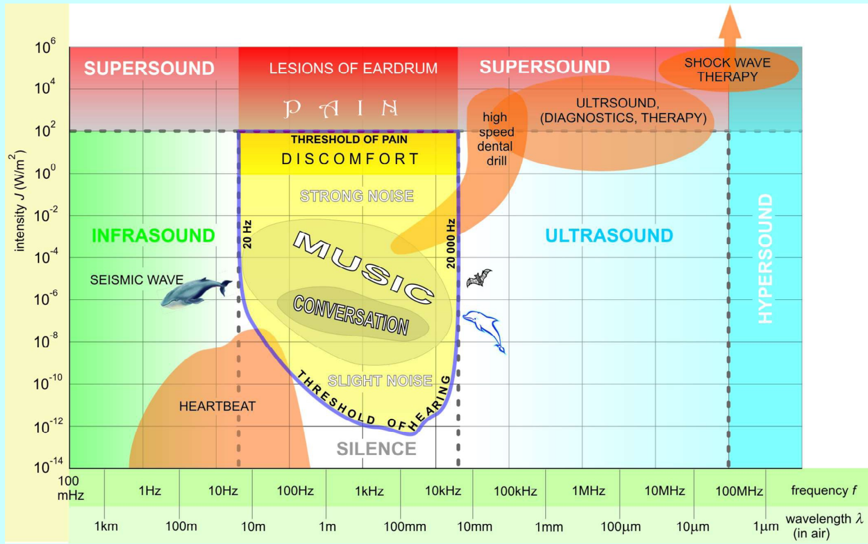
timbre
relative strengths of overtones/harmonics (spectrum)

tone colour

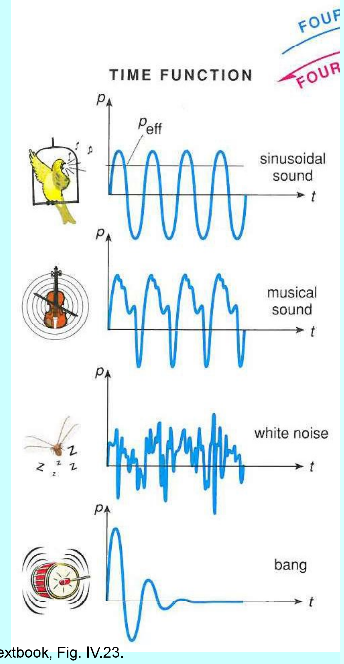
Intensity*
from pressure amplitude

loudness

Frequency and intensity regions of sounds



Lab. manual, Audiometry.



Textbook, Fig. IV.23.

pitch
frequency of the fundamental — hight

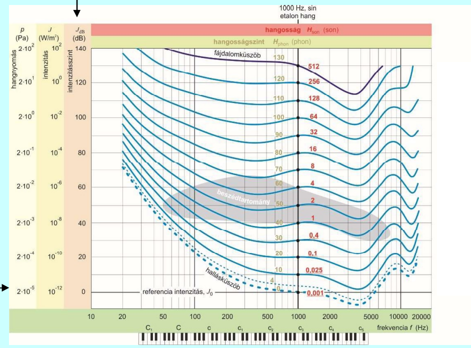
timbre
relative strengths of overtones/harmonics (spectrum) — tone colour

Intensity*
from pressure amplitude — loudness

$$J_{dB} = 10\lg\frac{J}{J_0}$$

$J_0 = 10^{-12}$ W/m^2

$$J_{dB} = 10\lg\frac{J}{J_0}$$



$J_0 = 10^{-12}$ W/m^2

Curves of similar loudness levels

What is the intensity of a 300 Hz sound that a person, who has 25 dB hearing loss at this frequency can hear? (The average hearing threshold at this frequency is 3•10-11 W/m^2)

$$J_{dB} = 10\lg\frac{J}{J_0}$$

$$25 = 10\lg\frac{J}{3*10^{-11}}$$

$$10^{2,5} = \frac{J}{3*10^{-11}}$$

$$J = 9,5*10^{-9}\left[Wm^{-2}\right]$$

Propagation of sound/ultrasound

The role of elastic medium – *speed* of propagation

$$c = f\lambda$$

$$\kappa = -\frac{1}{V}\left(\frac{\Delta V}{\Delta p}\right)\left[Pa^{-1}\right]$$

compressibility
relative volume decrease over pressure

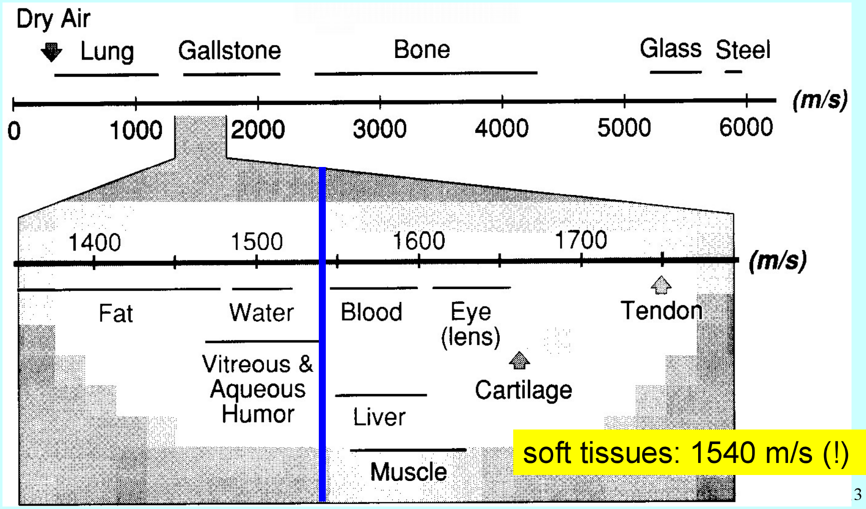
$$c = \frac{1}{\sqrt{\rho\kappa}}$$

ρ: density of medium

Speed of propagation is higher in solids than in liquids.

$\rho \uparrow \quad \kappa \downarrow$

Speed of sound/US in various media



Propagation of sound/ultrasound

$$c = \frac{1}{\sqrt{\rho\kappa}}$$

$$\kappa = \frac{-\Delta V / V}{\Delta p} \quad [\mathrm{Pa}^{-1}]$$

$$Z = \frac{p}{v} = \frac{p_{max}}{v_{max}}$$

acoustic **impedance** (definition)

$$Z_{el} = \frac{U}{I}$$

$$Z = c\rho = \sqrt{\frac{\rho}{\kappa}}$$

acoustic **impedance** (useful form)

$$[kg/m^2 s]$$

Propagation of sound/ultrasound

$$c = \frac{1}{\sqrt{\rho\kappa}}$$

$$\kappa = \frac{-\Delta V / V}{\Delta p}$$

$$Z = c\rho = \sqrt{\frac{\rho}{\kappa}}$$

material	ρ	κ	c	Z
	[kg/m ^{3}]	[1/GPa]	[m/s]	[kg/(m ^{2}·s)]
air	1,3	7650	331	$0,00043 \cdot 10^6$
water 20°C	998	0,45	1492	$1,49 \cdot 10^6$
aluminum	2700	0,009	6400	$17,28 \cdot 10^6$
quertz	2650	0,011	5736	$15,2 \cdot 10^6$

Problem:
A sound beam of 3 MHz frequency and 50 mW/cm ^{2} intensity propagates in blood.
What is the pressure? What is the maximal displacement and velocity of particles in this beam?

Z_{blood}=1,66x10 ^{6} kg/m ^{2} s

Solution:

Intensity:

$$J = \frac{p_{max}^2}{2Z}$$

$$p = \sqrt{2JZ} = 40.74\mathrm{kPa}$$

Velocity:

$$v = \frac{p}{Z} = \frac{40,74 \cdot 10^3}{1,66 \cdot 10^6} = 0,0245\mathrm{m/s} = 24,5\mathrm{mm/s}$$

Displacement:

$$A = \frac{v}{\omega} = \frac{24,5}{2 \cdot \Pi \cdot 3 \cdot 10^6} = 1,3 \cdot 10^{-6}\mathrm{mm} = 1,3\mathrm{nm}$$

Intensity of US

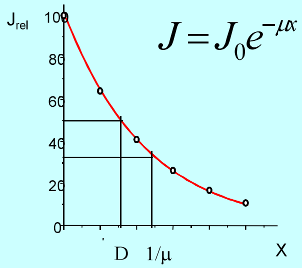
$$J = \frac{1}{Z}\Delta p_{eff}^{\ 2}$$

intensity = energy/current density

$$P_{el} = \frac{1}{Z_{el}} U_{eff}^{\ 2}$$

electric analogy

Loss of energy during propagation (absorption)

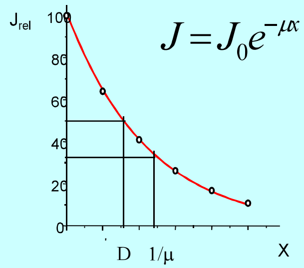


$$J = J_0 e^{-\mu x}$$

attenuation: $\alpha = 10 \cdot \lg \frac{J_0}{J}$ dB

$$\alpha = 10 \cdot \mu \cdot x \cdot \lg e \text{ dB}$$

Loss of energy during propagation (absorption)



$$J = J_0 e^{-\mu x}$$

attenuation: $\alpha = 10 \cdot \lg \frac{J_0}{J}$ dB

$$\alpha = 10 \cdot \mu \cdot x \cdot \lg e \text{ dB}$$

μ is proportional to frequency in the diagnostic range

At f = 1 MHz

D_{air} ~ 1 cm

D_{water} ~ 1 m

Specific attenuation: $\frac{\alpha}{f \cdot x}$

μ is proportional to frequency in the diagnostic range

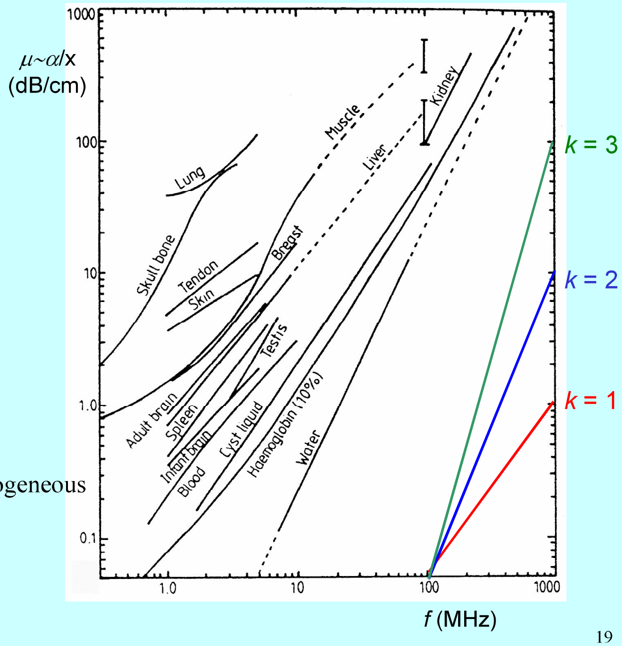
$\mu \sim f^k, \quad k \sim 1(?)$

$\log \mu \sim k \log f$

if the graph is a linear, the power function approximation is valid

specific attenuation for soft tissues (homogeneous tissue model):

$$\frac{\alpha}{f\,x} \sim 1 \frac{\text{dB}}{\text{cm MHz}}$$

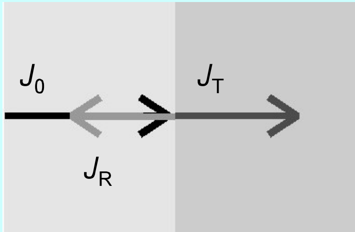


$$\frac{\alpha}{f\,x} \sim 1 \frac{\text{dB}}{\text{cm MHz}}$$

tissue	Specific attenuation
liver	0,6 – 0,9
kidney	0,8 – 1,0
fat	1,0 – 2,0
blood	0,17 – 0,24
bones	16 – 23

Phenomena at the boundary of different media

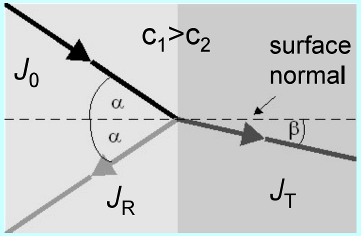
normal/perpendicular incidence



$J_0 = J_R + J_T$

reflection and transmission (penetration)

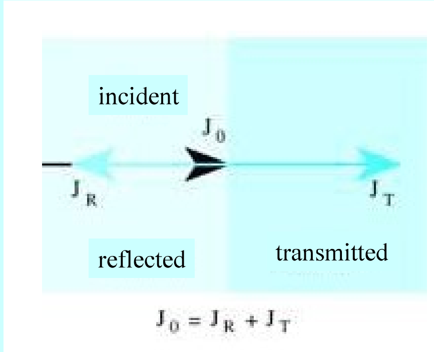
skew incidence



$$\frac{\sin\alpha}{\sin\beta} = \frac{c_1}{c_2}$$

Snellius-Descartes

Reflection of ultrasound



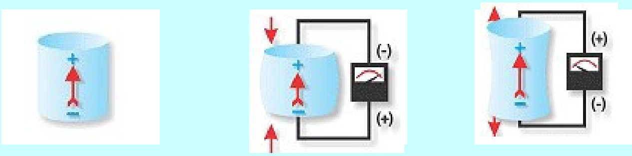
$$R = \frac{J_R}{J_0}$$

$$R = \left(\frac{Z_1 - Z_2}{Z_1 + Z_2}\right)^2$$

If $R \approx 1$ ⟶ Total reflection

Detection/Generation of US

Piezoelectric effect



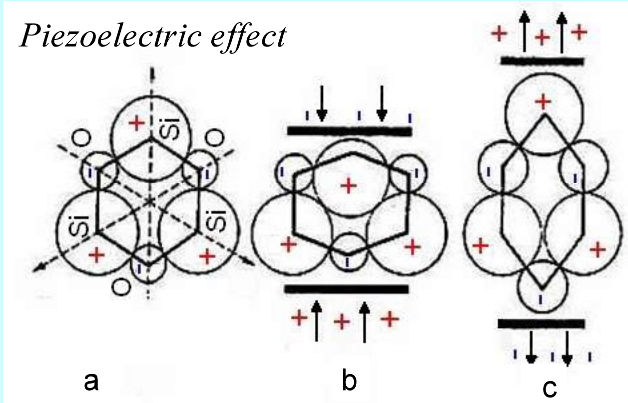
Pressure change

↓

Mechanical deformation of crystal

↓

Electric potential difference

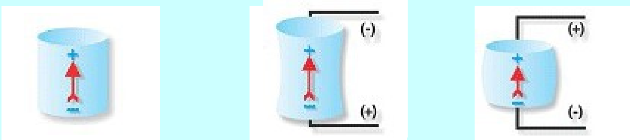


(a) Center of charge of positive and negative charges coincides.

(b) and (c) As a result of pressure , the charge centers are separated, i.e. a potential difference arises

Detection/Generation of US

Inverse piezoelectric effect



Periodic electric potential difference

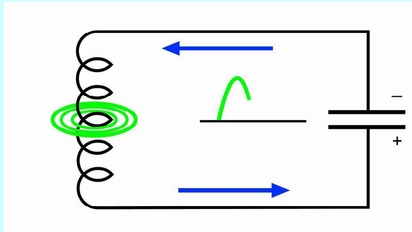
↓

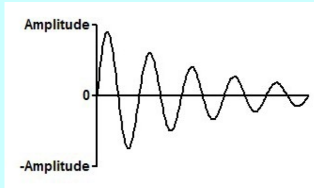
The crystal is deformed when voltage is applied

↓

Mechanical vibration

Oscillator circuit – LC circuit





$$f = \frac{1}{2\pi\sqrt{LC}}$$

L : self inductance $[s \cdot \Omega^{-1}]$

$L \sim A\,N^2$

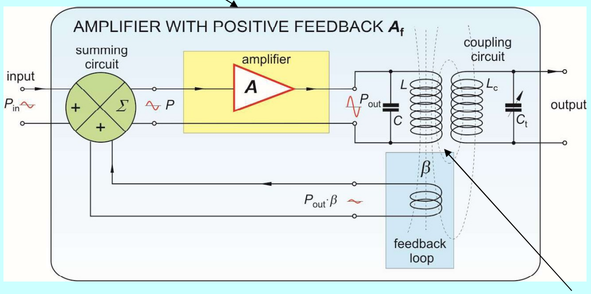
Source of electric signal: **sine wave oscillator**

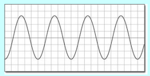
$$A_{feedback} = \frac{U_{out}}{U_{in}} = \frac{A}{1 - A \cdot \beta}$$

$A\beta = 1$

amplification = „infinity“

no input signal, output signal: sine voltage



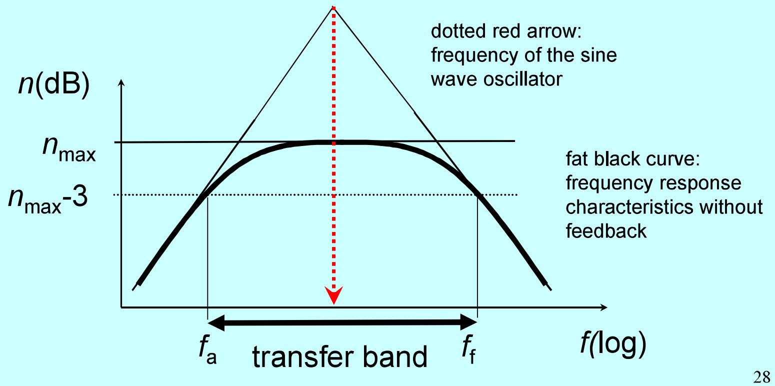


$$f = \frac{1}{2\pi\sqrt{LC}}$$

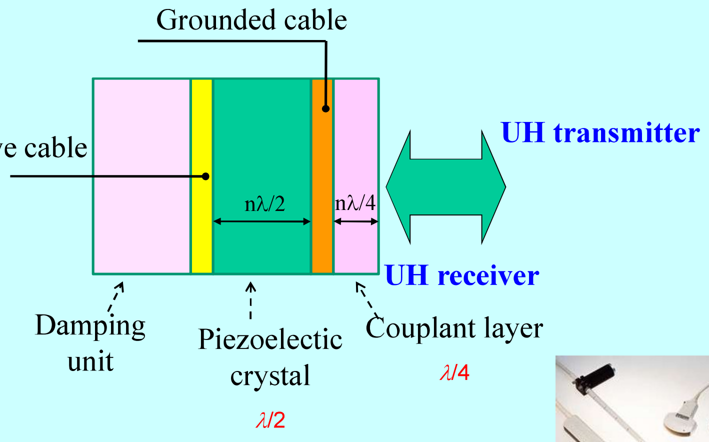
Source of electric signal: **sine wave oscillator**

amplifier with positive feedback

$$A_{U,\,feedback} = \frac{A_U}{1 - \beta A_U}$$



Detection/Generation of US
– Ultrasound transducer



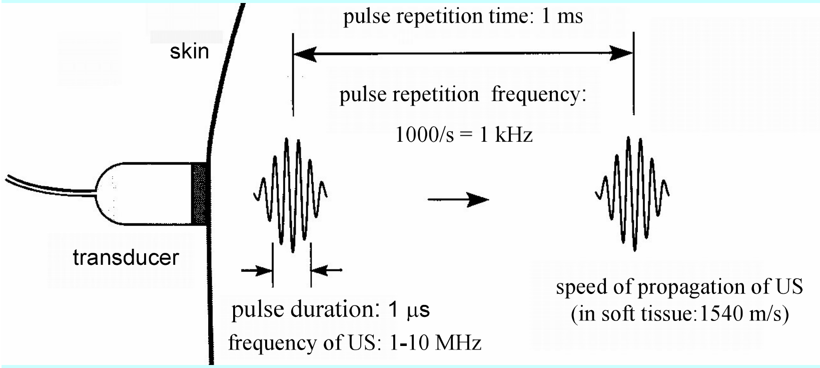
Question of the week

Why is the speed of propagation of US higher in bones than in soft tissues?

Characteristic of US pulses

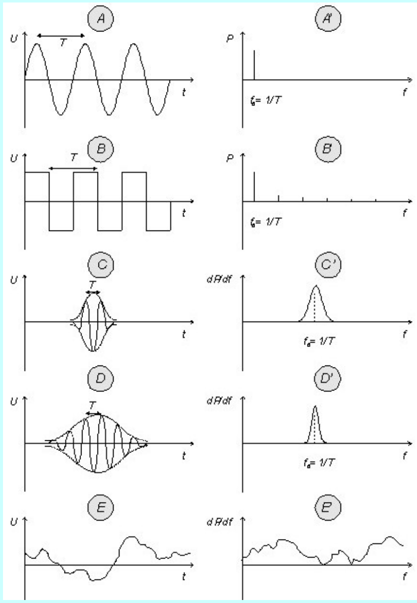
transducer: transmitter and receiver is the same unit

time sharing mode: pulses instead of continuous wave US

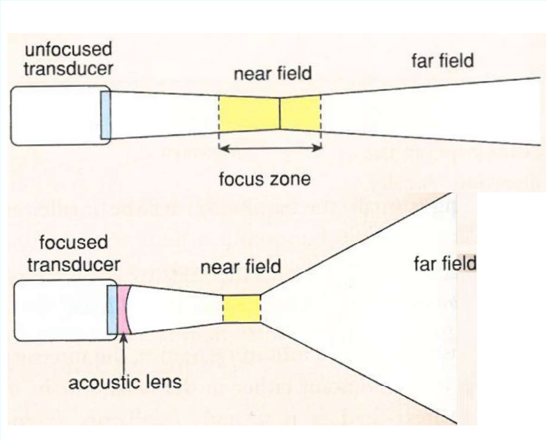


	Time function	Spectrum	
sine function	A	A'	line spectrum (1 line)
square function	B	B'	line spectrum
sine wave pocket (some „periods”)	C	C'	band spectrum
sine wave pocket (several „periods”)	D	D'	band spectrum
aperiodic function	E	E'	continuous spectrum

US pulse



Focusing of the beam



Focusing increases the divergence of the beam in the far field regime and reduces the depth sharpness.