Transport processes

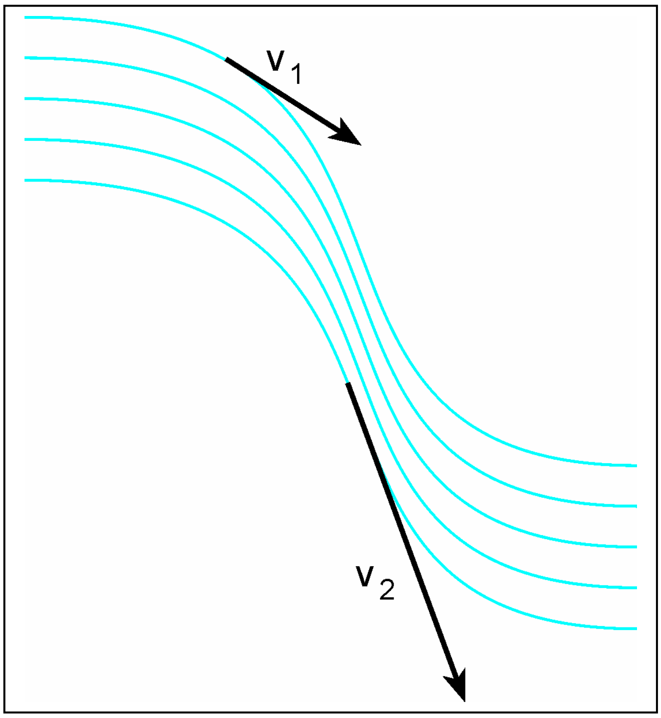
There are two distinct mechanisms.
The first is when particles of the quantity in question are translocated collectively and this is observable macroscopically, such as in a fluid flow. The other mechanism is where the detected macroscopic translocation is the result of individual motions of particles, which are not observable even microscopically. Diffusion is an example of the latter mechanism.

Flow of fluids and gases (blood-circulation, respiration)

Under ordinary pressure, fluids can be assumed to be incompressible (small compressibility κ).

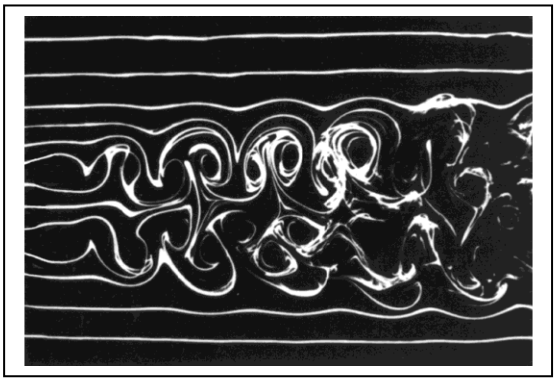
$$\kappa = -\frac{1}{V}\frac{\Delta V}{\Delta p}$$

V (volume)
p (pressure)
v_1, v_2 (velocity)



Gases are naturally compressible, but the pressure differences occuring in the body (for example a few hundred Pascals during breathing) are not normally sufficient to significantly alter gas density. In this sense gases can also be regarded as being essentially incompressible.

For the characterization of fluid flow we can use **streamlines**. At any given point, the tangent of a streamline shows the direction of fluid velocity and the density of streamlines shows the velocity of the flow.



layered flow, or laminar flow and turbulent flow

Flow can be characterized by the volumetric flow-rate:

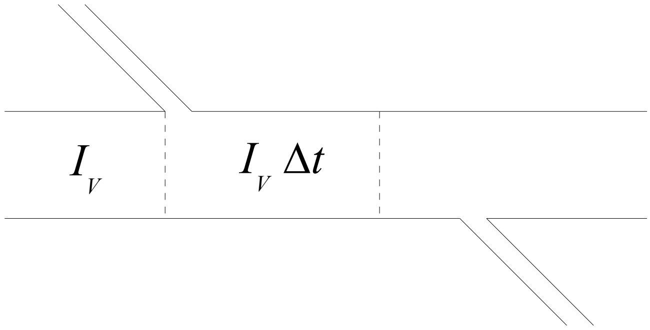
$$I_V = \frac{\Delta V}{\Delta t} \quad (V \text{ volume, } t \text{ time})$$

Methods for measuring the volumetric flow-rate:
ultrasound methods (Doppler-examination):
f frequency
υ velocity of the object
c ultrasound velocity

$$\boxed{f = f_0\left(1 \pm \frac{\upsilon}{c}\right)}$$

dilution techniques: ν amount of dye (mol); c concentration of dye

$$\frac{\Delta \nu}{\Delta t} = M$$

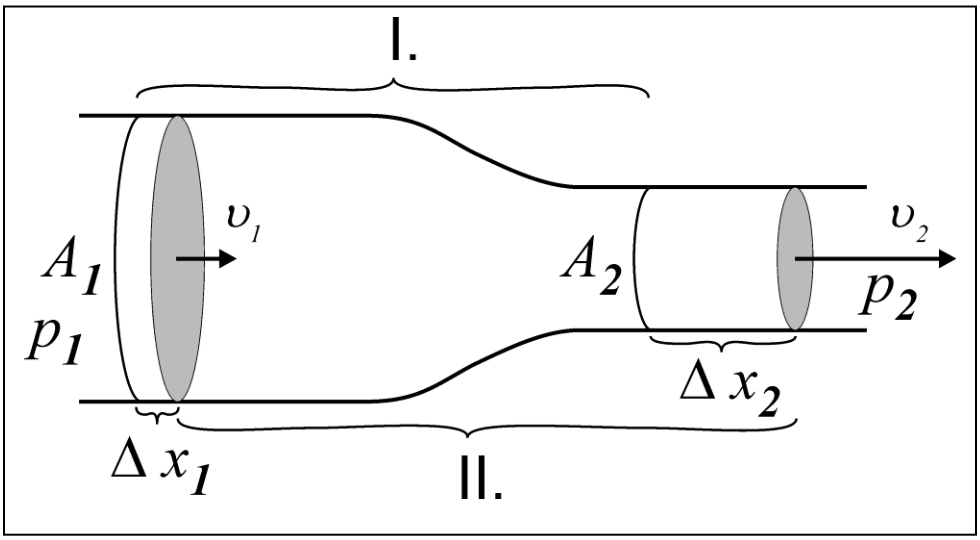


$$c = \frac{\Delta \nu}{\Delta V} = \frac{\Delta \nu}{I_V \Delta t}$$

Law of continuity

$$I_V = \frac{\Delta V}{\Delta t} = \frac{A \upsilon \Delta t}{\Delta t} = A\upsilon = \text{constant}$$

$$I_V = A\bar{\upsilon} = \text{constant}$$



Ideal and real fluids (question of friction)

Bernoulli's law (for ideal and real fluids)

Mechanical energy conservation:

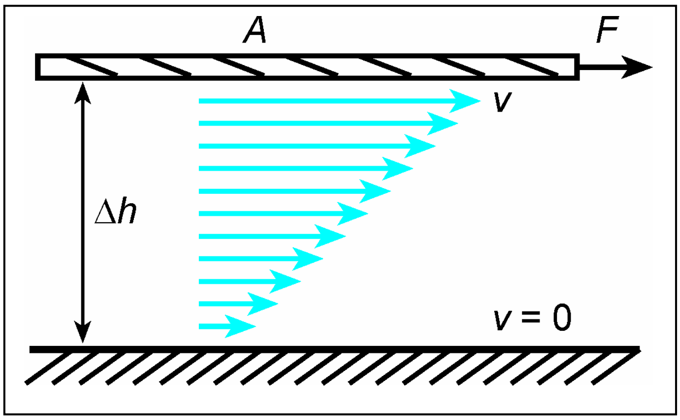
$$W = \Delta E_{\text{kinetic}}$$

$$p_1 + \frac{1}{2}\rho\upsilon_1^2 = p_2 + \frac{1}{2}\rho\upsilon_2^2 = \text{constant}$$

p (pressure)
υ (velocity)
ρ (mass density)

Consequences

Newton's law of friction



Let fluid flow between two close flat plates and examine what force F has to be exerted to slide the upper plate of surface A with constant velocity υ over the fixed lower plate.

$$F = \eta A \frac{\Delta\upsilon}{\Delta h}$$

η is the coefficient of viscosity or internal friction coefficient (unit: Pa·s)

velocity drop per unit length $\Delta\upsilon/\Delta h$ is constant

Fluids obeying the above equation are called **newtonian fluids**.

substance	η (mPa·s) 20 °C
air	(101 kPa) 0.019
water	1
ethanol	1.2
blood (37 °C)	2–8
glycerine	1490
honey	2000–14000

viscosity of gases increases with rising temperature

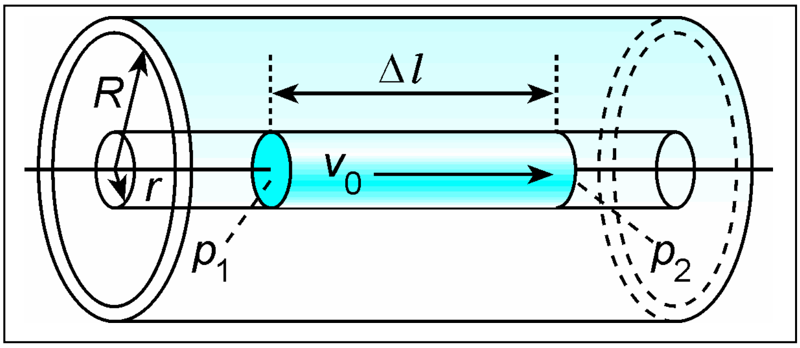
viscosity of fluids decreases with rising temperature

$$\eta \sim e^{\frac{E}{kT}}$$

Fluid flow in a tube

Real fluids show a **linear velocity profile** when flowing between two flat plates very close to each other.

Now, we are going to consider flow in a tube of finite (but not too large) diameter ($2R$).



Let us consider a tube of fluid of radius r, and length Δl within the original tube, at the ends of which the pressure is p_1 and p_2, ($p_1 > p_2$).

$$F = \Delta p r^2 \pi = \eta 2 r \pi \Delta l \frac{\Delta \upsilon}{\Delta r}$$

from where

$$\frac{\Delta \upsilon}{\Delta r} = \frac{\Delta p}{2\eta \Delta l} r = -Kr$$

Differential equation: $$\frac{d\upsilon}{dr} = -Kr$$

Solution: $$\upsilon = \upsilon_0 - \frac{K}{2} r^2$$

$\upsilon = \upsilon_0$ at the middle of the tube ($r = 0$) (but $K > 0$) and
$\upsilon = 0$ at the borders ($r = \pm R$)

The **velocity profile** ($\upsilon(r)$, the flow velocity of the fluid as a function of the radius) is **parabolic.**

$$\upsilon_0 = \frac{1}{2} KR^2$$

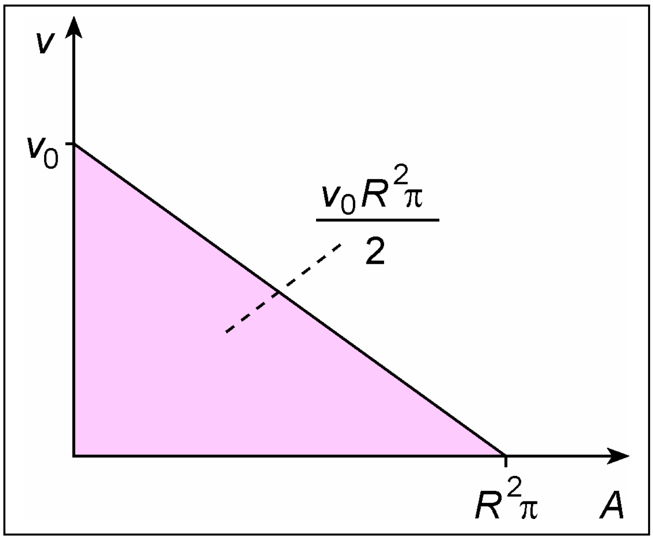
Hagen-Poiseuille law and its application to blood-circulation

The most important question is:
what is the volumetric flow rate in the tube?

$$I_V = A\overline{\upsilon}$$

$$I_V = \frac{\upsilon_0 R^2 \pi}{2} = \frac{1}{4} KR^4 \pi$$

$$I_V = -\frac{1}{8} \frac{\Delta p}{\eta \Delta l} R^4 \pi$$



Hagen-Poiseuille law:
only applies to newtonian fluids undergoing stationary and laminar flow. (The negative sign denotes that fluid flows from the site of higher pressure to the site of lower pressure.)

The volumetric flow rate is directly proportional to the drop of pressure ($\Delta p/\Delta l$) and to the fourth power of the radius.

Blood is a non-newtonian fluid, and **in the arteries** close to the heart, **the flow is not stationary** (away from the heart, stationary flow is a good approximation), **nevertheless the Hagen-Poiseuille law is still a useful approach for investigating blood-circulation**.

Hagen-Poiseuille law's analogy with Ohm's law
(See in the manuel: FLOW)

$$-\Delta p = R_{\text{tube}} I_V \quad , \quad U = RI$$

The potential difference corresponds to the pressure difference, the electric current intensity corresponds to volumetric flow rate, and the resistance is analogous to the frictional resistance (tube-resistance) defined by:

$$R_{\text{tube}} = 8\pi\eta \frac{\Delta l}{(r^2\pi)^2} \quad , \quad R = \rho \frac{\Delta l}{r^2\pi}$$

Moreover, the resistance of tubes connected in serial and parallel has to be calculated as the resultant resistance of resistors in electronics.

Turbulent Flow

Laminar flow **becomes turbulent if it exceeds a certain** critical velocity**.** Observations showed that this critical velocity depends on viscosity (η), fluid density (ρ) and the radius of the tube (r):

$$\upsilon_{\text{crit}} = \mathrm{Re}\frac{\eta}{\rho r}$$

The coefficient Re is the Reynolds-number, a dimensionless constant (Re ≈ 1160 for tubes).

Stokes' law
Spherical body moving in a viscous medium

What frictional force acts on a spherical body moving in a fluid?

$$F_s = 6\pi\eta r\upsilon$$

$(F \sim \upsilon)$
The coefficient that relates the two parameters is introduced in the following way:

$$u = \frac{\upsilon}{F} \quad , \quad u = \frac{1}{6\pi\eta r}$$

u is the mobility of the sphere, which is the value of the velocity resulting from a unity of force.