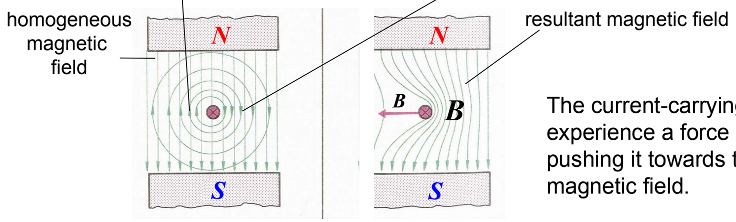
Electromagnetism III

Szabolcs Osváth

Semmelweis University
szabolcs.osvath@eok.sote.hu

Forces in magnetic fields

The magnetic field of the electric current and the static magnetic field superimpose. In some places they weaken, in others they strengthen each other.



The current-carrying wire will experience a force that is pushing it towards the weaker magnetic field.

induction: $F \sim B$

current intensity: $F \sim I$

length of the wire: $F \sim l$

force: $\boldsymbol{F = B \cdot I \cdot l}$

Lorenz force

Lorenz force: is the force acting on a charge Q moving with velocity v in a static magnetic field.

$F = B \cdot I \cdot l$, $I = \frac{Q}{t}$, $\frac{l}{t} = v$

$$F = B \cdot \frac{Q}{t} \cdot l = B \cdot Q \cdot v$$

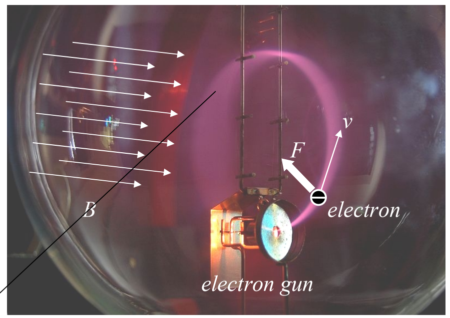
$B \perp v \perp F$

⇩

F will be a centripetal force

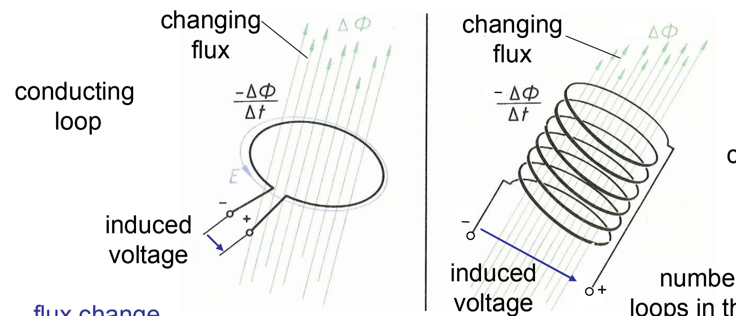
⇩

the charge will move on a circular path



Faraday's law of induction

The electromotive force (voltage) produced around a closed path is proportional to the rate of change of the magnetic flux through the surface bounded by that path.

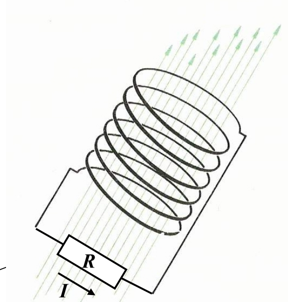


$U_i = -\Delta\Phi/\Delta t$ — flux change; time during which the flux change happened

$U = -N \cdot \Delta\Phi/\Delta t$ — number of loops in the coil

Lenz's law

An induced current is always in such a direction as to oppose the motion or change causing it.
(this is what the ”-” sign means in Faraday's law)



flux generated by the induced current

U induced electron motive force

R resistance, $\boldsymbol{I = U / R}$ induced current

Motional electron motive force

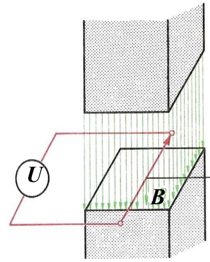
A straight conductor with length l is moved in a homogeneous magnetic field with a velocity v perpendicular to the magnetic induction vector.

$$Q{\cdot}E = B{\cdot}Q{\cdot}v$$

$$E = B{\cdot}v$$

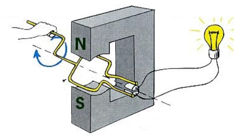
$$U = l{\cdot}E = l{\cdot}B{\cdot}v$$

induced voltage: $\boldsymbol{U = B{\cdot}l{\cdot}v}$



electric generator:

mechanic energy in



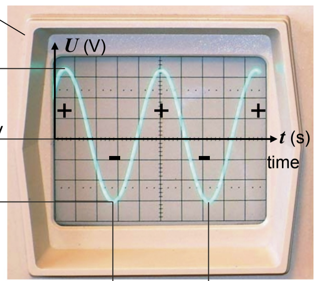
electric energy out

Alternating current (AC)

Parameters describing a sinusoidal alternating voltage:

voltage (V) — $U = U_{peak} \cdot \sin \omega t$ (voltage amplitude (V); angular frequency (rad/s); time (s))

oscilloscope screen



voltage amplitude (V)

period (s)

$\omega = 2\pi f = 2\pi/T$ (frequency (Hz=1/s); period time (s))

$f = 1/T$

peak-to-peak voltage (V) $U_{peak} = U_{peak\text{-}peak}/2$

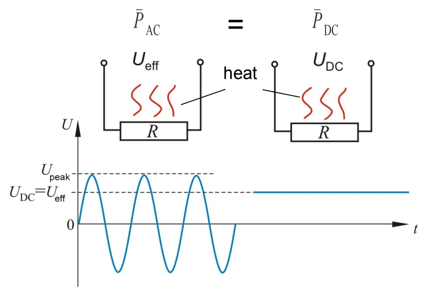
$f = 50$ Hz

e.g. for the voltage of the mains: $T = 1/f = 1/50\text{Hz} = 0.02\text{s} = 20\text{ms}$

Effective value

The effective value of an AC current (voltage) is equivalent to the DC current (voltage) that on average produces the same ammount of heat on a resistor.

For sinusoidal voltage and current:



$$U_{eff} = \frac{U_{peak}}{\sqrt{2}}$$

$U_{eff} = 0{,}707 \cdot U_{peak}$

$U_{peak} = 1{,}41 \cdot U_{eff}$

$$I_{eff} = \frac{I_{peak}}{\sqrt{2}}$$

Effective value of the mains electricity

230V sinusoidal voltage

Effective value is used and not the peak voltage!

$U_{eff} = 230\ V$

$U_{peak} = 1{,}41 \cdot 230\ V = 324\ V$

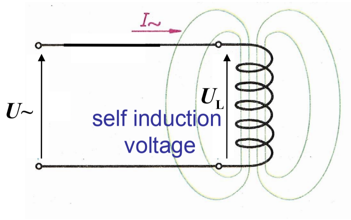
The advantage of using effective value rather than peak value is that the effective values can be directly used for calculating power.

$\bar{P} = U_{eff} \cdot I_{eff}$ but $\bar{P} \neq U_{cs} \cdot I_{cs}$

Self induction

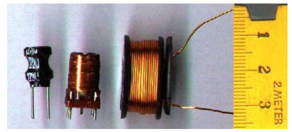
- We connect an AC voltage to a coil.
- A constantly changing magnetic field is genereted around the coil.
- The changing magnetic field induces a voltage in the coil that opposes the change of the external voltage.

self induction voltage $U_L \sim \frac{\Delta I}{\Delta t}$ — current change (ΔI); time during which the current change happenned (Δt)



$U_L = -L \cdot \frac{\Delta I}{\Delta t}$

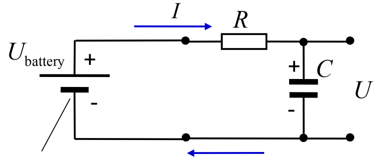
The proportionality faftor L is called self inductance



unit of self incuctance: henry, H

1H is the self inductance of the coil which responds to a change of 1A that happens uniformly during 1s with 1V self induction voltage.

Charging a capacitor through a resistor

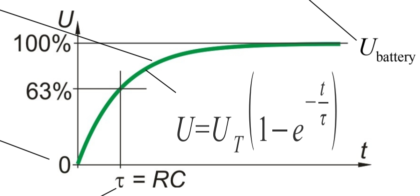


DC voltage generator (e.g. battery)

After long time the capacitor will be charged to the voltage of the battery: $U = U_{battery}$

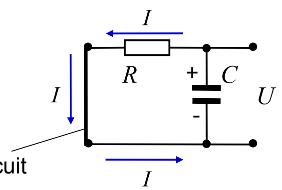
(Equilibrium is reachend and no current will flows.)

exponencial charging



At t=0 the capacitor is uncharged ($U = 0$). (In this moment the charging current is maximal: $I = U / R$.)

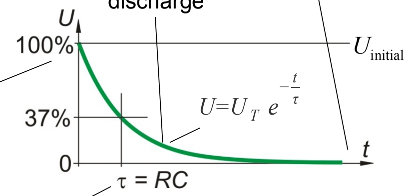
Discharging a capacitor through a resistor



short circuit ($R = 0$)

Aftre a long time the capacitor loses the stored charge, the voltage on it becomes $U = 0$, and the discharge current stops.

exponential discharge



At the first moment of the discharge the voltage on the capacitor is $U = U_{initial}$, the current is maxiamal $I = U_T / R$.

Capacitive reactance

Reactance is the opposition of a circuit element to a change of current, caused by the build-up of electric or magnetic fields in the element.

symbol: X_C unit: ohm, Ω $X_C = U / I_C$

Although capacitive reactance is analogous to the resistance, no heat is produced on an ideal capacitor because $R = \infty$, and $P = U^2/R = 0$.

$X_C \sim 1/f$ $X_C \sim 1/C$ $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f \cdot C}$

If $f = 0$ (DC case), $X_C = \infty$ ⟶ The capacitor can be substituted with a breakage.

If $f = \infty$, $X_C = 0$ ⟶ The capacitor is equivalent with a shortcircuit.

Inductive reactance

symbol: X_C unit: ohm, Ω $X_L = U / I_L$

Although inductive reactance is analogous to the resistance, no heat is produced on an ideal coil because $R = 0$, and $P = I^2 R = 0$.

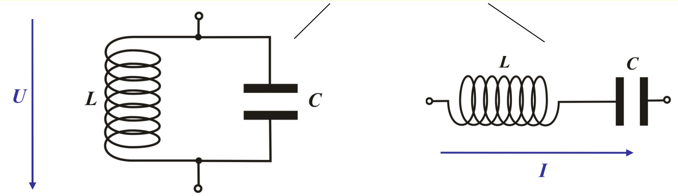
$X_L \sim f$ $X_L \sim L$ $X_L = \omega \cdot L = 2\pi f \cdot L$

If $f = 0$ (DC case), $X_L = 0$ ⟶ The coil can be substituted with a shortcircuit.

If $f = \infty$, $X_L = \infty$ ⟶ The capacitor is equivalent with a breakage.

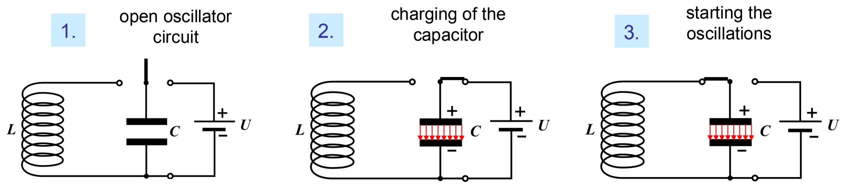
Oscillator circuit

The oscillator circuit consists of a coil and capacitor connected parallel or in series.



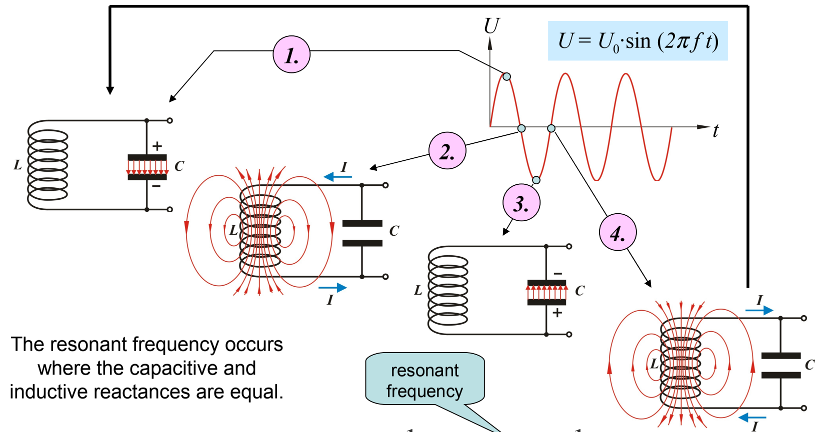
To start the oscillation we provide energy in the capacitor or in the coil.

The example below shows the start of a parallel oscallator circuit.



Oscillator circuit

The current and the voltage of a released ideal oscillator circuit will follow sinusoidal oscillations.



The resonant frequency occurs where the capacitive and inductive reactances are equal.

resonant frequency

$X_L = X_C \longrightarrow 2\pi f L = \frac{1}{2\pi f C} \longrightarrow f = \frac{1}{2\pi\sqrt{LC}}$