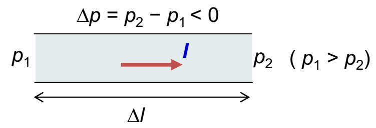
Biophysik für Pharmazeuten II.

16. 04. 2018.

Transportprozesse 2.

Strömungen, Diffusion, Wärmeleitung

- **Transportgesetz** (Hagen-Poiseuille-Gesetz):



G. H. L. Hagen 1797-1884 Wasseringenieur

J. L. M. Poiseuille 1799-1869 Physiologe

Bedingungen:
- inkompressible Fl.
- laminare Str.
- stationäre Str.
- newtonsche Fl.

$$\frac{\Delta V}{\Delta t} = -\frac{\pi}{8}\frac{1}{\eta}R^4\frac{\Delta p}{\Delta l}$$

Volumenstromstärke; Radius des Rohres; Viskosität; Druckgradient

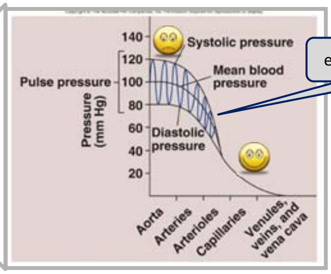
Alternativform: $$\frac{\Delta V}{A \cdot \Delta t} = -\frac{R^2}{8\eta}\frac{\Delta p}{\Delta l}$$

Volumenstromdichte; „Strömungsleitfähigkeit"

Ist das H-P-Gesetz anwendbar für die Blutströmung?

Gültigkeitsbedingungen?

- inkompresible Fl.?
- laminare Strömung?
- stationäre Strömung?
- newtonsche Fl.?

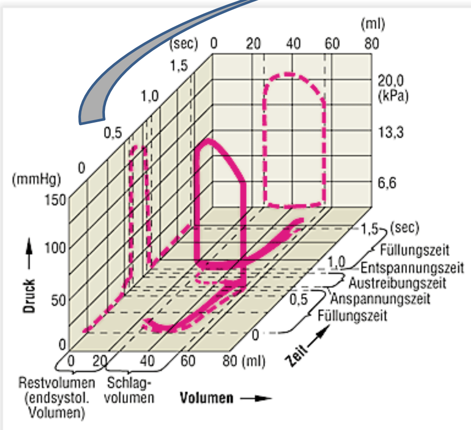


elastische Aderwände

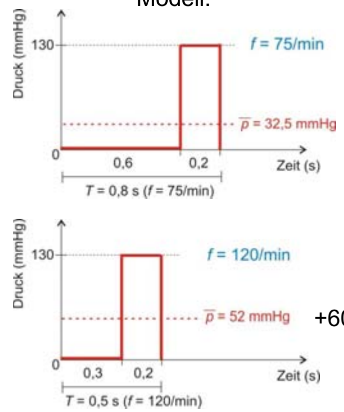
Folgerung: H-P nur qualitativ anwendbar!

Blutströmung

- Regulation der Volumenstromstärke laut Hagen-Poiseuille-Gesetzes:
  - **Druck**

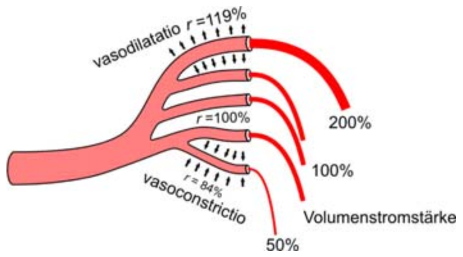


Modell:



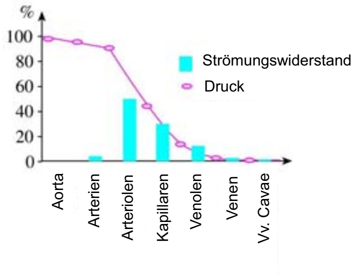
+60%!

- **Radius** (r^4 !)

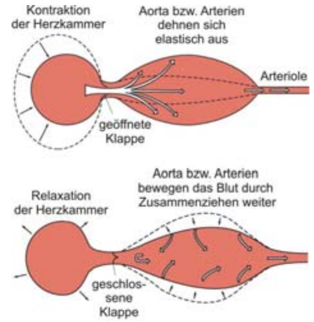


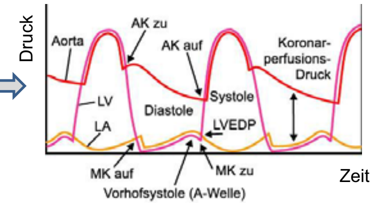
Elastizität!

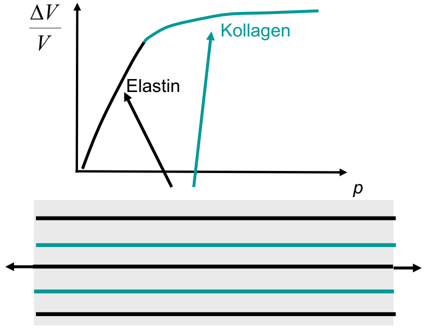
- Druck und Strömungswiderstand im Kreislauf:

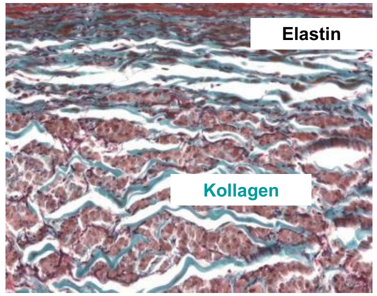


- Rolle der Elastizität von Aorta und Arterien (Windkesselfunktion):

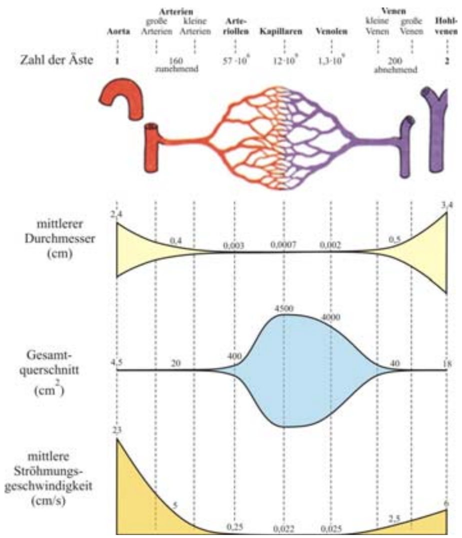


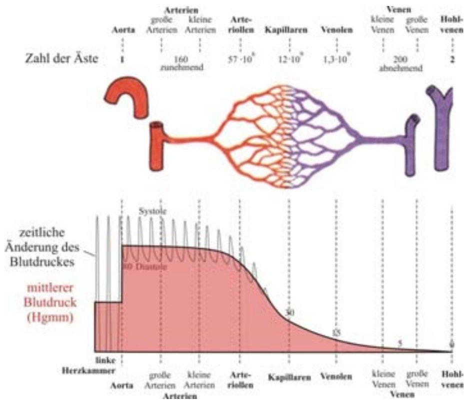






Zusammenfassend:



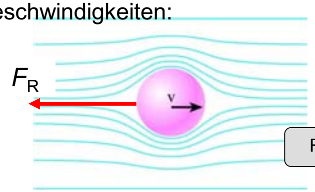


Analogie

	Was strömt?	Stärke?	Was treibt die Strömung?		Zusammenhang?
Ladungs-transport	q	$J_q = \frac{\Delta q}{A \cdot \Delta t}$	φ	$-\frac{\Delta\varphi}{\Delta l}$	$J_q = -\sigma \frac{\Delta\varphi}{\Delta l}$
Volumen-transport	V	$J_V = \frac{\Delta V}{A \cdot \Delta t}$	p	$-\frac{\Delta p}{\Delta l}$	$J_V = -\frac{R^2}{8\eta}\frac{\Delta p}{\Delta l}$

4. Bewegung von Teilchen in reellen Flüssigkeiten

Bei kleineren Geschwindigkeiten:



stokessches Reibungsgesetz:

Reibungskraft

Radius des Teilchens

$$F_R = 6\pi\eta r v$$

Viskosität

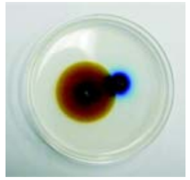
Geschwindigkeit des Teilchens

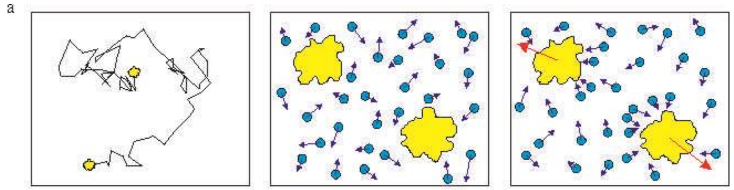
G. G. Stokes
1819-1903
Mathematiker
Physiker

Bei gleichmäßigen Bewegung: $F_{\text{Bewegung}} = F_R$

Beweglichkeit (u) eines Teilchens: $u = \dfrac{v}{F_{\text{Bewegung}}} \Rightarrow \quad u = \dfrac{1}{6\pi\eta r} \Longrightarrow$ s. Diffusion

III. Stofftransport (Diffusion)





1. Grundbegriffe

- Diffusion: Tendenz zur gleichmäßigen Verteilung von Molekülen durch die thermische Bewegung

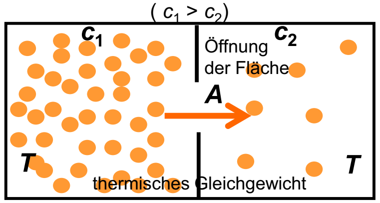
- Stoffstromstärke (I): $I = \dfrac{\Delta \nu}{\Delta t} \quad \left(\dfrac{\text{mol}}{\text{s}}\right)$
- Stoffstromdichte (J): $J = \dfrac{\Delta \nu}{A \cdot \Delta t} \quad \left(\dfrac{\text{mol}}{\text{s} \cdot \text{m}^2}\right)$
- stationäre Diffusion: zeitlich konstant

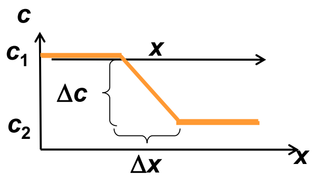
Adolf Fick
1829-1901
Physiologe

2. Transportgesetz — 1. ficksches Gesetz

$\Delta c = c_2 - c_1 < 0$

$(c_1 > c_2)$





$$\frac{\Delta \nu}{\Delta t} = -DA\frac{\Delta c}{\Delta x}$$

Konzentrationsgradient

$$\boxed{J = -D\frac{\Delta c}{\Delta x}}$$

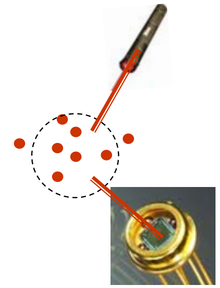
Stromdichte

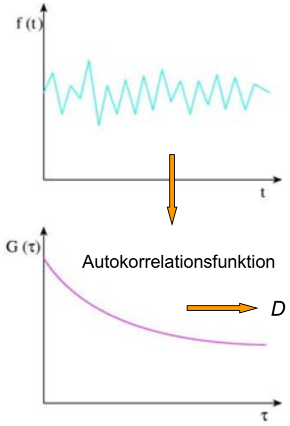
Diffusionskoeffizient

- **Diffusionskoeffizient:**
  - ➢ stoffspezifisch
    - — diffundierende Moleküle — Größe (r)
    - — Form
    - — Medium (η)
  - ➢ temperaturabhängig $\quad D \sim e^{-\frac{\Delta E}{RT}}$
  - ➢ $D = ukT$ (Beweglichkeit des Teilchens)
  - ➢ **Einstein-Stokes-Gleichung (für kugelförmige Teilchen)** $\quad D = \dfrac{kT}{6\pi\eta r}$

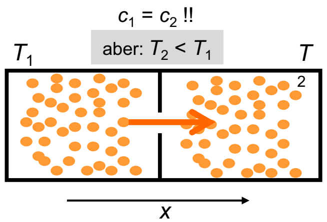
Diffundierendes Teilchen (Molmasse)	Medium	D (m²/s)
H_2 (2)	Luft	$6{,}4\cdot10^{-5}$
O_2 (32)	Luft	$2\cdot10^{-5}$
CO_2 (44)	Luft	$1{,}8\cdot10^{-5}$
H_2O (18)	Wasser	$2{,}2\cdot10^{-9}$
O_2 (32)	Wasser	$1{,}9\cdot10^{-9}$
Glyzin (75)	Wasser	$0{,}9\cdot10^{-9}$
Serum Albumin (69 000)	Wasser	$6\cdot10^{-11}$
Tropomiozin (93 000)	Wasser	$2{,}2\cdot10^{-11}$
Tabakmosaik-virus (40 000 000)	Wasser	$4{,}6\cdot10^{-12}$

- Messung:
  eine Möglichkeit – dynamische Lichtstreuungsmessung





- Im thermischen Nichtgleichgewicht:



Konzentration (c) $\Rightarrow$ chemisches Potenzial (μ)

$$\mu = \mu_0 + RT \ln \frac{c}{c_0}$$

Die Triebkraft der Diffusion ist: $-\dfrac{\Delta\mu}{\Delta x}$

3. Das 2. ficksche Gesetz:

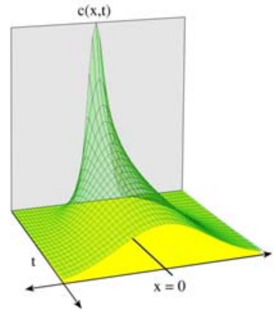
$$D\frac{\Delta\left(\frac{\Delta c}{\Delta x}\right)}{\Delta x} = \frac{\Delta c}{\Delta t} \qquad D\frac{\partial^2 c}{\partial x^2} = \frac{\partial c}{\partial t}$$

Lösungen:

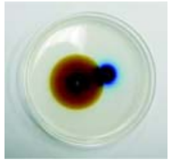
- Für eindimensionale Diffusion:

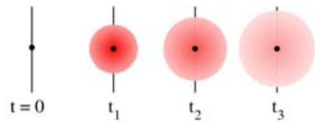
$$c(x) = \frac{c_0 \Delta x}{\sqrt{2\pi\sigma_x^{\,2}}}\, e^{-\frac{x^2}{2\sigma_x^{\,2}}}$$

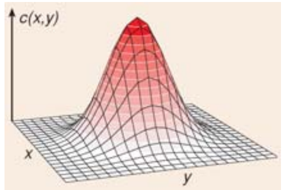
$$\sigma_x = \sqrt{2Dt}$$



- Für zweidimensionale Diffusion:

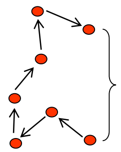




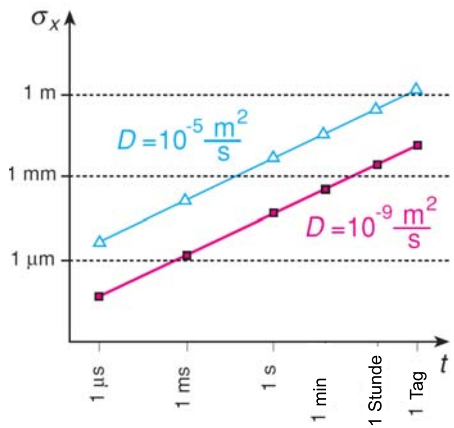


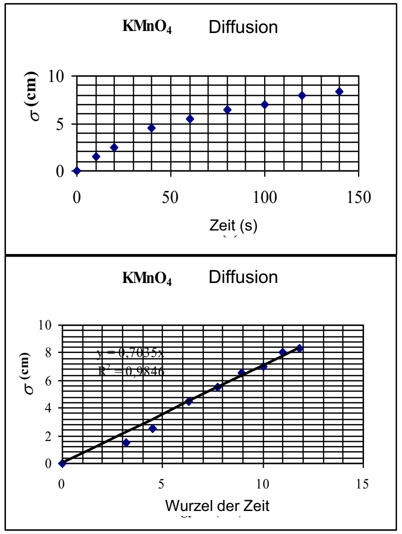
Siehe auch Praktikum!

4. Diffusion als Random Walk



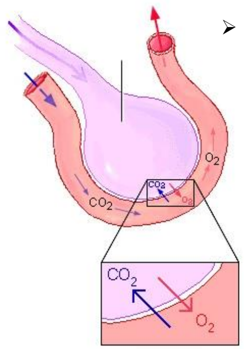
$\sigma = ?$ $\quad \boxed{\sigma \approx \sqrt{2D \cdot t}}$



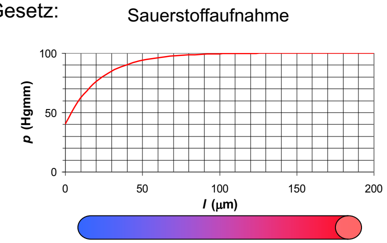


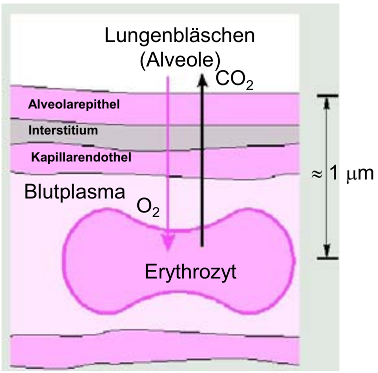
5. Anwendungen

- O_2/CO_2-Diffusion Lunge-Blut



- 1. ficksches Gesetz:



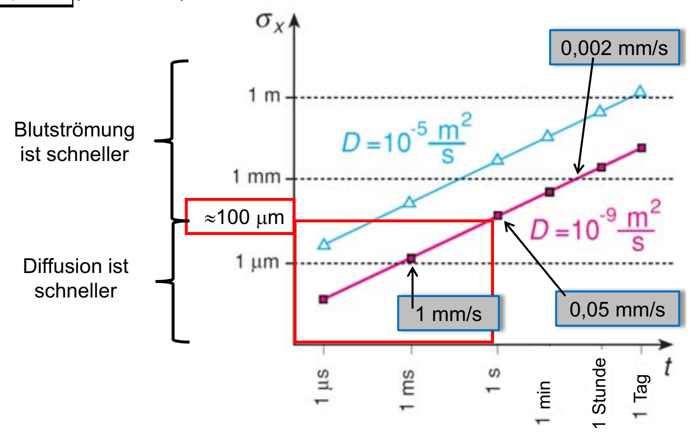


- Random Walk:
  1 µm $\Rightarrow$ 1 ms

- Welche ist „schneller” für O_2-Transport im Gewebe? Diffusion ↔ Blutströmung

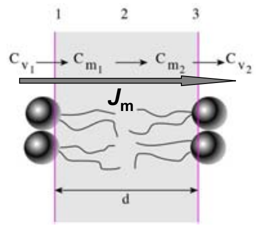
Gefäß	Kapillaren
A (cm²)	4500
v (cm/s)	0,022

(= 0,22 mm/s)



- Diffusion durch eine Membran (passiver Transport)

Für neutrale Teilchen:



$$J_m = -D \cdot \frac{\Delta c}{\Delta x} = -D \cdot \frac{c_{m2} - c_{m1}}{d} = -p(c_{v2} - c_{v1})$$

$$J_m = -p(c_{v2} - c_{v1})$$

Permeabilitätskoeffizient (m/s)