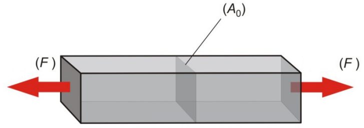
Biomechanics
Biomolecular and tissue mechanics

Zsolt Mártonfalvi

Physical bases of biomechnaics

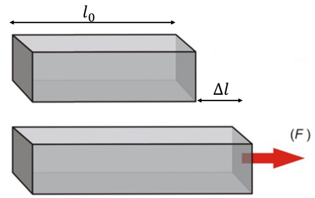
Stress

$\sigma = \dfrac{F}{A_0}$ $\left[\dfrac{N}{m^2} = Pa\right]$



Strain (deformation)

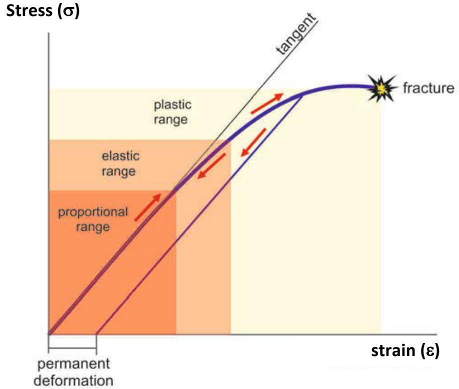
$\varepsilon = \dfrac{\Delta l}{l_0}$ $\left[\dfrac{m}{m}\right]$ *no dimension*



Strain is proportional to stress!

$\sigma \backsim \varepsilon$

Stress-strain diagram



Elastic range
Range of reversible deformation. Unloaded length (l_0) recovers when released. Hysteresis may occour.

Proportional range
(part of elastic range)
Deformation is linearly proportional to the load. No hysteresis.

Plastic range
After a critical stress value, object undergoes irreversible change of its structure. Unloaded length (l_0) does not recover. Permanent deformation of object.

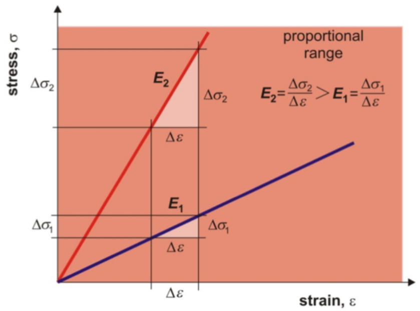
Hooke's law

$\sigma = E\varepsilon$

$\dfrac{F}{A_0} = E\dfrac{\Delta l}{l_0}$ Hooke's law

$F = \dfrac{EA_0}{l_0}\Delta l$

$F = D\Delta l$



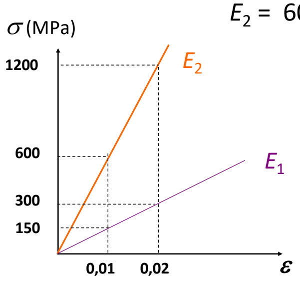
Young's modulus
(material stiffness)

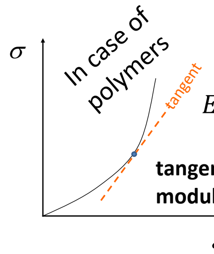
$E = \dfrac{\sigma}{\varepsilon} = \dfrac{F}{A_0}\dfrac{l_0}{\Delta l}$ $E = \left[\dfrac{N}{m^2} = Pa\right]$

Spring constant
(body stiffness)

$k = \dfrac{F}{\Delta l}$ $k = \left[\dfrac{N}{m}\right]$

Two examples: E_1 = 15 GPa
E_2 = 60 GPa





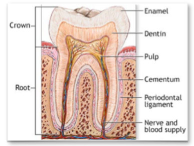
$$E = \frac{\Delta\sigma}{\Delta\varepsilon}$$

tangential modulus

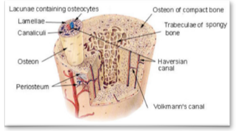
5

Hard tissues

Tooth



Bone

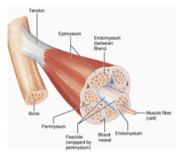


Composite of collagen (organic) and apatite (inorganic).

Organic component provides: toughness
Inorganic component provides: strength

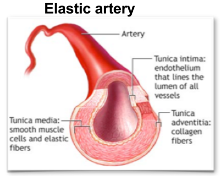
Soft tissues

Muscle



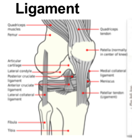
titin, desmin, collagen

Elastic artery



Collagen and elastin

Ligament

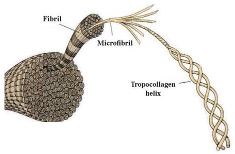


Tendon

Cartilage

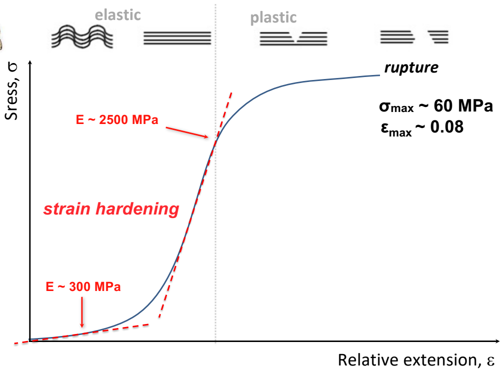
Collagenn, proteoglycans (water)

Collagen



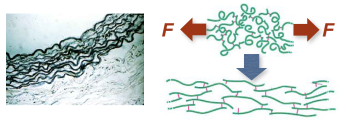
1400 amino acids/chain
3 chains (triple helix)
Glicine (1/3)
Proline (1/10)
Hidroxiproline

Posttranslational modification is required for mechanical stability: Prolin-hidroxilase and vitamin-C

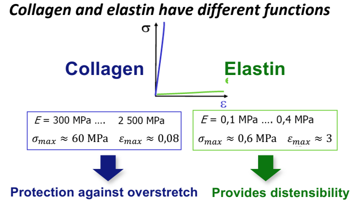


Biomechanics of elastic arteries

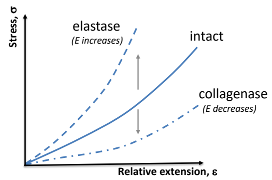
Elastin – elastic protein network



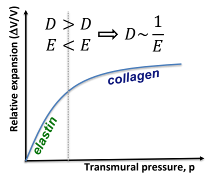
Collagen and elastin have different functions



Effect of proteases on the mechanics of vessel wall



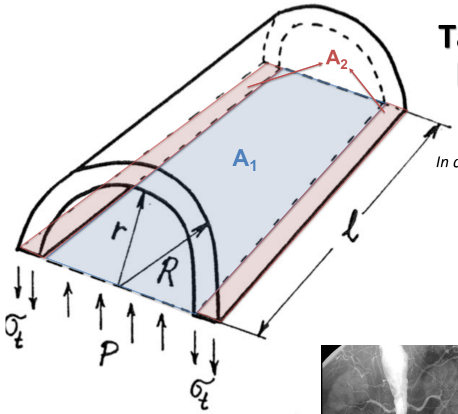
Aorta expansion



Distensibility
The change in vessel volume under pressure

$$D = \frac{\Delta V}{\Delta p \cdot V_0}$$

Tangential stress of blood vessel wall



In case of equilibrium, the forces acting on the two surfaces are equal

$$F_{A_1} = F_{A_2}$$

$$p \cdot A_1 = \sigma_t \cdot A_2$$

$$p \cdot 2r \cdot l = \sigma_t \cdot 2(R - r) \cdot l$$

p: transmural pressure
σ_t: tangential stress
r: inner radius of blood vessel
R: outer radius of blood vessel
R-r: wall thickness

$$\sigma_t = \frac{r}{R-r} \cdot p$$

Laplace-Frank equation

Significance: High blood pressure, aneurism

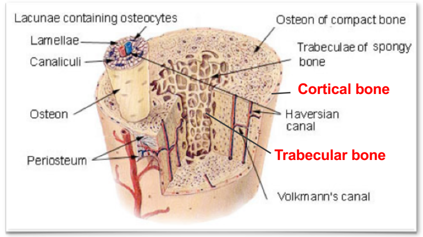
Bone

Due to the different structure of bone tissue along the cross section of long bones, the **Young's modulus distribution is anisotropic**. Denser cortical bone has greater Young's modulus vs. the trabecular bone..

Young's-modulus: 5-20 GPa
Decalcified bone (acid treatment): flexible
Removal of organic compounds (heating): brittle

30 % Organic matrix: collagen

70 % Inorganic salt: Hydroxylapatite



Bone has an average Young's modulus of 18 GPa. Under compression, it can withstand a stress of about 1.6x10[8] Pa before breaking. Assume that a femur (thigh-bone) is 46 cm long, and calculate the amount of compression this bone can withstand before breaking.

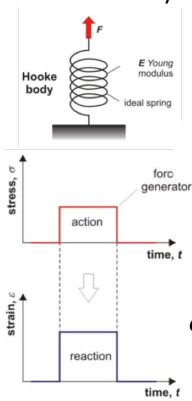
$$\sigma = E\frac{\Delta l}{l} \rightarrow \Delta l = \frac{\sigma}{E} l$$

$$\Delta l = \frac{1{,}6 \cdot 10^8\ Pa}{18 \cdot 10^9\ Pa} \cdot 46\ cm \cong 0{,}4\ cm$$

Viscoelasticity

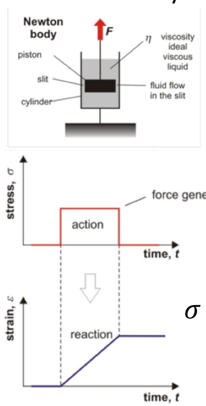
(mechanical model)

Elastic body



$$\sigma = E\varepsilon$$

Viscous body



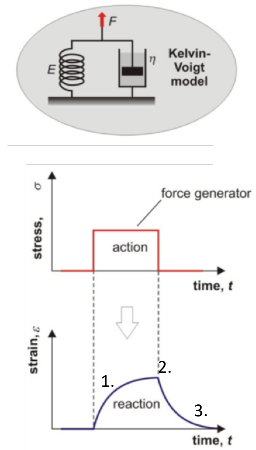
viscosity

$$\sigma = \eta\frac{\Delta\varepsilon}{\Delta t}$$

Strain rate

Viscoelasticity

(mechanical model)



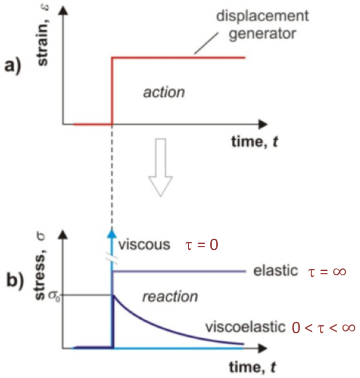
Viscoelasticity means the co-appearance of viscous and elastic behavior.
model: parallel connection of spring and dashpot (Kelvin-Voight model)

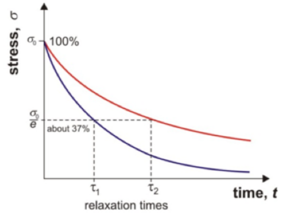
Spring: ideal elastic (Hooke) body
Dashpot: ideal viscous (Newton) body

1. Upon stretch, the extension of the spring is slowed down by the dashpot.
2. Extension stops when the elastic spring force equals the external force.
3. When the external force is quenched, the contraction of the spring is slowed down by the dashpot.

Stress-relaxation in viscoelastic system

Decrease in stress while strain remains constant



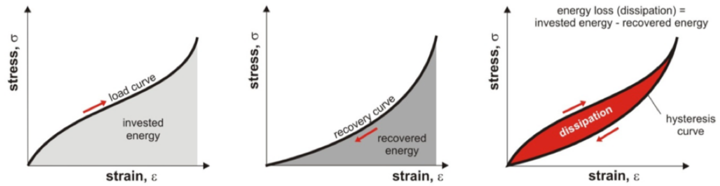


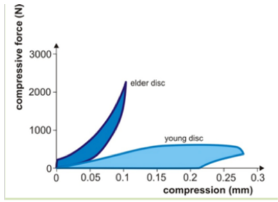
$$\sigma = \sigma_0 \cdot e^{-\frac{t}{\tau}}$$

Stretching

Energy dissipation in viscoelastic system

(hysteresis)

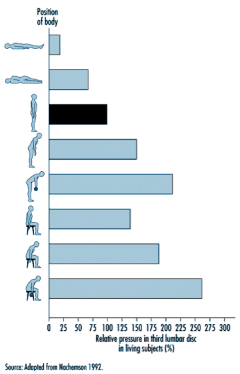


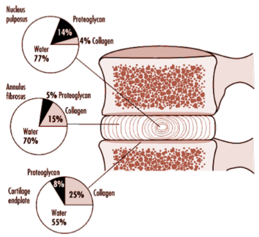


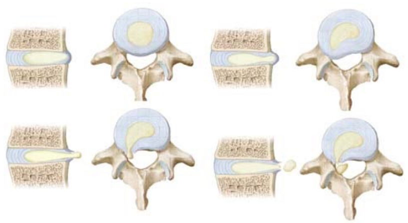
Example I: Consequence of mechanical stress on intervertebral discs

(*discus hernia*)

Stress on the L3 disc at various positions

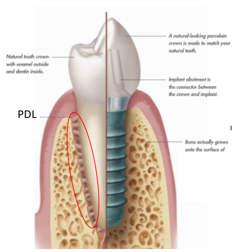


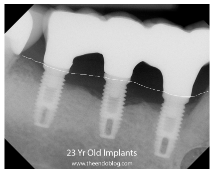




Example II: Implant vs. Natural tooth?

The difference is the periodontal ligament (PDL)!





Absence of PDL results in:
loss of masticatory force perception
loss of viscoelastic dampening
loss of force sensory mechanism
no implant movement

Implant is in direct contact with bone tissue

⬇

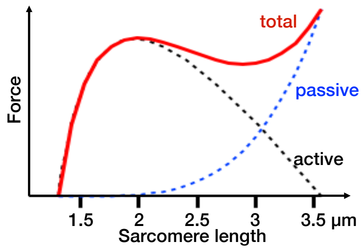
Increased compressive stress during mastication

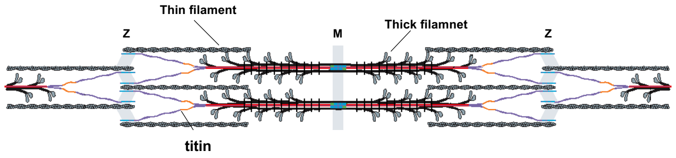
⬇

Bone loss (0.2 mm /year)
Loss of gingival height

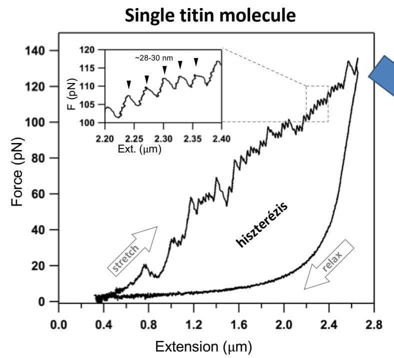
Implants ⬄ **Root canal treatment**

Titin: the elastic filament of the sarcomere

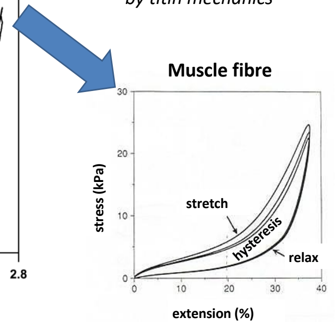




Titin is the main determinant of muscle's passive elasticity



From molecule to tissue
Muscle mechanics can be explained by titin mechanics



Calculations (required for the final exam!)

To stretch a relaxed biceps muscle 3 cm requires a force of 25 N. To do the same stretch of a contracted muscle at its maximal tension requires a force of 500 N. Find the Young's modulus for both relaxed and tense muscle tissue. Assume the biceps is a uniform cylinder of length 20 cm and diameter 6 cm. (59 kPa, 1.18 MPa)

Collagen fiber is stressed with 12 N force. The cross-sectional area of the fiber is 3 mm2, its Young's modulus is 500 MPa. Give the percentage of relative extension. (0.8 %)

The length of an elastic thread used in orthodontics is 6 cm, its cross-sectional area is 1 mm2, its Young's modulus is 5 MPa. We extend the thread with 40 %. How large is the retracting force and what is the amount of elastic energy stored in the thread?(2 N, 24 mJ)

Bone has an average Young's modulus of 18 GPa. Under compression, it can withstand a stress of about 2.7×10^8 Pa before breaking. Assume that a femur (thigh-bone) is 46 cm long, and calculate the amount of compression this bone can withstand before breaking. (6.9 mm)