

Mathematical and Physical Basis of Medical Biophysics

Lecture 1

Mathematics Necessary for Understanding Physics
Physical Quantities and Units

10th September 2018

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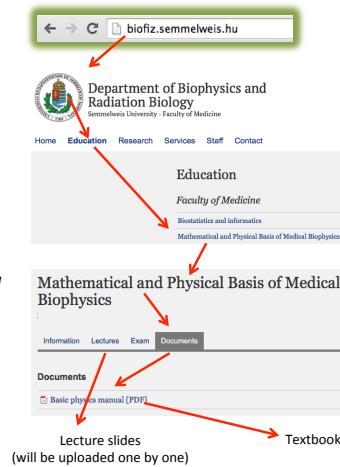
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How to Get Prepared?

- university = autonomous learning

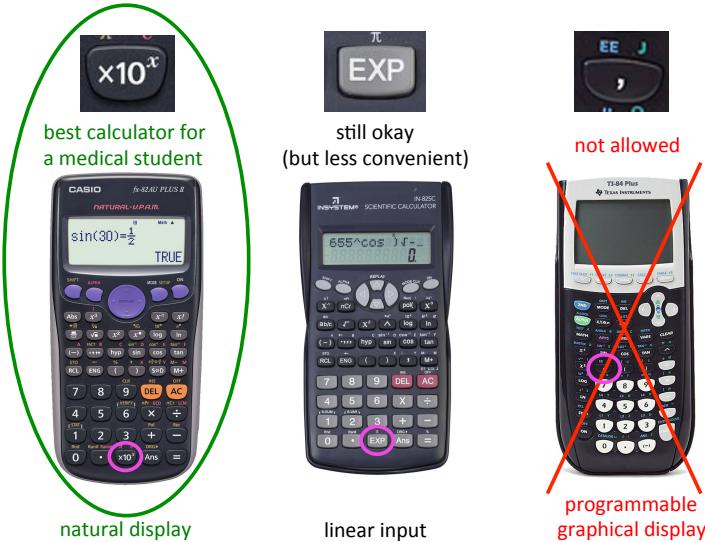
- sources:

- your notes made in the lectures (*Monday 19³⁰–20⁵⁰; Friday 16¹⁰–17³⁰; EOK "Szent-Györgyi Albert" lecture hall; only in the first four weeks*)
- Tölgyszi: *Mathematical and Physical Basis of Medical Biophysics* (2016)
- homepage: biofiz.semmelweis.hu
 - subject requirements
 - lecture schedule and slides
 - textbook



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How to Use Scientific Notation?

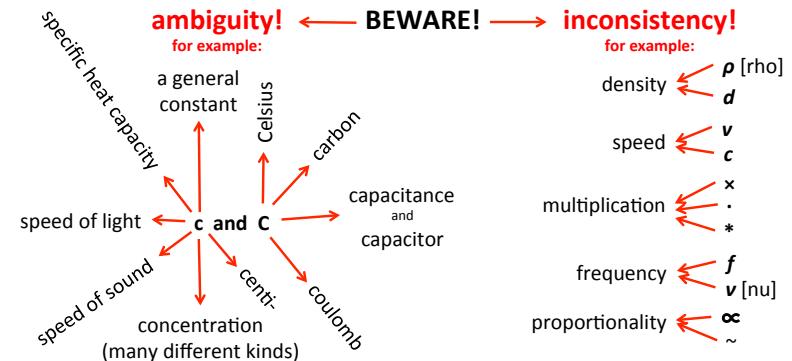


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Use of Symbols in Science

In science we use a large number of Latin and Greek letters (and their combinations) as symbols, so it is inevitable to learn the Greek alphabet.

However, the number of quantities and units is much greater than the number of available letters, and this can lead to confusion. Your help: CONTEXT

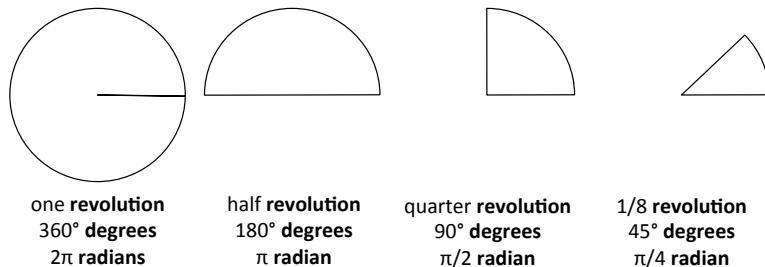
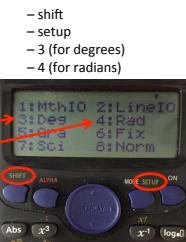


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Angles

D: degrees mode
R: radians mode

revolution: one turn
degree: practical, traditional unit
radian: scientific unit, arc/radius
1 revolution = $360^\circ = 2\pi$ rad
 $1^\circ = 60' = 3600''$



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What is a Function?

Unambiguous assignment of one set of values to another set of values

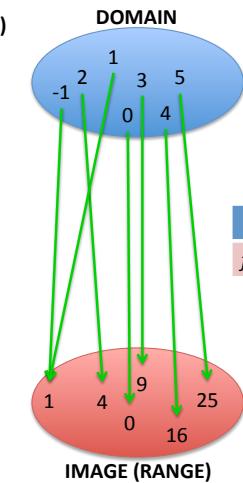
INPUT (ARGUMENT, INDEPENDENT VARIABLE)

x
-1 1 3 5
2 0 4

function as a "machine"

1 4 0 9 25
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OUTPUT (VALUE, DEPENDENT VARIABLE)
 $f(x)$ or y



$x \mapsto f(x)$ or $y = f(x)$

x	-1	0	1	2	3	4	5
$f(x)$	1	0	1	4	9	16	25

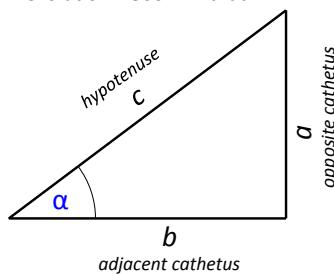
$x \mapsto f(x)$ or $y = f(x)$

f is the function defining the relationship between x and $f(x)$

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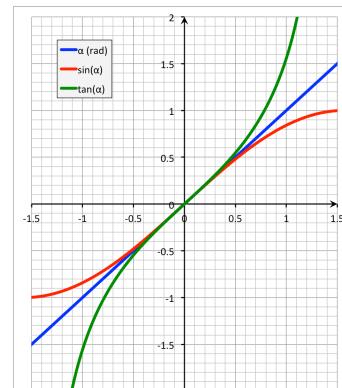
Trigonometric Functions

degree: practical, traditional unit
radian: scientific unit, arc/radius
1 revolution = $360^\circ = 2\pi$ rad



sine: $\sin(\alpha) = a/c$
cosine: $\cos(\alpha) = b/c$
tangent: $\tan(\alpha) = \operatorname{tg}(\alpha) = a/b$

for small angles ($< 10^\circ \approx 0.2$ rad):
 $\sin(\alpha) \approx \alpha$ [rad] $\approx \tan(\alpha)$



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Linear Function

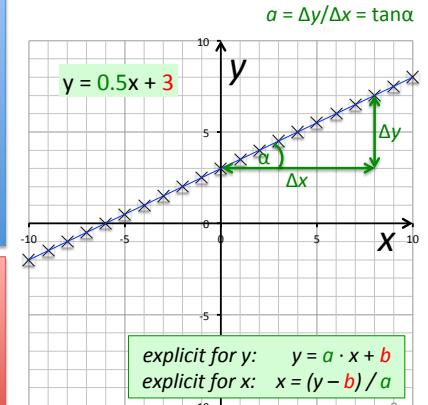
INTEGRAL FORM
VARIABLES: dependent variable
independent variable
 $y = a \cdot x + b$
PARAMETERS: slope (gradient, increment)
y-axis intercept

"DIFFERENTIAL" FORM

$\Delta y \propto \Delta x$

The change of the dependent variable is proportional to the change of the independent variable

if $x = 0$
then $y = b$
if $\Delta x = 1$
then $\Delta y = a$



explicit for y : $y = a \cdot x + b$
explicit for x : $x = (y - b) / a$

Linear Function: Some Examples from the Biophysics Formula Collection

#1: The ideal gas law
(I.35)
 $pV = nRT$ (if n & V are constant)

$$y = a \cdot x + b$$

Diagram showing variables: p , V , n , R , T pointing to the slope a and the intercept b .

#2: Photoelectric effect
(II.37)

$$E_{\text{kin}} = h \cdot f - W_{\text{em}}$$

$$y = a \cdot x + b$$

Diagram showing variables: E_{kin} , h , f , $-W_{\text{em}}$ pointing to the slope a and the intercept b .

#3: Attenuation coefficient
(II.85)

$$\mu = \mu_m \cdot \rho$$

$$y = a \cdot x + b$$

Diagram showing variables: μ , μ_m , ρ pointing to the slope a and the intercept b .

#4: Ohm's law

$$R = U/I$$

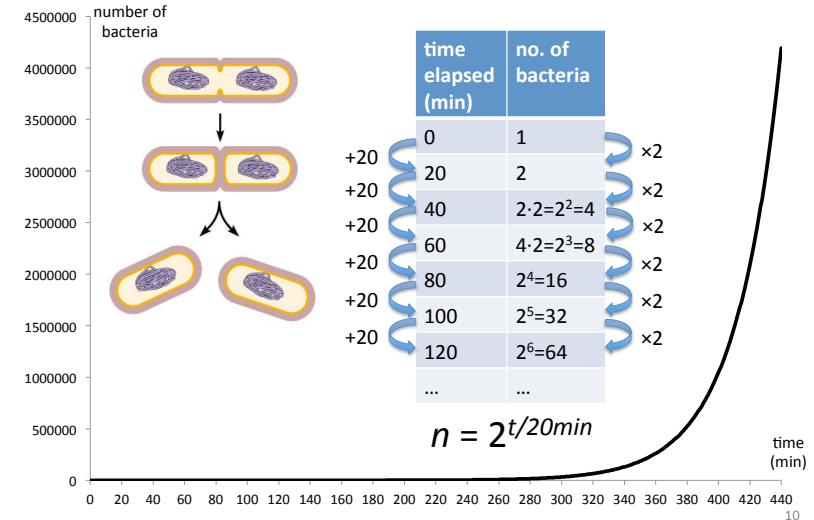
$$I = 1/R \cdot U + 0$$

$$y = a \cdot x + b$$

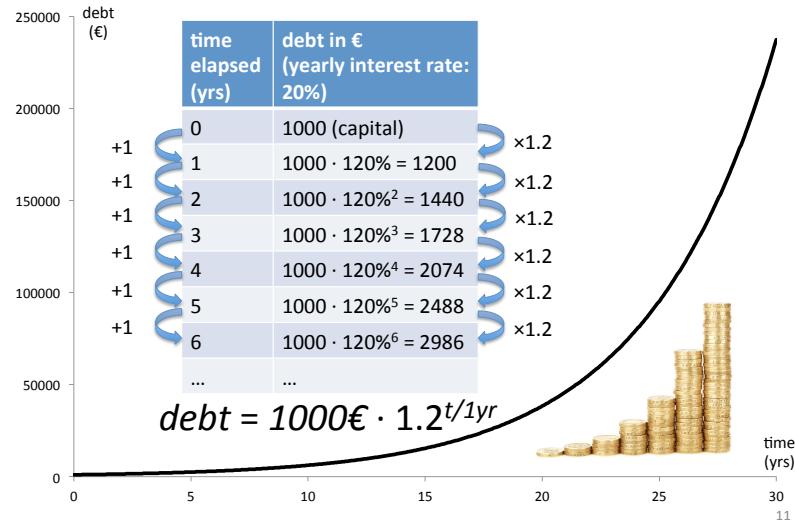
Diagram showing variables: R , U , I pointing to the slope a and the intercept b .

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Exponential Function: Example #1

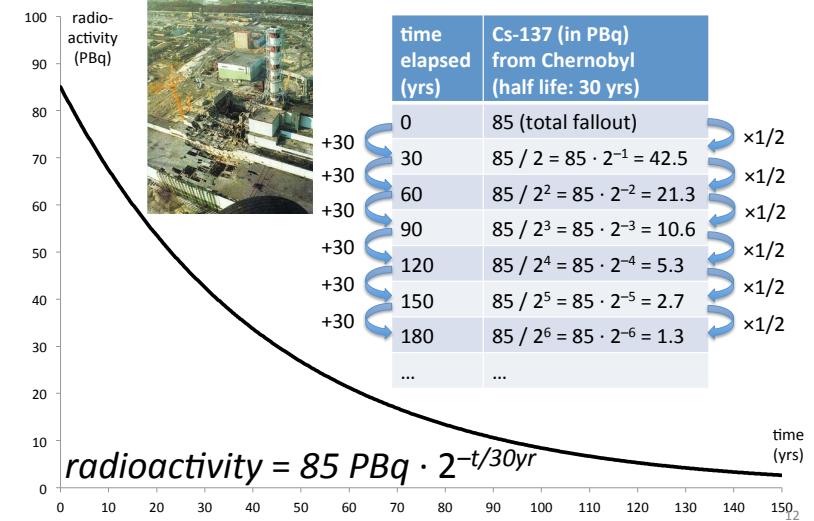


Exponential Function: Example #2



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Exponential Function: Example #3



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Exponential Function

INTEGRAL FORM

$$y = b \cdot a^x$$

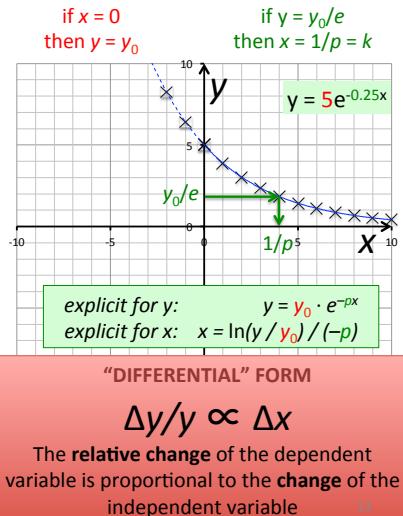
PRACTICAL MODIFICATIONS:

- the base number is preferred to be e
- a new factor parameter p (or $1/k$) is necessary in the exponent
- use a negative sign in the exponent
- b is rather denoted by y_0

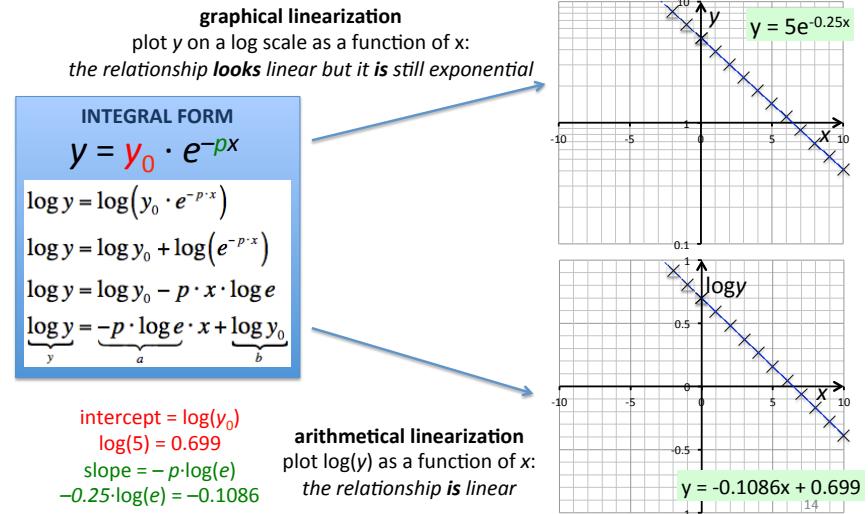
VARIABLES: dependent variable independent variable

$$y = y_0 \cdot e^{-px} = y_0 \cdot e^{-x/k}$$

PARAMETERS: exponential coefficient exponential coefficient



Exponential Function: Linearization



Exponential Function: Some Examples from the Biophysics Formula Collection

#1: Law of radiation attenuation
(II.11)

$$J = J_0 \cdot e^{-\mu x}$$

$$y = y_0 \cdot e^{-px}$$

#2: Boltzmann's distribution
(I.25)

$$n_i = n_0 \cdot e^{-\Delta E/(kT)}$$

$$y = y_0 \cdot e^{-x/k}$$

#3: Decay law
(II.96)

$$N = N_0 \cdot e^{-\lambda t}$$

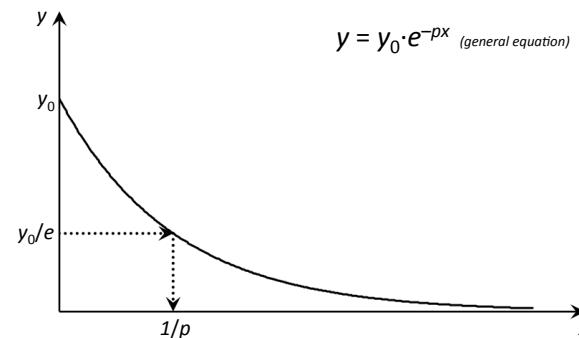
$$y = y_0 \cdot e^{-pt}$$

#4: Discharging an RC circuit
(VII.2)

$$U = U_0 \cdot e^{-t/(RC)}$$

$$y = y_0 \cdot e^{-x/k}$$

Graph of Exponential Functions from the Biophysics Formula Collection



Graph of Exponential Functions

from the Biophysics Formula Collection

