# Mathematical and Physical Basis of Medical Biophysics

Lecture 1

Mathematics Necessary for Understanding Physics
Physical Quantities and Units

10th September 2018

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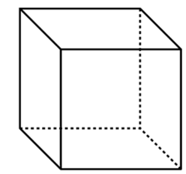
# Power Function: Example

mass ∝ volume ∝ [body]length$^3$
surface area ∝ [body]length$^2$





# Power Function

**INTEGRAL FORM**

**VARIABLES:** dependent variable ($y$); independent variable ($x$)

$y = b \cdot x^a$

**PARAMETERS:** pre-exponential coefficient ($b$); exponent ($a$)

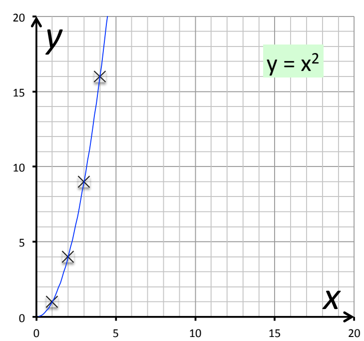
*explicit for y:* $y = b \cdot x^a$
*explicit for x:* $x = (y / b)^{1/a}$

**"DIFFERENTIAL" FORM**

$\Delta y/y \propto \Delta x/x$

The **relative change** of the dependent variable is proportional to the **relative change** of the independent variable

if $x = 1$
then $y = b$




**inverse proportionality** and **square root** functions are also power functions

$$y = \frac{b}{x} = b \cdot x^{-1}$$

$$y = \sqrt{x} = x^{\frac{1}{2}}$$

# Power Function: Linearization

**graphical linearization**
plot both $y$ and x on log scales:
*the relationship **looks** linear but it **is** still power function*

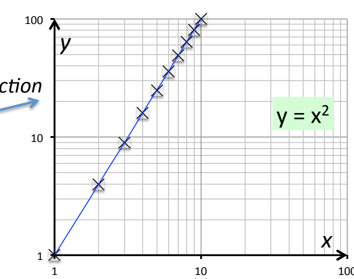
**INTEGRAL FORM**

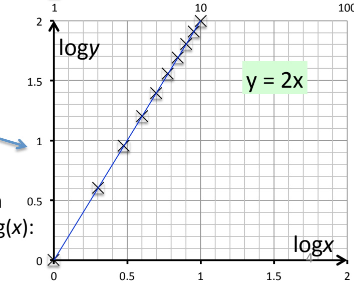
$y = b \cdot x^a$

$$\log y = \log\left(b \cdot x^a\right)$$
$$\log y = \log b + \log\left(x^a\right)$$
$$\log y = \log b + a \cdot \log x$$
$$\underbrace{\log y}_{y} = a \cdot \underbrace{\log x}_{x} + \underbrace{\log b}_{b}$$

intercept = log$b$
log1 = 0
slope = $a$
$a$ = 2

**arithmetical linearization**
plot log($y$) as a function of log($x$):
*the relationship **is** linear*

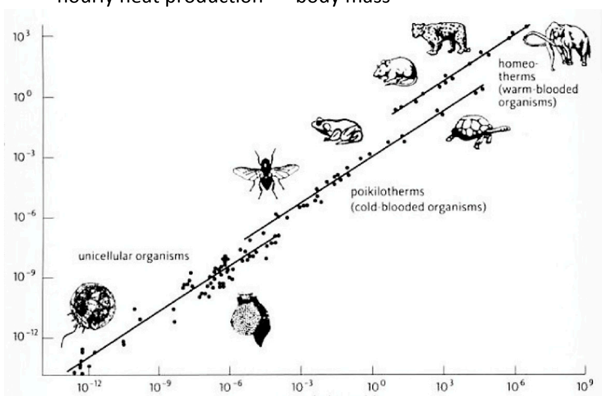


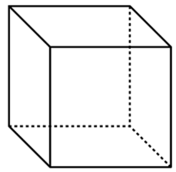
# Power Function: Example

Allometric scaling
(E.g. Kleiber's law)

hourly heat production ∝ body mass$^{3/4}$




mass ∝ volume ∝ [body]length$^3$
surface area ∝ [body]length$^2$





# Power Function: Some Examples
from the Biophysics Formula Collection

#1: The de Broglie wavelength
(I.3)
$\lambda = h/p$

$$\lambda = h \cdot p^{-1}$$
$$y = b \cdot x^{a}$$

#2: Stefan–Boltzmann law
(II.41)

$$M_{\text{black}} = \sigma \cdot T^{4}$$
$$y = b \cdot x^{a}$$

#3: Duane–Hunt law
(II.80)

$$\lambda_{\min} = \frac{hc}{eU_{\text{anode}}}$$
$$\lambda_{\min} = hc/e \cdot U^{-1}$$
$$y = b \cdot x^{a}$$

#4: Mass dependence of eigenfrequency
(Resonance 6)

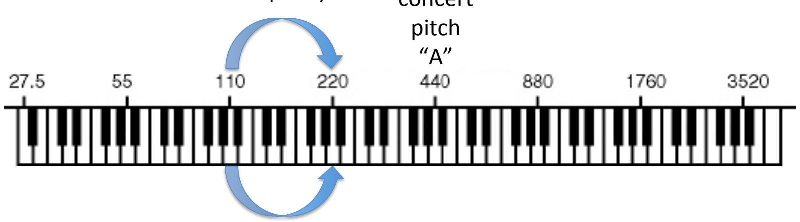
$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$
$$f_0 = k^{1/2}/(2\pi) \cdot m^{-1/2}$$
$$y = b \cdot x^{a}$$



# Logarithmic Function: Example

×2
frequency

concert
pitch
"A"




+12
semitones

# Logarithmic Function

**INTEGRAL FORM**

$$y = b \cdot \log_a(x)$$

**PRACTICAL CONSIDERATIONS:**
- base is 10 (sometimes *e* or 2)
- if the base is fixed this will modify the factor paramater according to the following identity:

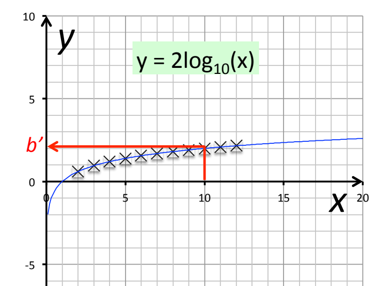
$$b \cdot \log_a(x) = b/\log_{10}(a) \cdot \log_{10}(x) = b' \cdot \log_{10}(x)$$

**VARIABLES:** dependent variable — independent variable

$$y = b' \cdot \log_{10}(x)$$

**PARAMETERS:** factor parameter

if $x = 10$
then $y = b'$



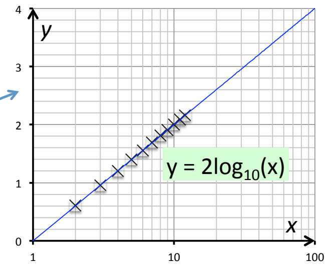

**„DIFFERENTIAL" FORM**

$$\Delta y \sim \Delta x / x$$

The **absolute change** of the dependent variable is proportional to the **relative change** of the independent variable

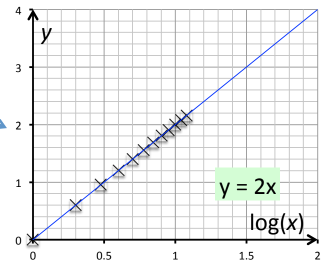
# Logarithmic Function: Linearization

**graphical linearization**
plot *y* on lin and x on log scales:
*the relationship **looks** linear but it **is** still a log function*




**INTEGRAL FORM**

$y = b' \cdot \log_{10}(x)$




**arithmetical linearization**
plot *y* as a function of log(*x*):
*the relationship **is** linear*

# Logarithmic Function: Some Examples

from the Biophysics Formula Collection ...and elsewhere

#1: The statistical definition of entropy
(III.72)
$S = k \ln \Omega$

$S = k \cdot \log_e(\Omega)$

$y = b \cdot \log_a(x)$

#2: The decibel (dB) scale
(VII.10)
$n = 10 \log A_p$

$n = 10 \cdot \log_{10}(A_p)$

$y = b \cdot \log_a(x)$

#3: The definition of absorbance
(VI.34)
$A = \lg(J_0/J)$

$A = 1 \cdot \log_{10}\underbrace{(J_0/J)}$

$y = b \cdot \log_a(x)$

#4: The pH scale

$\mathrm{pH} = -\log[\mathrm{H}^+]$

$\mathrm{pH} = -1 \cdot \log_{10}\underbrace{([\mathrm{H}^+]/(1\ \mathrm{M}))}$

$y = b \cdot \log_a(x)$



# Derivative and Integral: Example #1

Δ Δ

| x | y = x² | y' = Δy/Δx | y'' = Δ(Δy/Δx)/Δx |
|---|---|---|---|
| 0 | 0 | | |
| 1 | 1 | 1 | |
| 2 | 4 | 3 | 2 |
| 3 | 9 | 5 | 2 |
| 4 | 16 | 7 | 2 |
| 5 | 25 | 9 | 2 |
| 6 | 36 | 11 | 2 |
| 7 | 49 | 13 | 2 |
| 8 | 64 | 15 | 2 |
| 9 | 81 | 17 | 2 |
| 10 | 100 | 19 | 2 |

Σ Σ

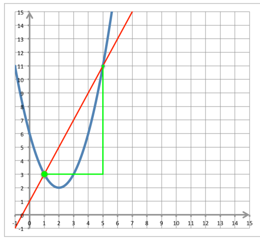


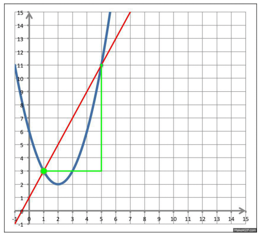
# Derivative: slope of tangent line

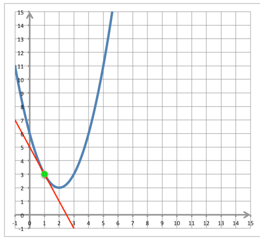
difference quotient:
$\Delta y/\Delta x$
slope of **secant** line

$\Delta \longrightarrow \mathrm{d}$

derivative:
$dy/dx$
slope of **tangent** line

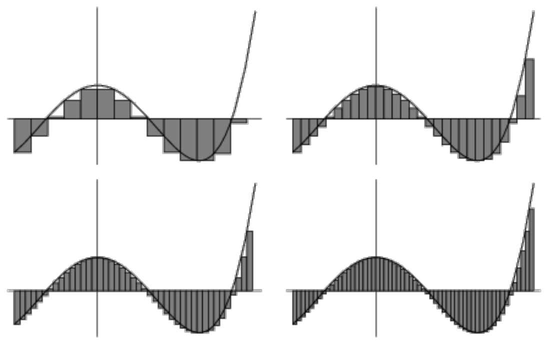






# Integral: Area Under the Curve (AUC)

$$\Sigma \rightarrow \int$$





# Rectilinear Motions

## Quantities, Units, and Equations

| | |
|---|---|
| *displacement*: $\Delta s = s_2 - s_1$ | $[\Delta s]$ = m |
| *velocity*: $v = \Delta s/\Delta t$ | $[v]$ = m/s |
| *acceleration*: $a = \Delta v/\Delta t$ | $[a]$ = m/s$^2$ |

*Uniform rectilinear motion*

- $s_t = s_0 + v \cdot t$
- $v$ = parameter
- $a = 0$

*Uniform rectilinear acceleration*

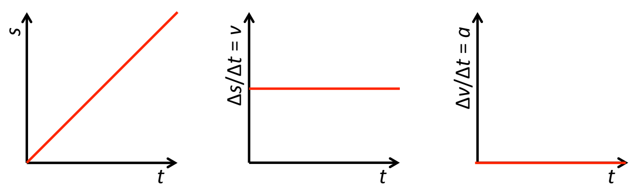
- $s_t = s_0 + v_0 \cdot t + a/2 \cdot t^2$
- $v_t = v_0 + a \cdot t$
- $a$ = parameter



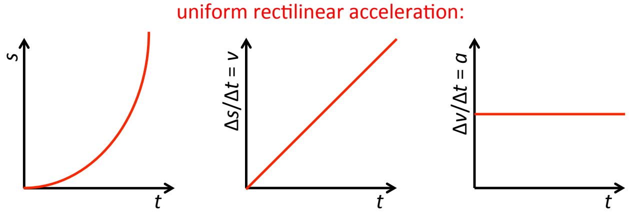
# Derivative and Integral: Application

## Rectilinear Motion

uniform rectilinear motion:




uniform rectilinear acceleration:

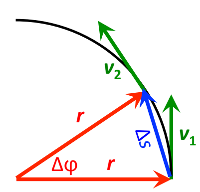


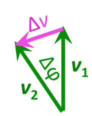



# Circular Motion

## Quantities, Units, and Equation

| | |
|---|---|
| *angular displacement:* $\Delta\varphi = \varphi_2 - \varphi_1$ | $[\Delta\varphi]$ = rad |
| *angular velocity, angular frequency*: $\omega = \Delta\varphi/\Delta t$ | $[\omega]$ = rad/s |
| *tangential velocity*: $v = r \cdot \Delta\varphi/\Delta t = r \cdot \omega$ | $[v]$ = m/s |
| *centripetal acceleration*: $a_{cp} = v^2/r = r \cdot \omega^2$ | $[a]$ = m/s$^2$ |



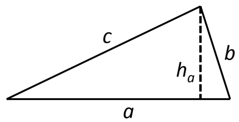

(1) approximation in case of small angles:
displacement = arc length = $v \cdot \Delta t \approx \Delta s$

(2) due to similarity:
$\Delta v/v = \Delta s/r$

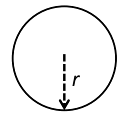
(1) + (2):
$\Delta v/v = v \cdot \Delta t/r$

$a_{cp} = v^2/r$

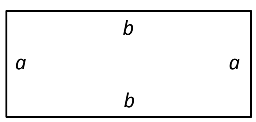
# Perimeter & Area



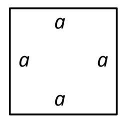

TRIANGLE
perimeter: $a+b+c$
area: $a*h_a/2$



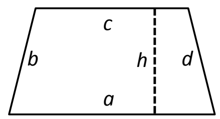

CIRCLE
perimeter: $2r\pi$
area: $r^2\pi$




RECTANGLE
perimeter: $2*(a+b)$
area: $a*b$



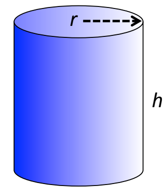

SQUARE
perimeter: $4a$
area: $a*a=a^2$



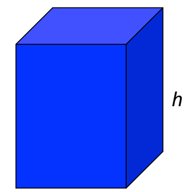

TRAPEZOID
perimeter: $a+b+c+d$
area: $(a+c)/2*h$



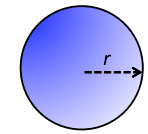
# Surface & Volume




CYLINDER (open)
surface (wall only): $2r\pi*h$
volume: $r^2\pi*h$




PRISM (open)
surface (wall only):
(perimeter of base)*$h$
volume: (area of base)*$h$




SPHERE
surface: $4r^2\pi$
volume: $4r^3\pi/3$



# Units – SI Base & Derived Units

| physical quantity | symbol | unit | symbol |
|---|---|---|---|
| length | *l, x, s, d* | meter | m |
| mass | *m* | kilogram | kg |
| time | *t* | second | s |
| temperature | *T* | kelvin | K |
| electric current | *I* | ampere | A |
| amount of substance | *n, N*, ν [nu] | mole | mol |
| luminous intensity | $I_v$ | candela | cd |

The SI base units

| physical quantity | symbol | unit | symbol | derivation |
|---|---|---|---|---|
| speed | *v, c* | – | – | $m\cdot s^{-1}$ |
| acceleration | *a* | – | – | $m\cdot s^{-2}$ |
| force | *F* | newton | N | $kg\cdot m\cdot s^{-2}$ |
| energy | *E* | joule | J | $kg\cdot m^2\cdot s^{-2}$ |
| power | *P* | watt | W | $kg\cdot m^2\cdot s^{-3}$ |
| intensity | *I* | – | – | $kg\cdot s^{-3}$ |
| pressure | *p* | pascal | Pa | $kg\cdot m^{-1}\cdot s^{-2}$ |

Some SI derived units



# Units – SI Prefixes

| prefix | symbol | meaning | etymology |
|---|---|---|---|
| exa | E | $\times10^{18} = \times1000^6$ | Greek 6 (ἕξ = hex) |
| peta | P | $\times10^{15} = \times1000^5$ | Greek 5 (πέντε = pente) |
| tera | T | $\times10^{12} = \times1000^4$ | Greek 4 (τέτταρες = tettares), originally: monster (τέρας = teras) |
| giga | G | $\times10^{9} = \times1000^3$ | Greek giant (γίγας = gigas) |
| mega | M | $\times10^{6} = \times1000^2$ | Greek great (μέγας = megas) |
| kilo | k | $\times10^{3} = \times1000^1$ | Greek 1000 (χίλιοι = khilioi) |
| hekto | h | $\times10^{2}$ | Greek 100 (ἑκατόν = hekaton) |
| deca | da (dk) | $\times10^{1}$ | Greek 10 (δέκα = deka) |
| | | | |
| deci | d | $\times10^{-1}$ | Latin 10 (decem) |
| centi | c | $\times10^{-2}$ | Latin 100 (centum) |
| milli | m | $\times10^{-3} = \times1000^{-1}$ | Latin 1000 (mille, *pl.* milia) |
| micro | µ | $\times10^{-6} = \times1000^{-2}$ | Greek small (μικρός = mikros) |
| nano | n | $\times10^{-9} = \times1000^{-3}$ | Greek dwarf (νᾶνος = nanos) |
| pico | p | $\times10^{-12} = \times1000^{-4}$ | Spanish small, bit (pico) |
| femto | f | $\times10^{-15} = \times1000^{-5}$ | Danish 15 (femten) |
| atto | a | $\times10^{-18} = \times1000^{-6}$ | Danish 18 (atten) |

# Units – Conversion

**from "with prefix" to "no prefix":**
15 km = 15 · $10^3$ m
15 cg = 15 · $10^{-2}$ g

**from "no prefix" to "with prefix":**
15 m = 15 / $10^3$ km
15 g = 15 / $10^{-2}$ cg

**from "with prefix" to "with prefix":**
15 km = 15 · $10^3$ m = 15 · $10^3$ / $10^{-2}$ cm

**when the unit has an exponent:**
15 $km^3$ = 15 · $(10^3\ m)^3$ = 15 · $(10^3)^3\ m^3$
15 $m^3$ = 15 / $(10^3)^3\ km^3$

**liters to and from cubic meters:**
1 $m^3$ = 10 hL = 1000 L
1 $dm^3$ = 1 L
1 $cm^3$ = 1 mL
1 $mm^3$ = 1 µL

**time to seconds:**
2 days 3 h 12 min 30 s = ((2·24+3)·60+12)·60+30 s

**degrees, minutes of arc, seconds of arc:**
45° 40′ 30″ = $(45+40/60+30/60^2)$°

**degrees to and from radians:**
1 rad = (360/2π)°
1° = (2π/360) rad

**compound units:**
15 $kg/m^3$ = 15 · $10^3$ / $(1/(10^{-2})^3)$ $g/cm^3$
45 km/h = 45 · $10^3$ / 3600 m/s

**degrees Celsius to and from kelvins:**
T = 15 °C = (15+273) K
T = 15 K = (15–273) °C
ΔT = 15 °C = 15 K
ΔT = 15 K = 15 °C