Mathematical and Physical Basis of Medical Biophysics

Chapter 5

Mechanics –Work and Energy

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Energy-Work

Work and energy describes the interactions between objects, can be applied more widely than force
(e.g. for thermal or chemical interaction)

Energy:

- Describes the state of an object or a system
- Ability of a system to perform work

Work: is done on an object when you transfer energy to that

Different kinds of energies

- Kinetic energy
- Potential energy
- Internal energy
- Chemical energy
- Nuclear energy
- Electrical energy

… they can be converted into each other

Work

For mechanical interactions:

- Object accelerated by force
- Lifting up an object
- Extending a spring

$$W = F \cdot s$$

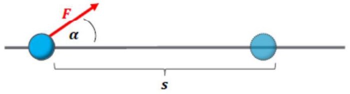
(if force and displacement have the same direction)

W: Work [N ·m=Joule=J] (scalar)

F: force $\left[\text{N=Newton=}\frac{kg \cdot m}{s^2}\right]$

s: displacement of the object [m]

$$W = F \cdot s \cdot \cos\alpha$$



α: angle between the force and the displacement

Power

- Rate of doing work

$$P = W/t$$

P: power [J/s=watt=W]
W: Work [N ·m=Joule=J]
t: time [s]

Energy and work have the same unit: J

Unit conversions

	Joule	Electronvolt	Calorie
1 J=	1	$6.25 \cdot 10^{18}$	0.239
1 eV=	$1.6 \cdot 10^{-19}$	1	not relevant
1 cal=	4.19	not relevant	1

Problem V/2

A car (m=1,2 t) is uniformly accelerating from rest for 12 seconds to reach a velocity of 100 km/h.

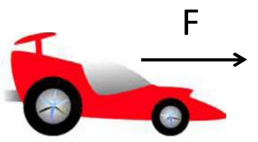
a) Calculate the force necessary for this acceleration.

b) Calculate the distance run by the car during acceleration.

c) Calculate the work done by the accelerating force.

d) Calculate the average power of the car.

e) Calculate the kinetic energy of the car at the end of the acceleration.



Types of mechanical energy

- Kinetic energy

  (accelerating an object)

- Gravitational potential energy

  (lifting up an object)

- Elastic energy

  (extending a spring)

Kinetic energy

- Related to motion

$$E_{kin} = \frac{1}{2} \cdot m \cdot v^2$$

E_{kin}: kinetic energy [J=Joule]

m: mass [kg]

v: velocity [m/s]

Work done during acceleration: $W = F \cdot s = m\frac{v}{t} \cdot \frac{v}{2}t = \frac{1}{2}m \cdot v^2.$

Potential energy

- Results from a position or configuration

Depending on the force field it can be:

- Gravitational
- Magnetic (later)
- Electric (later)

Elastic energy: configuration dependent potential energy

Gravitational potential energy

- A capacity for doing work as a result of the object position in a gravitational field

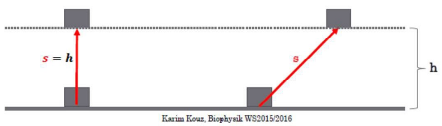
$$E_{pot} = m \cdot g \cdot h$$

E_{pot}: potential energy [J=Joule]

m: mass[kg]

g: gravitational acceleration=9.81 [m/s^2]

h: height above a reference level [m]



Work done during elevation:

$$W = F \cdot s = mgh$$

Problem V/4

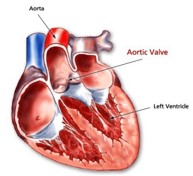
The left ventricle pump about 70 g of blood in a contraction into the aorta. This amount of blood reaches the aortic arch that is located approximately 15 cm above the ventricle and has a flow velocity of 30 cm/s.

Calculate:

a) the work needed to lift the blood,

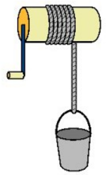
b) the work needed to accelerate the blood.

c) the power of the left ventricle during a contraction that lasts for 0,2 s!



Problem V/5

Someone is pulling a bucket full of water (m=12 kg, including the mass of 10 litres of water in it) to the top of an 8 m deep well, with uniform velocity of 50 cm/s. Calculate



a) the force acting on the bucket

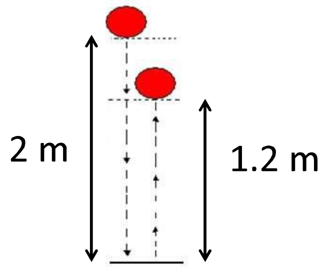
b) the work done

c) the power

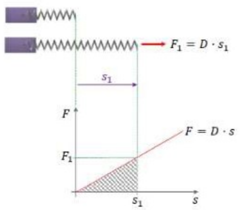
d) How many kcal of energy equals the work of a man who is lifting up a total of 4,8 m^3 water from the well during one day?

Problem V/9

A ball (m=0,8 kg) hits the floor from a height of 2 m and bounces back to the height of 1,2 m. Calculate the amount of energy lost due to air drag and the collision with the ground.



Elastic energy



$$E_{elastic} = \frac{1}{2} \cdot k \cdot s^2$$

$E_{elastic}$: elastic potential energy [J=Joule]

k: spring constant [N/m]

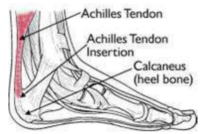
s: deformation of an elastic object [m]

Work done during the strech:

$$W = F \cdot s = \frac{1}{2}ks \cdot s = \frac{1}{2}ks^2$$

Problem V/7

Calculate the amount of energy stored in the Achilles tendon with a spring constant of $3 \cdot 10^5$ N/m that is extended by 2mm.



Conservation of energy for mechanics

- Total amount of energy in an isolated system remains constant
- Isolated system: neither matter nor energy can pass

$$E_{kin} + E_{pot} + E_{elastic} = constant$$

$$\frac{1}{2} \cdot m \cdot v^2 + m \cdot g \cdot h + \frac{1}{2} \cdot k \cdot s^2 = constant$$

Mass-energy equivalence

- Every object of mass m, has a rest energy:

$$E = mc^2$$

E: rest energy[J=Joule]

m: mass [kg]

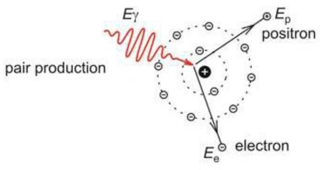
c: speed of light in vacuum =$3 \cdot 10^8$ [m/s]

Mass and energy can be transferred into each other.

e.g. PET

Problem V/10

Calculate the rest energy of an electron ($m_e = 9{,}11 \cdot 10^{-31}$ kg)! Convert your result to eV unit !

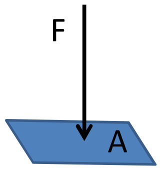


Chapter 6

Mechanics –Pressure

Pressure

$$p = \frac{F}{A}$$



p: pressure [N/m^2=pa=pascal]

F: force applied perpendicular to the surface [N]

A: surface of an object [m^2]

Pressing something with palm or finger cause different deformation

Units of pressure

SI unit: pascal (pa=N/m^2)

Other units:

1 bar = 10^5 pa = 100 kpa

1 atm=1.01·10^5 pa = 101 kpa=1.01 bar = 760 mmHg

1 mmHg=1 torr=133 pa=0.133 kpa

Problem VI/2

Masticatory forces of a human can reach up to 100 N (for crocodiles it is 1000 N!). When someone bites on a bone chip in the burger or on the seed of a fruit, this force is concentrated on a surface area of 1 mm^2. Calculate the pressure!

Problem VI/3

a) Calculate the pressure that a 70 kg standing man exerts on the floor. (The total surface of the two soles is 200 cm^2)

b) Calculate the pressure this man exerts on the ice surface during skating! (The total surface of the blades is 4 cm^2)

Density

- Pressure in gases and fluids depends on density

$$\rho = \frac{m}{V}$$ if it is homogenous

ρ: density [kg/m^3]
m: mass [kg]
V: volume [m^3]

Density of materials

material	ρ (g/cm^3)
air (at 0°C and 101 kappa)	0,00129
water (at 4°C and 101 kPa)	1
water (at 100°C and 101 kPa)	0,958
ice	0,92
aluminium	2,7
mercury	13,6
gold	19,3
human body (averaged)	1,04

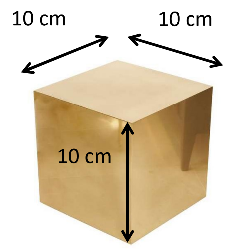
$1 g/cm^3 = 1 kg/dm^3 = 1000\ kg/m^3$

$ml = cm^3$ $liter = dm^3$

Problem VI/5

a) Calculate the mass of a gold cube with the width of 10 cm!

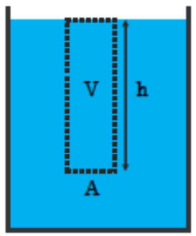
b) Calculate the pressure exerted by this cube on a horizontal shelf, that holds it!



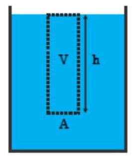
Hydrostatic pressure

Spontaneously arising in fluids and gases caused by the gravitational field of Earth

The ‚body' (gas or fluid) with height ‚h' is pressing the underneath surface ‚A' with its weight.



Hydrostatic pressure



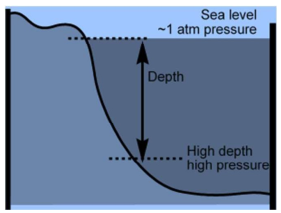
$$W = m \cdot g = \rho \cdot V \cdot g = \rho \cdot A \cdot h \cdot g$$

means weight, not work

$$p = \frac{F}{A} = \frac{W}{A} = \frac{\rho \cdot A \cdot h \cdot g}{A} = \rho \cdot h \cdot g$$

means pressure, not power

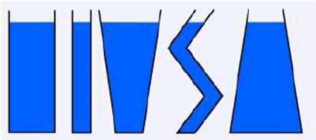
Pressure (in fluids and gases) is directly proportional to depth



The hydrostatic paradox

At the bottom of which container we have the highest pressure?

p=$\rho \cdot g \cdot h$



Hydrostatic pressure depends on:

- density
- height of fluid

Does NOT depend on the shape of the container