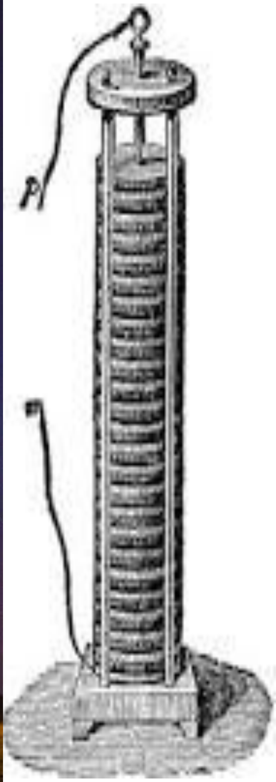
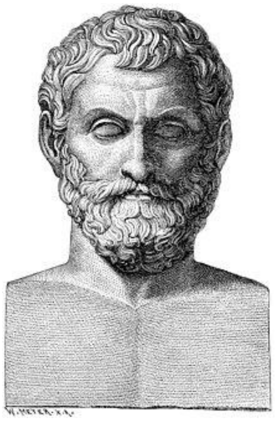
Electricity theory



G.Schay

Electrostatics: charges in resting state (not moving)



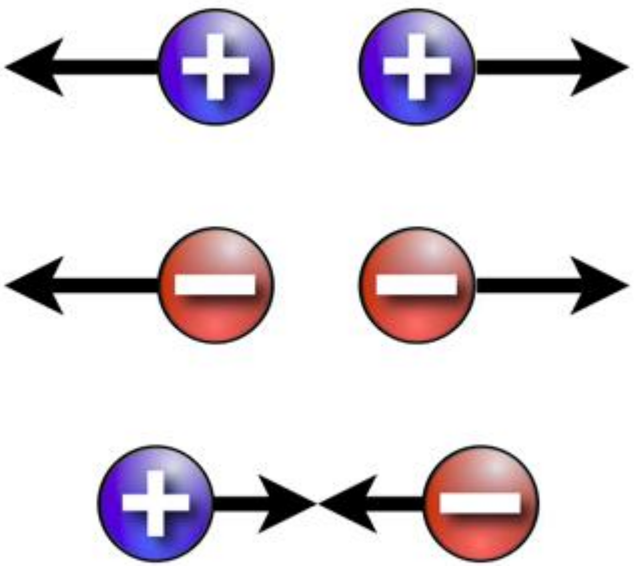
Discovery:

Thales of Mile (624BC-546BC)

Rubbed amber can attract fur/feather

“electron” was the name of amber in greek ☺

Experiments:
When we rub materials then they accumulate “charge” and attract or repel each other.
(triboelectric effect)



Triboelectric series:

Most positively charged

+

Polyurethane foam
Hair, oily skin
Nylon, dry skin
Glass
Acrylic, Lucite
Leather
Rabbit's fur
Quartz
Mica
Lead
Cat's fur
Silk
Aluminium
Paper (Small positive charge)
Cotton
Wool (No charge)

0

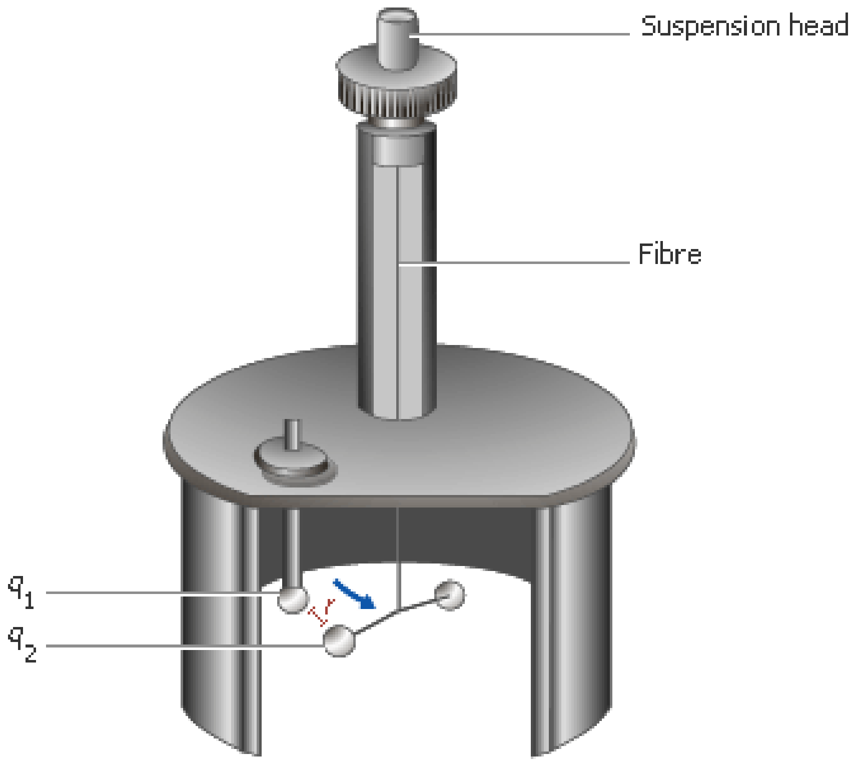
Steel (No charge)
Wood (Small negative charge)
Amber
Sealing wax
Polystyrene
Rubber balloon
Resins
Hard rubber
Nickel, Copper
Sulfur
Brass, Silver
Gold, Platinum
Acetate, Rayon
Synthetic rubber
Polyester
Styrene and polystyrene
Orlon
Plastic wrap
Polyethylene (like Scotch tape)
Polypropylene
Vinyl (PVC)
Silicon
Teflon (PTFE)
Silicone rubber
Ebonite

–

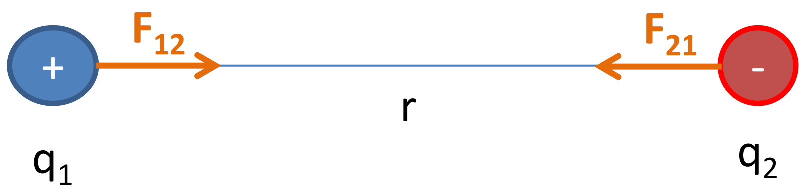
Most negatively charged

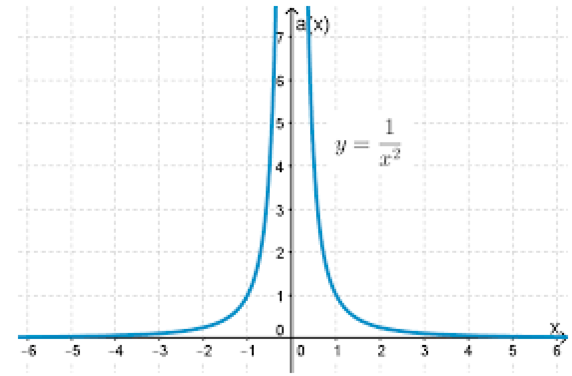
First quantitative measurements:

Coulomb's Force



Charles A. de Coulomb
1736-1806





In size $F_{12}=F_{21}$

$$F = k\frac{q_1 q_2}{r^2}$$

Very similar to gravity,
inverse square law

There exists a minimum charge

$$e = 1{,}6 \cdot 10^{-19}\mathrm{C}$$

There is no charge on it's own, it is always carried by some material

The modern way of defining the constant:

$$k_e = \frac{1}{4\pi\varepsilon_0} = \frac{c_0^2\mu_0}{4\pi} = c_0^2 \times 10^{-7}\ \mathrm{H \cdot m^{-1}}$$
$$= 8.987\,551\,787\,368\,176\,4 \times 10^9\ \mathrm{N \cdot m^2 \cdot C^{-2}}$$

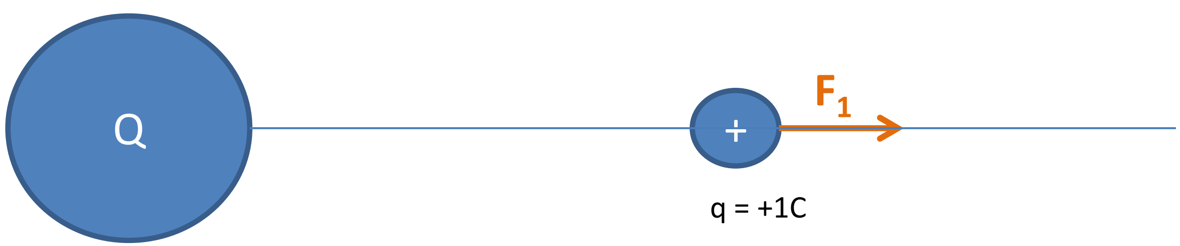
What transmits the force?

Electrical **field**

Force field: a type of vector, which is present in all points in space, but may depend on the position. The vector is related to the force acting on an object at the same position.

Let's define the **electric field** as the force acting on +1C charged positive testing object

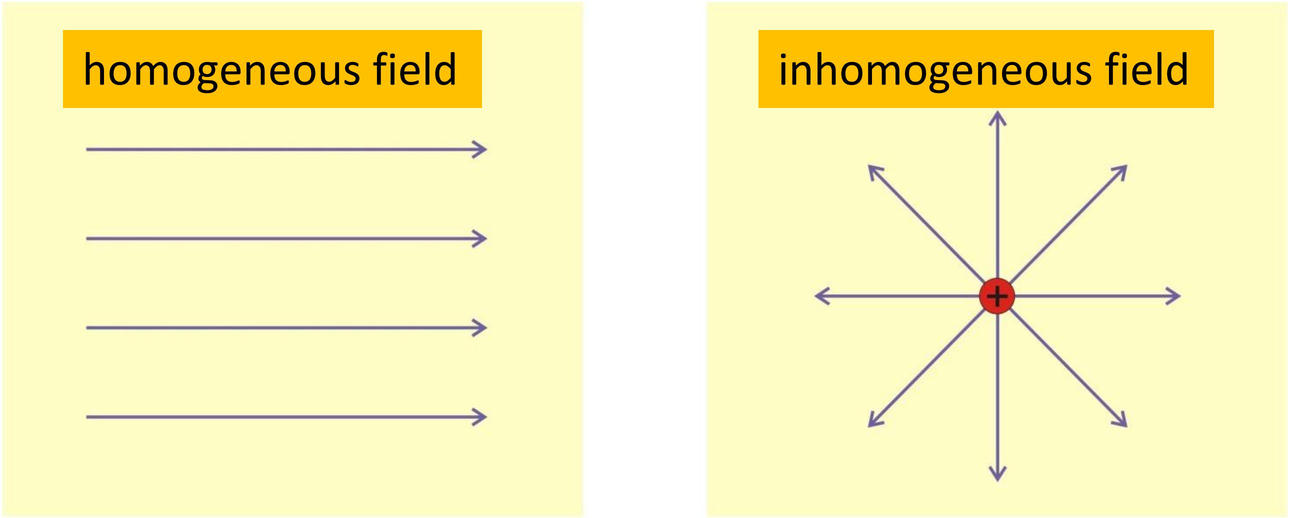
E



With this F=q*E
And $E=k*Q/r^2$

Unit: [F]/[q] = N/C

In a field, we can draw the directions of the force with lines: **field lines**



Rules:
1: the direction of the force is parallel to the tangential of the field lines.
2: the size of the force is proportional to the density of lines.

Work in an electric field:

$W=F*s$, so here we also have a work if we move parallel to the field lines.
$F=q*E$
So
$W=q*E*s$

Here it is convenient to have $W=q*\Delta\phi$, and define the electric potential ϕ.
(just like $E_{pot}=mgh$)

So $\phi = E*s$, BUT we need a 0=point.
Let's define ϕ =0 IF we are infinitely far away

Now we can say that **the electric potential ϕ is equal to the work needed to move +1C charge from infinity to the given position.**

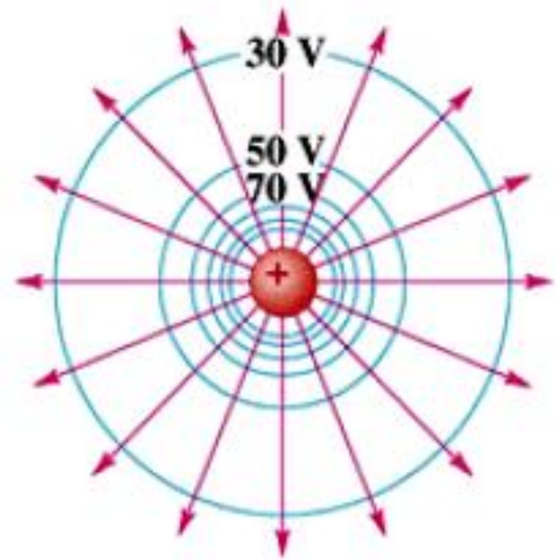
Since E is a conservative force field (just like gravity) the way is not important, just the start and end positions.

U= $\Delta\phi$, electric voltage. Unit: [W]/[q] = J/C = Volt [V].

So we have **$W=q*U$**

We have now field lines and equipotential lines

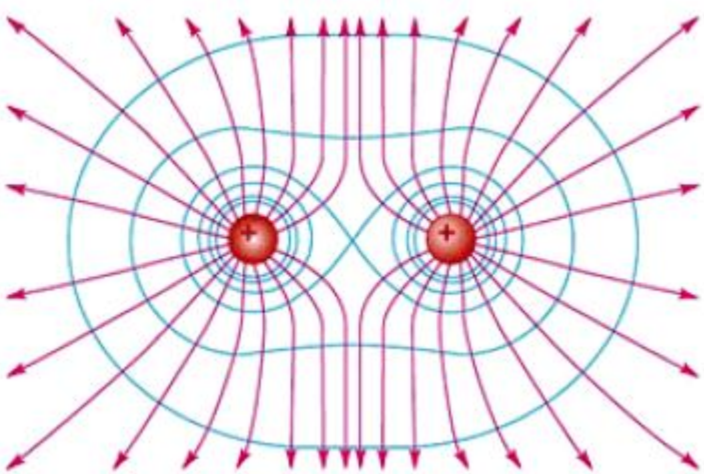
Notice that they are always perpendicular!
($\Delta\phi$ = 0 if F*Δs=0 -> cosα = 0, so α=90 deg.)

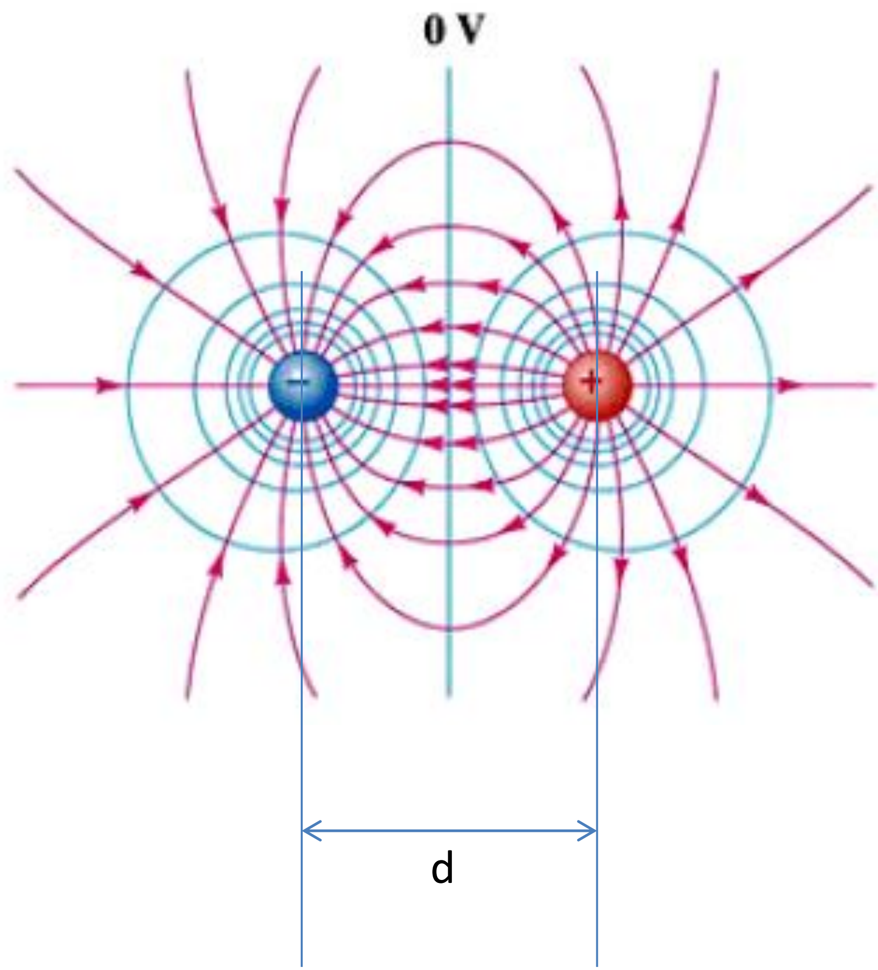


We can make geometrical constructs of charges…

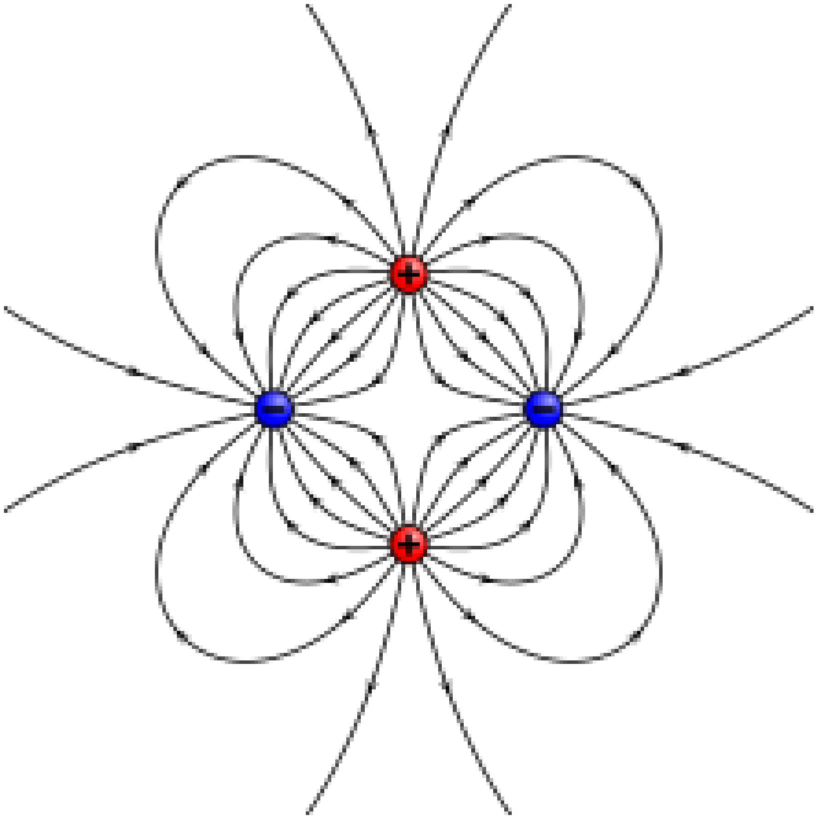
Dipole

Dipole moment: p=q*d





Quadrupole…



All kinds of charge distributions can be modelled by a series of geometrical objects together. (multipole expansion)

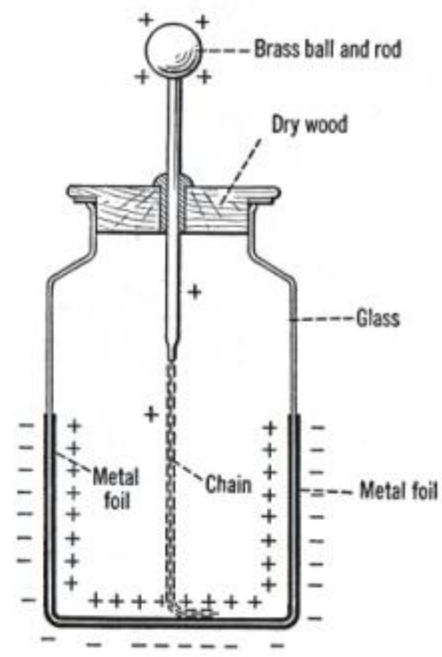
Storing charges: the capacitor

We need work to move the charge into the capacitor.
(like charges repel each other…)

The CAPACITY is a measure of the storing capability of the device:

C=Q/U

Unit: C/V = F (Farad)



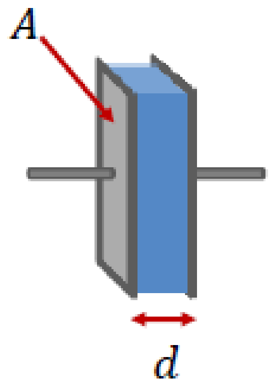
Leyden jar

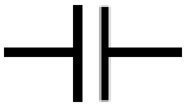
Simplest case for calculation:
Flat panel capacitor

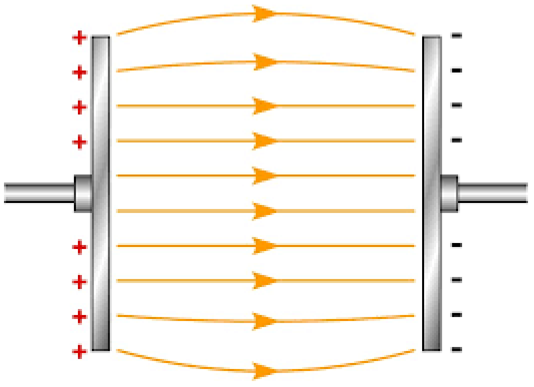
Here the E is constant, homogeneous field.
So U=E*d
But
$E=\sigma/\varepsilon_0$, where σ is the charge density ($\sigma=Q/A$)
And ε_0 is the vacuum permittivity (see later)

With this $C=Q/U = Q/(Q/A * 1/ \varepsilon_0 *d) = 1/ \varepsilon_0 *A/d$

If we have some material in the gap then:
$C= 1/ \varepsilon_0 \varepsilon_r *A/d$
Where ε_r is the permittivity of the material.







Electric field in materials:

The electric field creates (induces) dipoles in a material, which has it's own field. This adds up to the original, so we get a modified field.

$D = P + \varepsilon_0 * E$
Where D is the electric displacement vector, and P is the polarization.

Usually $P=\chi * \varepsilon_0 * E$, so
$D = \varepsilon_r * \varepsilon_0 * E$

If we have no material, just classical vacuum then $D = \varepsilon_0 * E$.

Permittivity: how much charge is needed to generate a given flux.
(flux measures the electric field going through a surface)

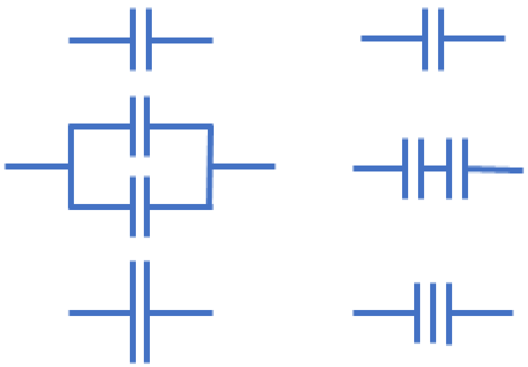
Circuits of capacitors:

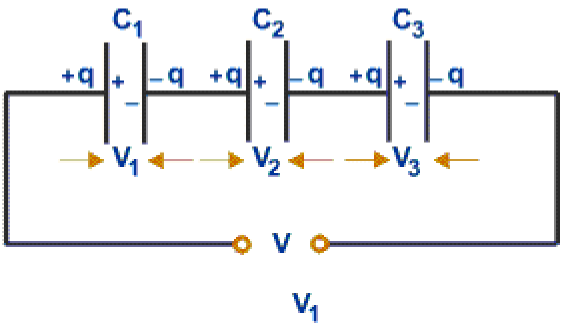
Parallel: more surface, more capacity

C_{tot} = C1+C2

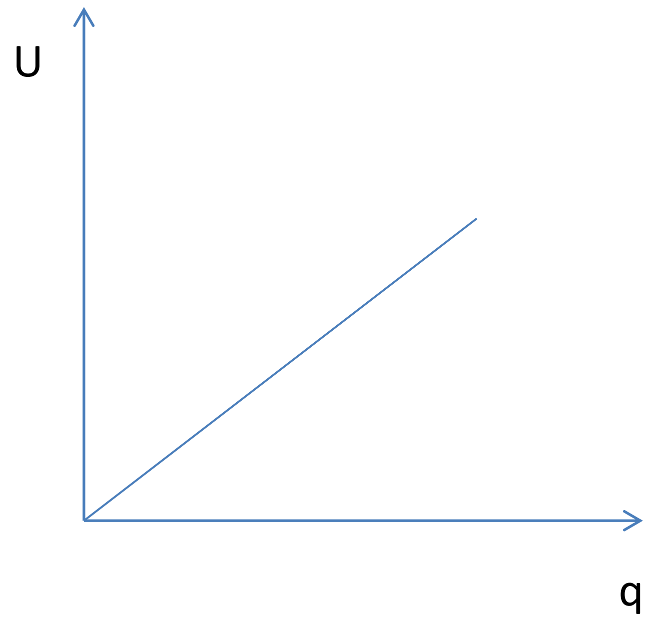
Series: more distance, less capacity

1/ C_{tot} = 1/C1 + 1/C2





We can calculate the work needed to charge up a capacitor



$\Delta W = U^* \Delta q$

But

$C = Q/U$, so the slope is $1/C$ here.

The total work is

$U_{average}{}^*Q$, but $U = Q/C$

Which is

$\frac{1}{2} Q^2/C = \frac{1}{2} CU^2$

Some calculation problems:

A Cl^--ion and a Na^+-ion are exactly on the opposite sides of a cell membrane. By what force do they attract one other if the thickness of the membrane is 6 nm?

The membrane potential measured between the two sides of an excitable cell is –90 mV. Suppose that the electric field in the 10 nm thick membrane is homogeneous. Find the field strength.

A capacitor of 50 nF capacitance has a charge of 30 μC. Determine
a) the voltage of the capacitor and
b) the energy stored in the capacitor.

Moving charges

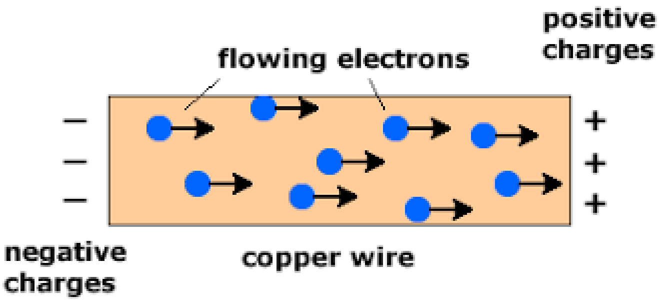
We measure how much charge flows in a given time

$$I = \frac{\Delta q}{\Delta t} \quad [A]\ Amper$$

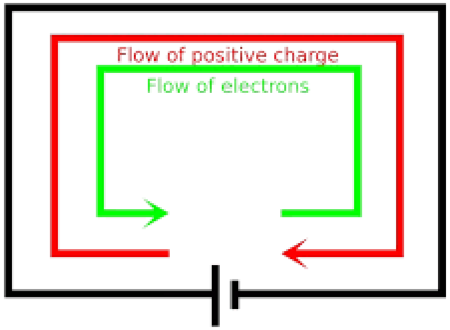
1A = 1C/1s

The traditional direction is for the POSITIVE charge,
Which is the lack of electrons.

So the real flow is always opposite to the
“technical” direction!



What can move is the electrons.
(if there are free, movable electrons in a **conductor**
If not then it is an **insulator**.)

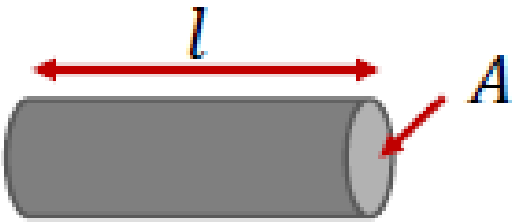


Resistors

When electrons move in materials they hit the atoms and loose speed.
Just like friction!

Ohm's law: the flow is determined by the voltage
and the material:
I = U/R

$$R = \frac{U}{I} \qquad [\Omega]\ Ohm$$



$$R = \rho \frac{l}{A}$$

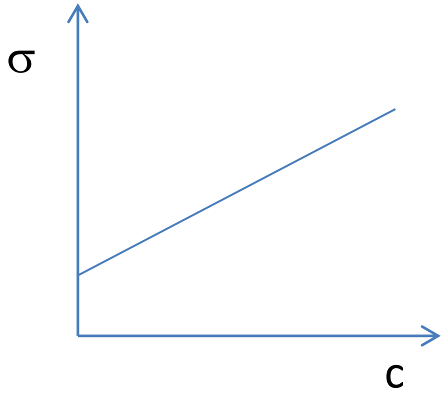
Specific resistance or resistivity

Conductance / conductivity: the inverse of the ohmic resistivity

$G = 1/R$: conductance (1/Ohm = Siemens)

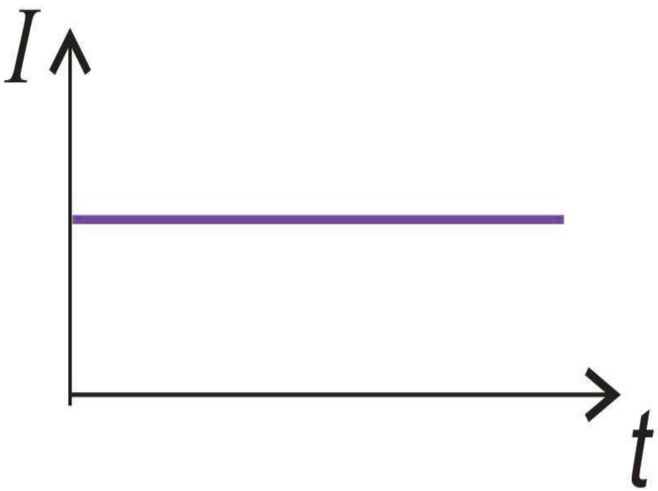
$\sigma = 1/\rho$: conductivity

We usually use this for electrolytes, etc.
(σ is proportional to the concentration)



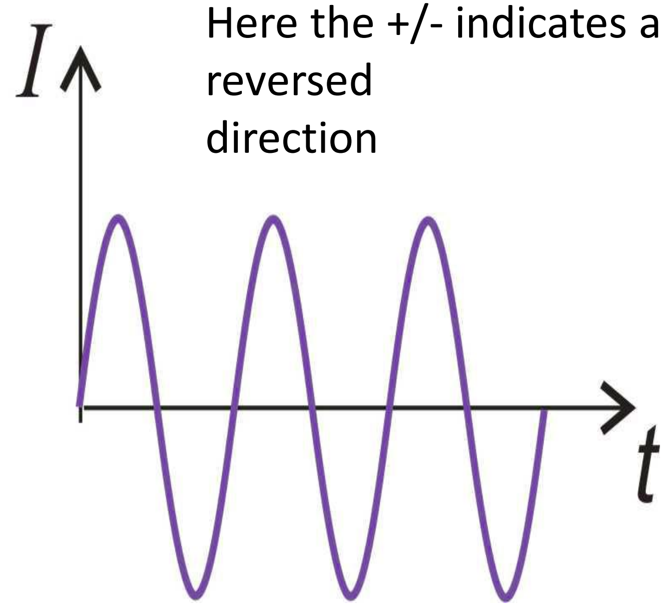
The movement can be unidirectional or changing:

DC: direct current



AC: alternating current

Here the +/- indicates a reversed direction



$$I = I_{max} \cdot \sin \omega t$$

$$U = U_{max} \cdot \sin(\omega t + \varphi)$$

$$I_{eff} = \frac{I_{max}}{\sqrt{2}} \qquad U_{eff} = \frac{U_{max}}{\sqrt{2}}$$

Irms Urms

Circuits of resistors:

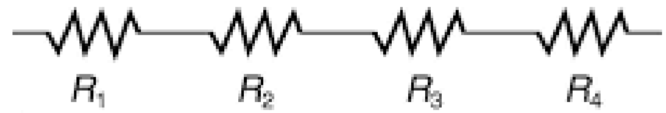
Series: more length => more R

R = R1+R2

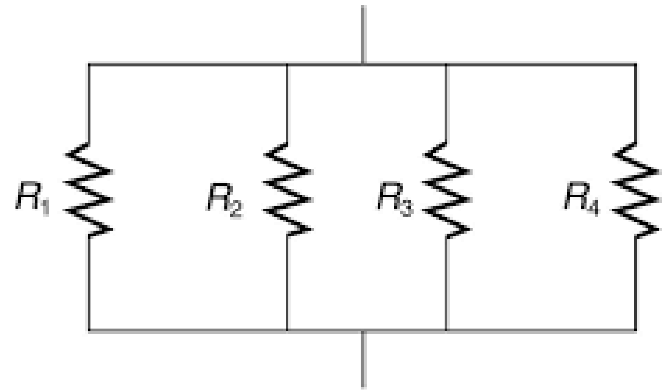
Parallel: more space to go => less R

1/R = 1/R1 + 1/R2

More length to go



(a)



(b)

More ways to go,
more area for the flow

Heat of the friction: Joule's work / Joule's heat

$W = U*I*t$. Unit:joule (J)

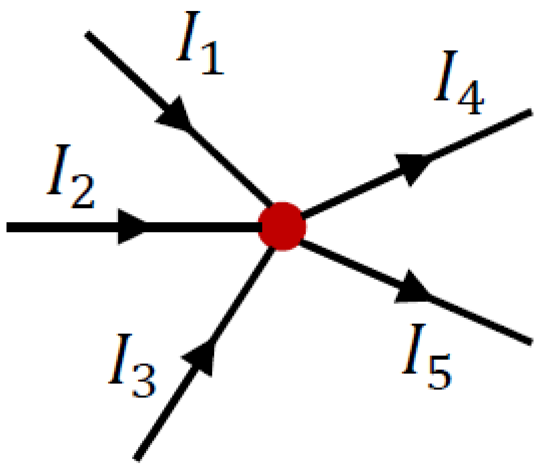
It is general: the work done by the electric field to move the charges is transformed into heat if the current flows through a resistor.
BUT
The electrical field can do other types of work too (electric motors, LED, etc).

Do not forget the energy conservation law!

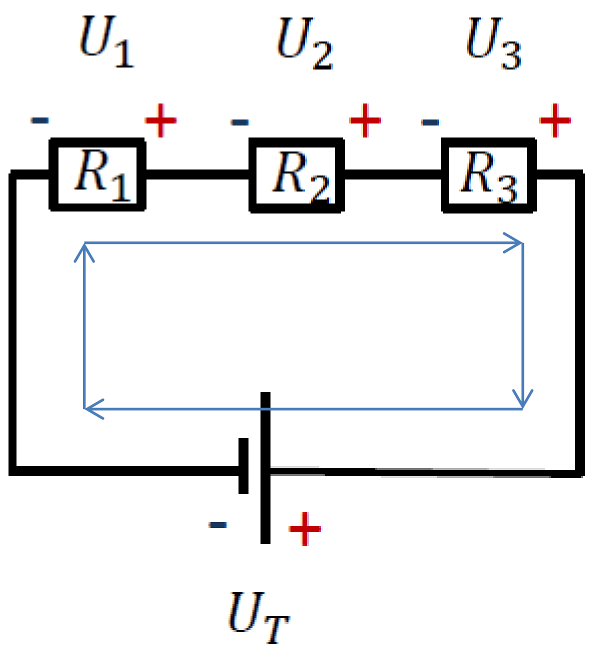
Power: $P = U*I$, unit: Watt (W)

Kirchoff's laws

I. Junction law: the sum of currents must match the storage in the junction. If there is no storage, then the sum is 0 (taking direction into account by +/-)

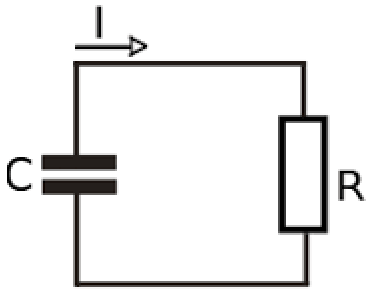


II. Loop law: in a closed loop the sum of electrical potential changes is 0. Here we have to take the +/- again into account, and use a single direction for the loop.



Gustav Robert Kirchhoff
1824-1887

RC circuit



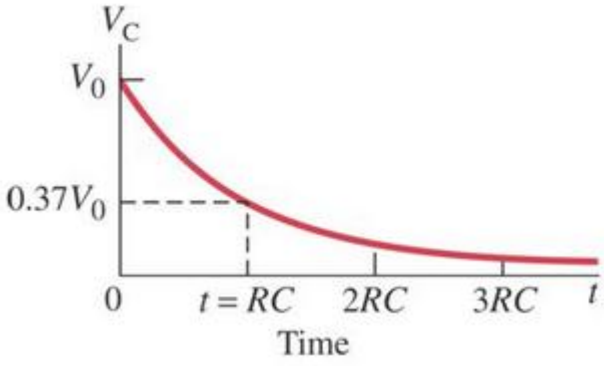
Discharge a capacitor through a resistor

C=Q/Uc , so Uc=Q/C, but Q=I*t, and I=Uc/R.

What we get for the loop eq. is:
I*R+Uc=0

$\Delta Q/ \Delta t = -Q/RC$
$\tau=RC$

$$U_C = U_0 \cdot e^{-\frac{t}{R \cdot C}}$$



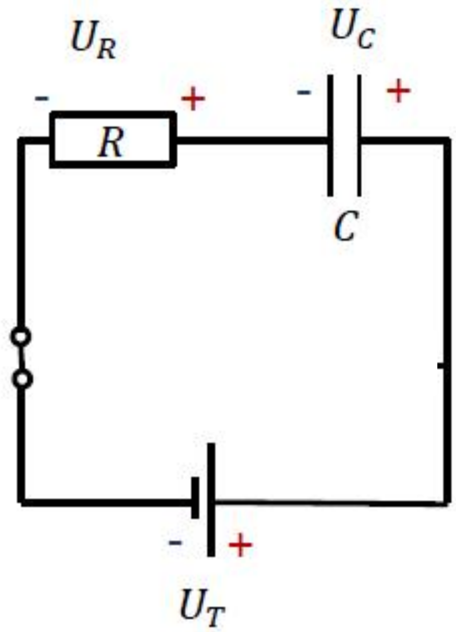
Charging up:

Here the loop equation includes the battery:

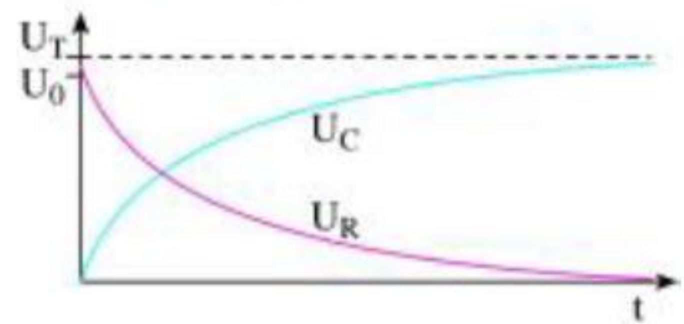
I*R+Uc=Ut

So

ΔQ/ Δt = -Q/RC + Ut/R



$$U_C = U_T \cdot \left(1 - e^{-\frac{t}{RC}}\right)$$



Capacitor in a circuit

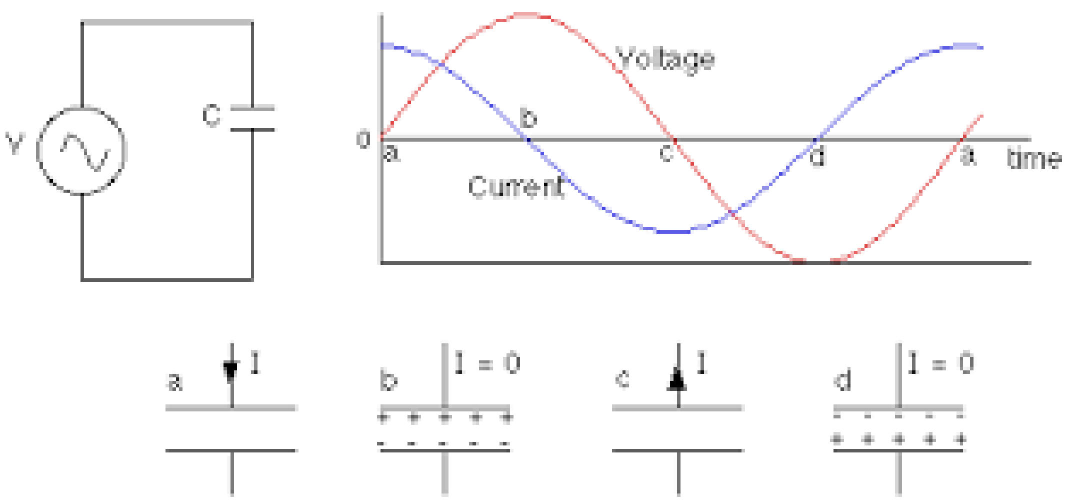
After charging/discharging in a DC circuit it is like a break.

In an AC it is periodically charged/discharged
So it acts as a resistor-like object,
BUT it is in a different phase than pure resistors, here **there is a phase shift!**

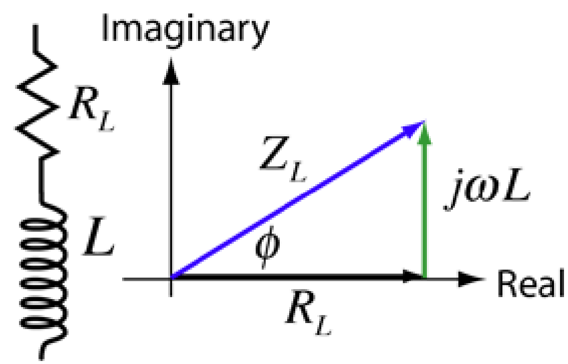
Capacitive reactance

$$X_C = \frac{1}{\omega C}$$

More capacity, less reactance
More freq, less reactance



Extension material: since the phase shift causes a shift between I and U, the AC resistance or IMPEDANCE can not be calculated so easily, but if we let the reactance behave like a vector then it is straightforward.

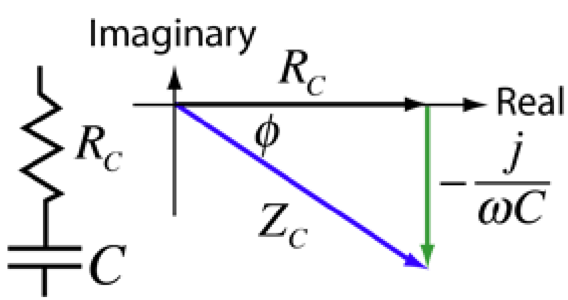


Cartesian form: $Z_L = R_L + j\omega L$

Polar form: $Z_L = |Z_L| e^{j\phi}$

where $|Z_L| = \sqrt{R_L^2 + \omega^2 L^2}$

$$\phi = \tan^{-1} \frac{\omega L}{R_L}$$

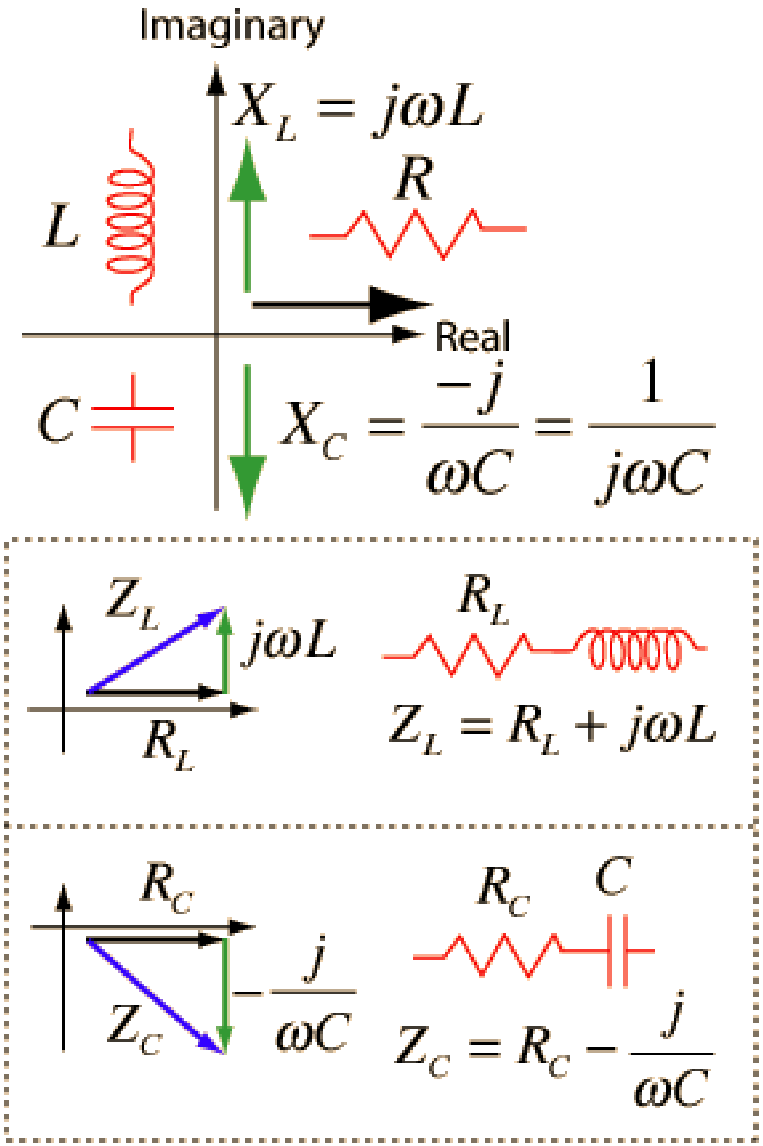


Cartesian form: $Z_C = R_C - \frac{j}{\omega C}$

Polar form: $Z_C = |Z_C| e^{j\phi}$

where $|Z_C| = \sqrt{R_C^2 + \left[\frac{-1}{\omega C}\right]^2}$

$$\phi = \tan^{-1} \frac{-1}{\omega C R_C}$$



Imaginary
$X_L = j\omega L$
R
L
Real
C
$X_C = \frac{-j}{\omega C} = \frac{1}{j\omega C}$
Z_L
$j\omega L$
R_L
R_L
$Z_L = R_L + j\omega L$
R_C
Z_C
$-\frac{j}{\omega C}$
R_C
C
$Z_C = R_C - \frac{j}{\omega C}$