

X-ray

X-ray

Generation, properties

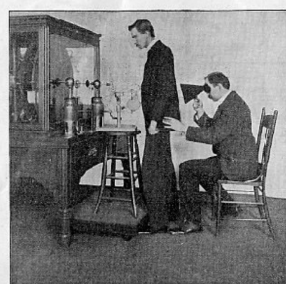
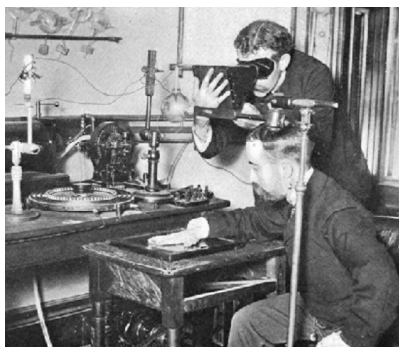


Wilhelm Konrad Röntgen
(1845-1923)
Nobel prize, 1901



Hand mit Ringen (Hand with Ring): print of Wilhelm Röntgen's first "medical" X-ray, of his wife's hand, taken on 22 December 1895 and presented to Professor Ludwig Zehender of the Physik Institut, University of Freiburg, on 1 January 1896. The dark oval on the third finger is a shadow produced by her ring.

Paper funnel radioscope



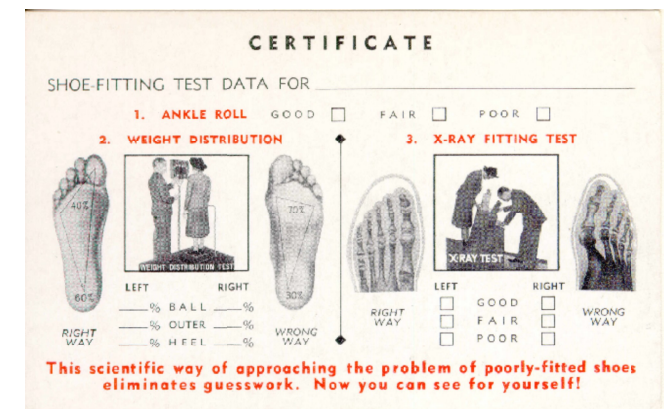
Free X-Ray Examination to Patients



Late 1890s

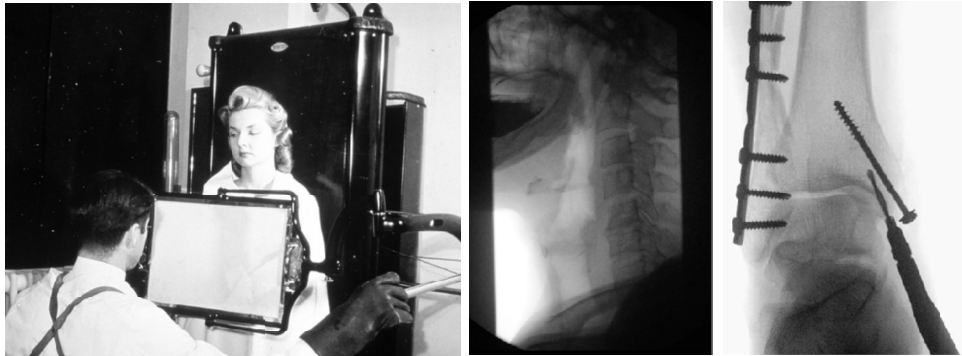
I. World war

Shoe-fitting fluoroscope (1930-50)



This scientific way of approaching the problem of poorly-fitted shoes eliminates guesswork. Now you can see for yourself!

Medical diagnostics



1940

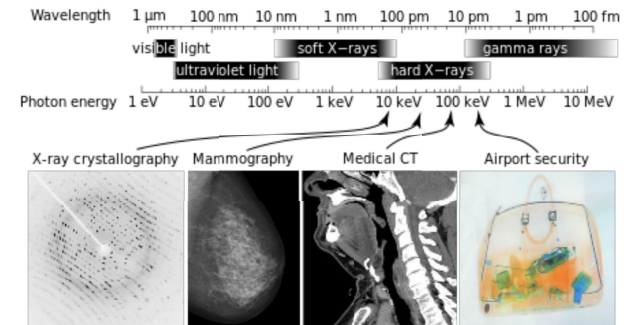
1950

today

X-rays

- Generation of X-rays
- X-ray spectrum
- Interaction with matter 1: diffraction
- Interaction with matter 2: absorption
- X-ray absorption mechanisms:
Photoelectric effect
Compton scatter
Pair production

X-rays are electromagnetic waves



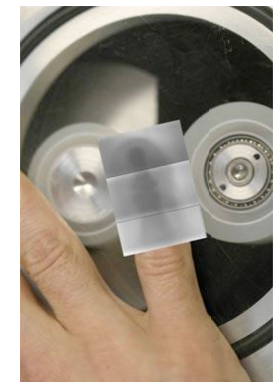
Wavelength 10 - 0.01 nm. **Frequency** 30×10^{15} - 30×10^{18} Hz. **Energy** 120 eV - 120 keV.
(petahertz - exahertz)

Generation of X-ray (non-conventional)

Triboluminescence: light emission evoked by scratching or rubbing. Francis Bacon, 1605.



Peeling away sticky tape emits light...



...and X-rays. (Nature News, October 2008)

Generation of X-ray: in Cathode Ray Tube

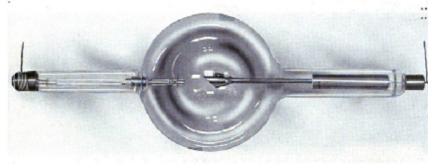
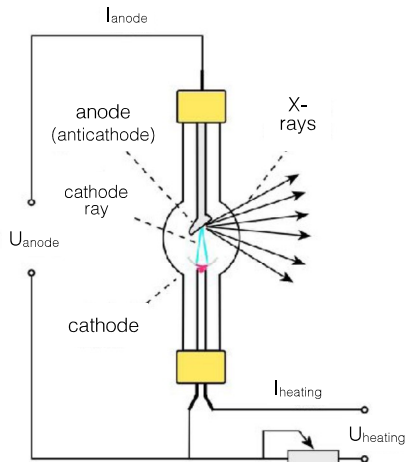
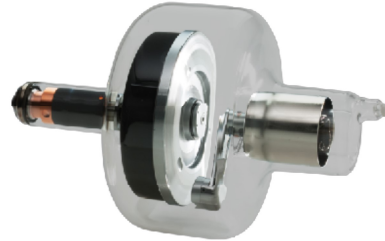
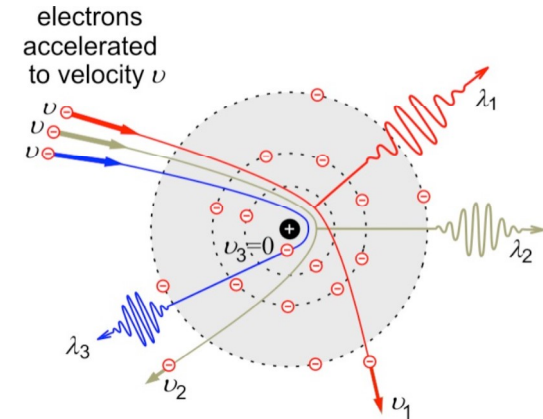


Photo of a Coolidge x-ray tube, from the early 1900s. The heated cathode is on the left, the anode target is on the right. The x-rays are emitted in a downward direction.



Rotating-anode X-ray tube. Anode rotation is used for cooling.

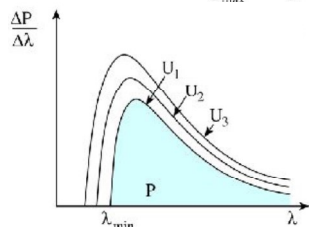
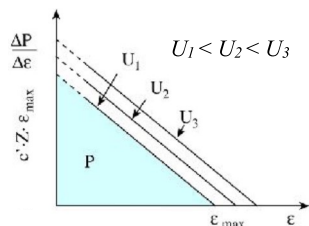
“Bremsstrahlung”



Electrons decelerate, thereby lose their kinetic energy, when interacting with the atoms of the anode (“braking radiation”).

Spectrum of Bremsstrahlung

Continuous spectrum



$$eU_{anode} = \epsilon_{max} = hf_{max}$$

Maximal photon energy (ϵ_{max})
N.B.: Total kinetic energy of electron is transformed in one step (rare event).
 e : electron's charge;
 U_{anode} : accelerating voltage;
 eU_{anode} : acceleration work;
 h : Planck's constant;
 f_{max} : limiting frequency

$$\lambda_{min} = \frac{hc}{e} \cdot \frac{1}{U_{anode}}$$

Limiting wavelength (λ_{min})
(Duane-Hunt Law)
N.B.: Limiting wavelength is inversely proportional to accelerating voltage.
 c : light speed;
 hc/e : constant (1.2398 kV·nm)

$$\frac{\Delta P}{\Delta \epsilon} = c' \cdot Z \cdot (\epsilon_{max} - \epsilon)$$

Energy spectrum
(energy dependence of power)

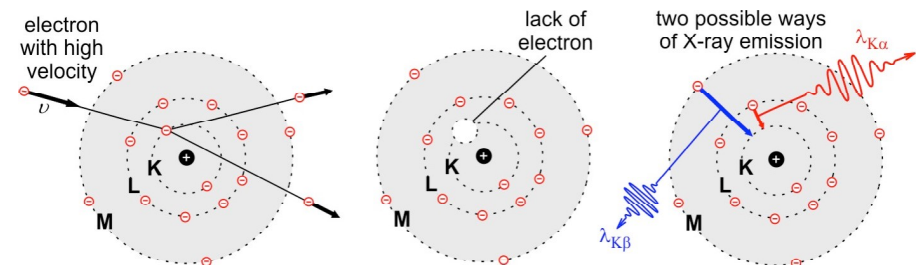
$$P_{tot} = \frac{1}{2} c' \cdot Z \cdot \epsilon_{max}^2 = c \cdot Z \cdot U_{anode}^2 \cdot e^2$$

Total power (P_{tot})
(based on the area of the triangle)
 C_{Rtg} : coefficient ($1.1 \times 10^{-9} \text{ V}^{-1}$);
 I_{anode} : anode current (number of electrons hitting the anode per unit time);
 Z : atomic number of the anode atoms

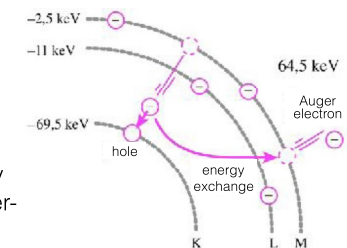
$$\eta = \frac{P_{tot}}{P_{in}} = \frac{C_{Rtg} \cdot I_{anode} \cdot U_{anode}^2 \cdot Z}{I_{anode} \cdot U_{anode}} = C_{Rtg} \cdot U_{anode} \cdot Z$$

Efficiency (η)
 P_{in} : invested power
N.B.: Typically, $\eta < 1\%$.

Characteristic X-ray

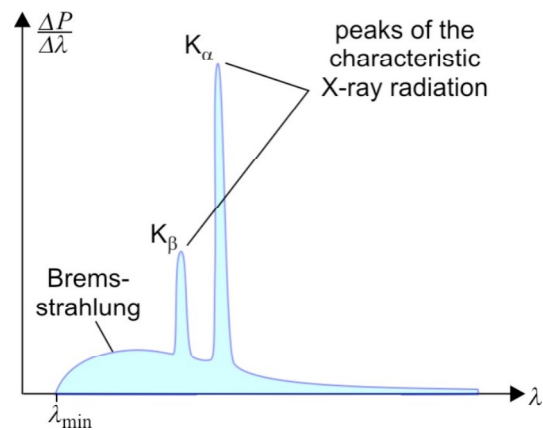


Knocked-out inner-shell electron is replaced by one on a higher-energy shell



Energy of electron transition may be used for the escape of an outer-shell electron: Auger electron

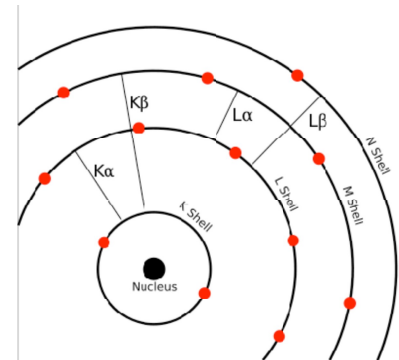
Spectrum of characteristic X-ray



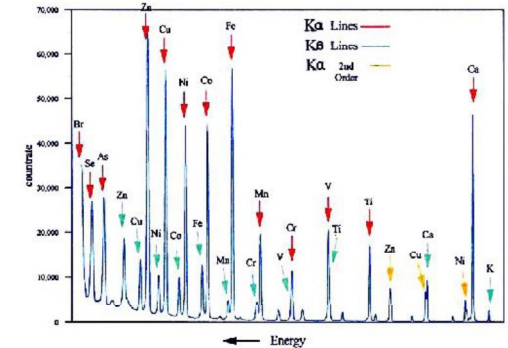
Line spectrum

X-ray spectrum characterizes the atomic composition

Because inner-shell electrons participate in characteristic X-radiation, only the atomic (and not the molecular) properties are revealed



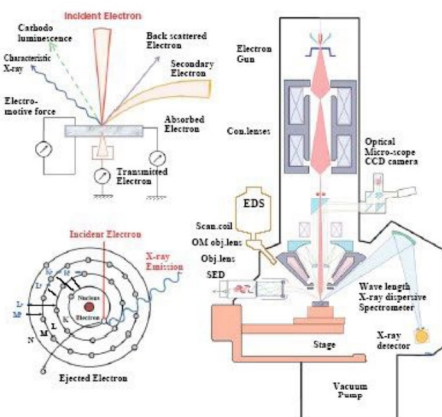
Electronic transitions in a calcium atom.



Energy dispersive X-ray fluorescence spectrum.

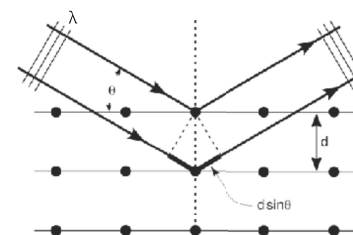
Detection of characteristic X-ray

Electron probe microanalyzer

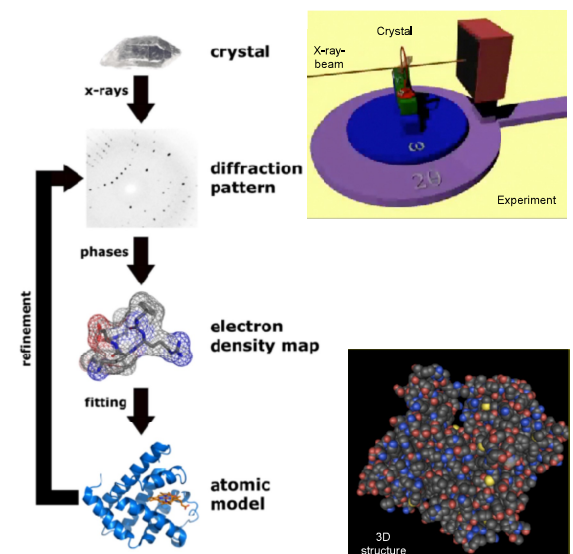
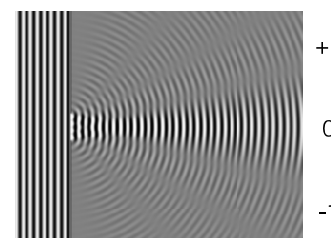


N.B.: electron microscope!

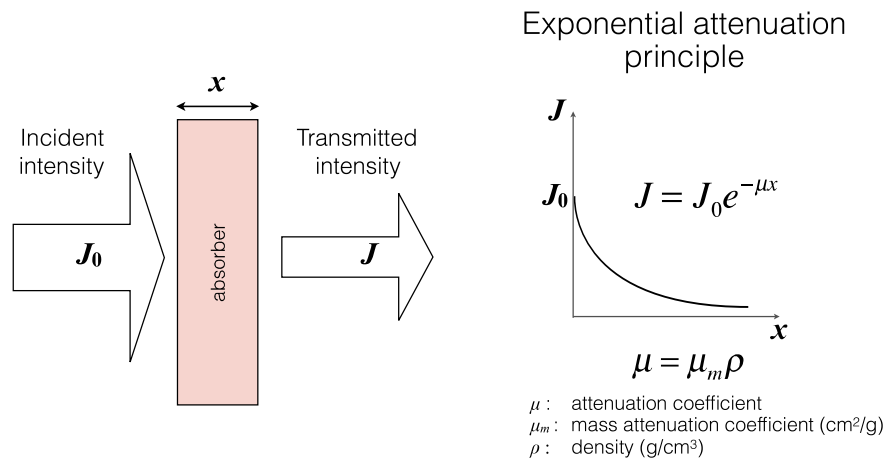
X-ray diffraction



Condition of constructive interference: $2d \sin \theta = n\lambda$

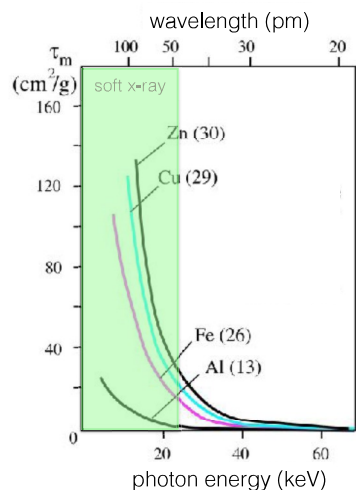


X-ray absorption



μ_m is the sum of the mass attenuation coefficients of the different absorption mechanisms.

Photoeffect attenuation depends strongly on the atomic number



$$\tau_m = \text{const} \cdot \frac{Z^3}{\epsilon^3} = C \cdot \lambda^3 \cdot Z^3$$

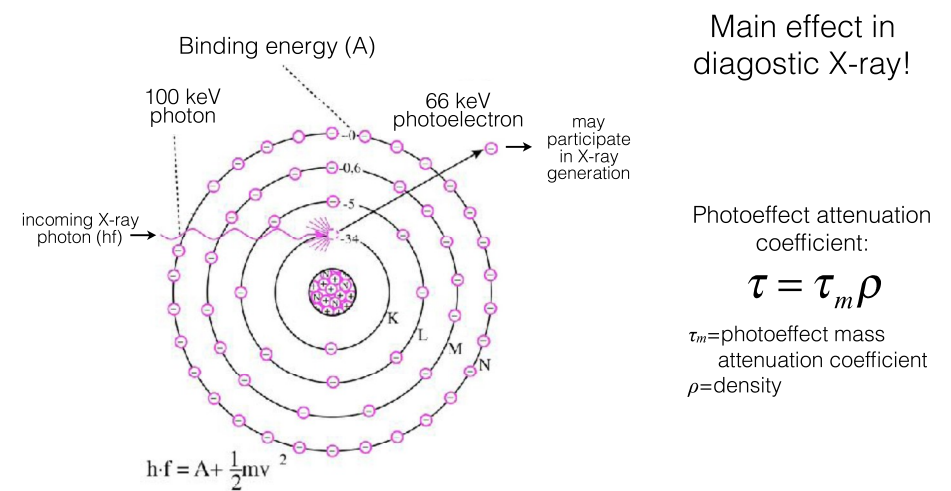
For multi-component system:
 "effective atomic number" (Z_{eff})

$$Z_{\text{eff}} = \sqrt[3]{\sum_{i=1}^n w_i Z_i^3}$$

ϵ =photon energy
 Z =atomic number
 w =mole fraction
 n =number of components

| Material | Z_{eff} |
|-------------|------------------|
| Air | 7.3 |
| Water | 7.7 |
| Soft tissue | 7.4 |
| Bone | 13.8 |

X-ray photoeffect



Compton scatter



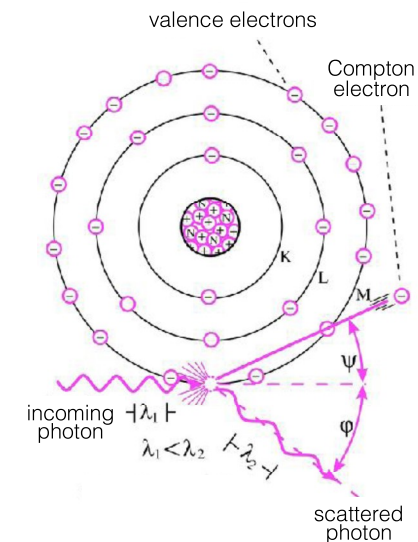
Arthur Holly Compton
 (1892-1962)

$$hf = A + hf_{\text{scatt}} + E_{\text{kin}}$$

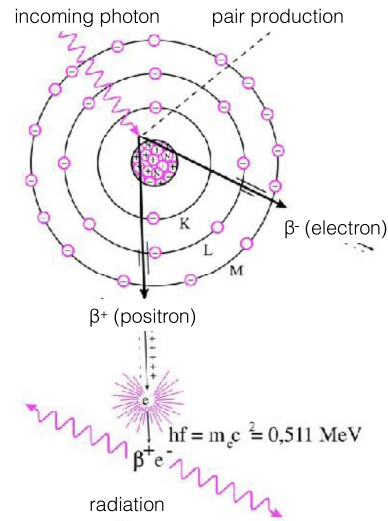
A =work function
 hf_{scatt} =energy of scattered photon
 E_{kin} =kinetic energy of Compton-electron

Compton-effect attenuation coefficient:

$$\sigma = \sigma_m \rho$$



Pair production



(relevant only in therapeutic x-ray)

Energy balance:

$$hf = 2m_e c^2 + 2E_{kin}$$

m_e =mass of electron
 c =speed of light

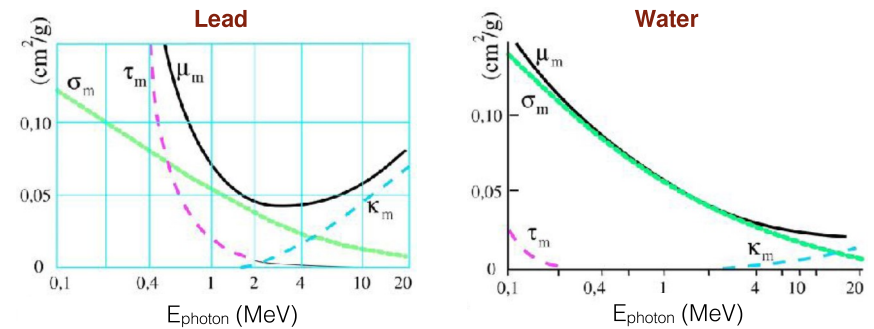
Pair production attenuation coefficient:

$$\kappa = \kappa_m \rho$$

Pair production relevant in high-energy X-ray photons, γ -radiation.

Attenuation mechanisms

Dependence on photon energy and material



$$\mu = \tau + \sigma + \kappa$$

μ_m =mass attenuation coefficient
 σ_m =Compton effect mass attenuation coefficient

τ_m =photoeffect mass attenuation coefficient
 κ_m =pair production mass attenuation coefficient

Summary of attenuation mechanisms

| Mechanism | Photon energy (ϵ) dependence of the mass attenuation coefficient | Atomic number (Z) dependence of the mass attenuation coefficient | Relevant energy range in soft tissue |
|------------------|---|--|---|
| Rayleigh scatter | $\sim 1 / \epsilon$ | $\sim Z^2$ | 1 - 30 keV |
| Photoeffect | $\sim 1 / \epsilon^3$ | $\sim Z^3$ | 10 - 100 keV |
| Compton scatter | falls gradually with ϵ | $\sim Z/A$ (A: mass number) | 0.5 - 5 MeV |
| Pair production | rises slowly with ϵ | $\sim Z^2$ | > 5 MeV |

Diagnostic X-ray:

1. Contrast mechanism between soft tissue and bone: photoeffect ($\sim Z^3$)
2. Contrast mechanism within soft tissue: Compton-scatter ($\sim \rho$)