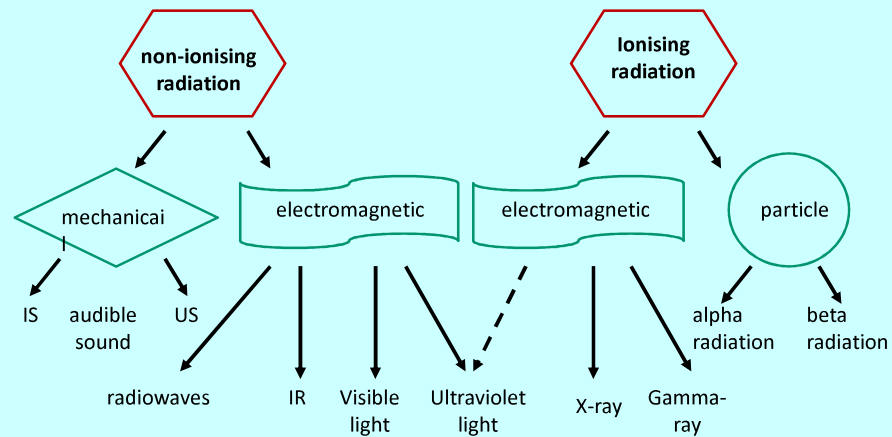


# Radiation



1

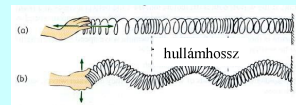
# Sound and ultrasound



2

## Physics of sound

### Longitudinal vs. Transverse wave



**longitudinal wave**  
(in the interior of liquids and gases only this type)



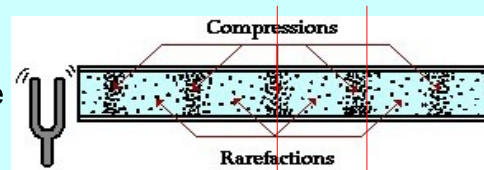
**transverse wave**  
can be generated in solid materials and at liquid surfaces

3

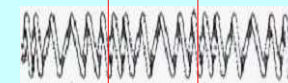
## Physics of sound

Sound: mechanical wave (model)

whistle

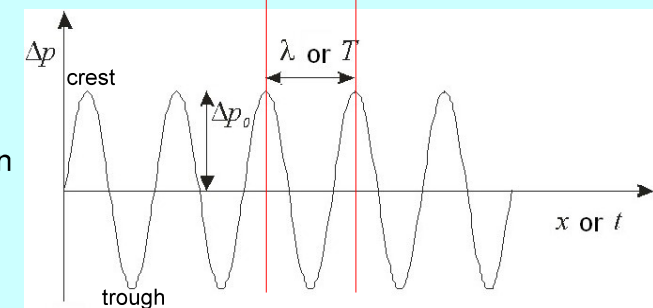


spring

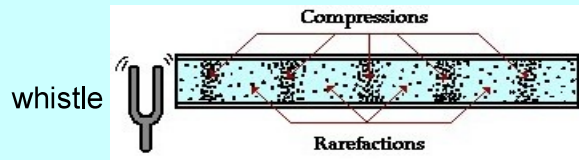


spatial and temporal periodicity

function



4



hydrostatic pressure      pressure change, sound pressure

$$p_{\text{total}} = p_{\text{hydrostat}} + \Delta p$$

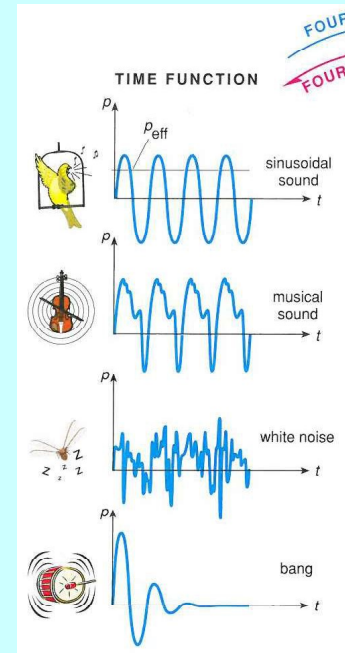
pressure DC + AC      amplitude      phase

$$\Delta p(t, x) = \Delta p_{\text{max}} \sin \left[ 2\pi \left( \frac{t}{T} - \frac{x}{\lambda} \right) \right]$$

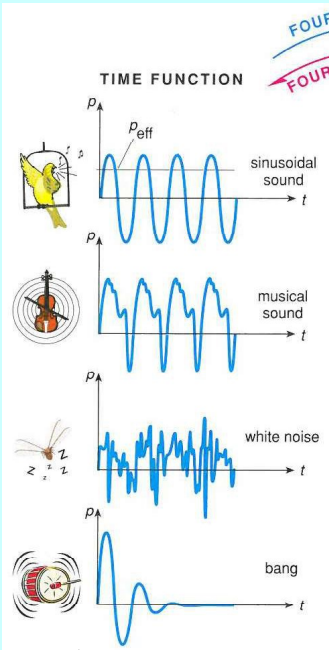
$$c \cdot T = \lambda, \quad c = f \cdot \lambda$$



5



$$p(t) = p_1 \sin(\omega t) + p_2 \sin(2\omega t) + p_3 \sin(3\omega t) + \dots$$



Textbook, Fig. IV.23.

**pitch**  
frequency of the fundamental

high

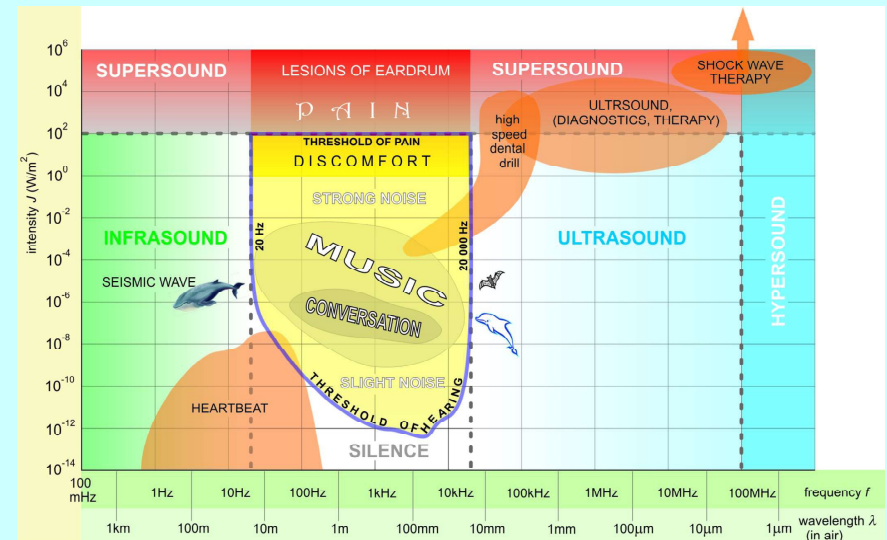
**timbre**  
relative strengths of overtones/harmonics (spectrum)

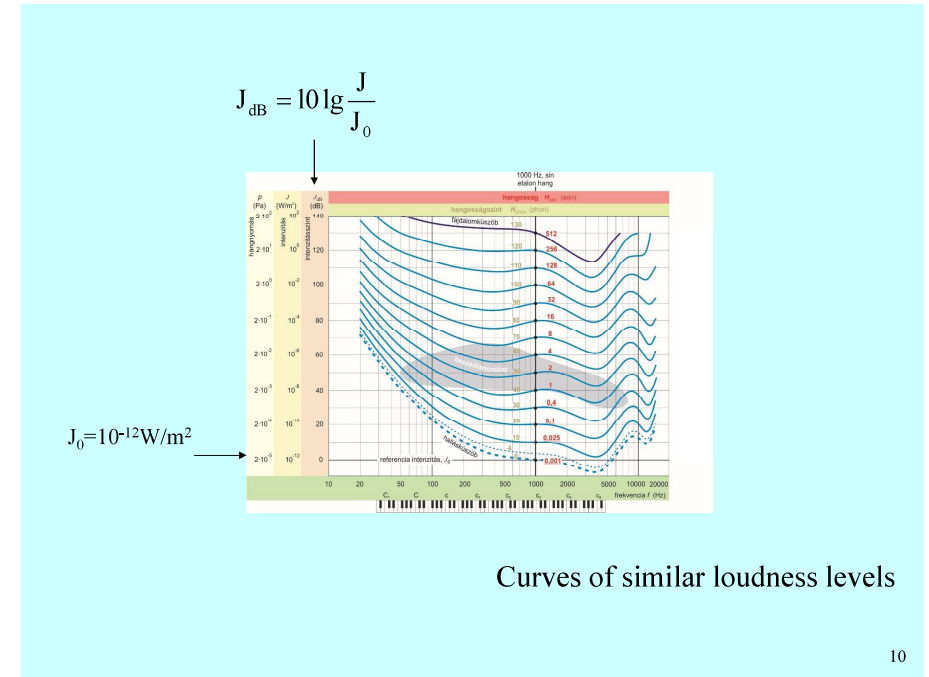
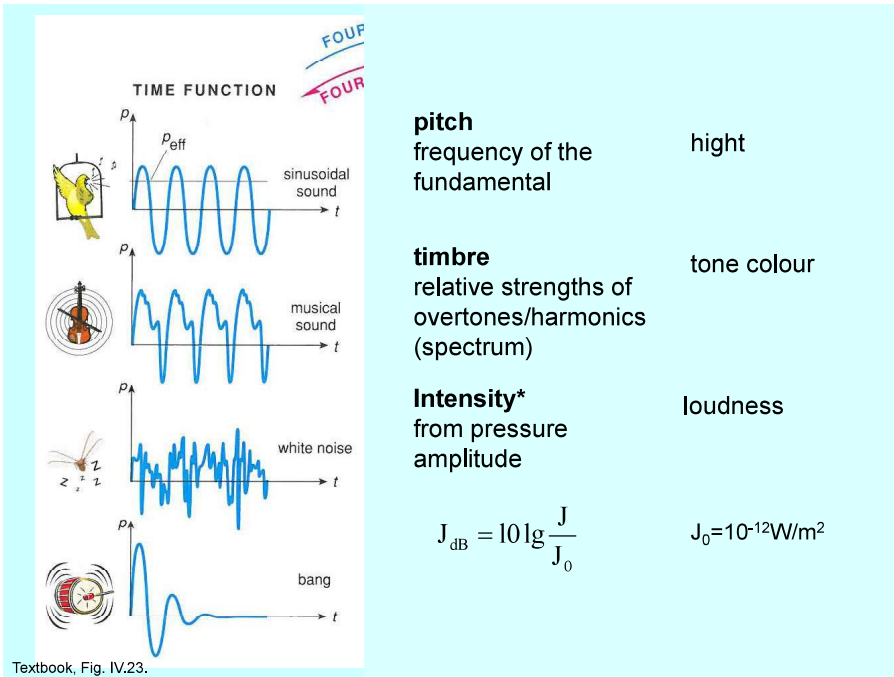
tone colour

**Intensity\***  
from pressure amplitude

loudness

## Frequency and intensity regions of sounds





What is the intensity of a 300 Hz sound that a person, who has 25 dB hearing loss at this frequency can hear? (The average hearing threshold at this frequency is  $3 \cdot 10^{-11} \text{ W/m}^2$ )

$$J_{dB} = 10 \lg \frac{J}{J_0}$$

$$25 = 10 \lg \frac{J}{3 \cdot 10^{-11}}$$

$$10^{2.5} = \frac{J}{3 \cdot 10^{-11}}$$

$$J = 9,5 \cdot 10^{-9} [\text{Wm}^{-2}]$$

## Propagation of sound/ultrasound

The role of elastic medium – *speed* of propagation

$$c = f\lambda$$

$$c = \frac{1}{\sqrt{\rho\kappa}}$$

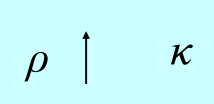


$\rho$ : density of medium

$$\kappa = -\frac{1}{V} \left( \frac{\Delta V}{\Delta p} \right) [\text{Pa}^{-1}]$$

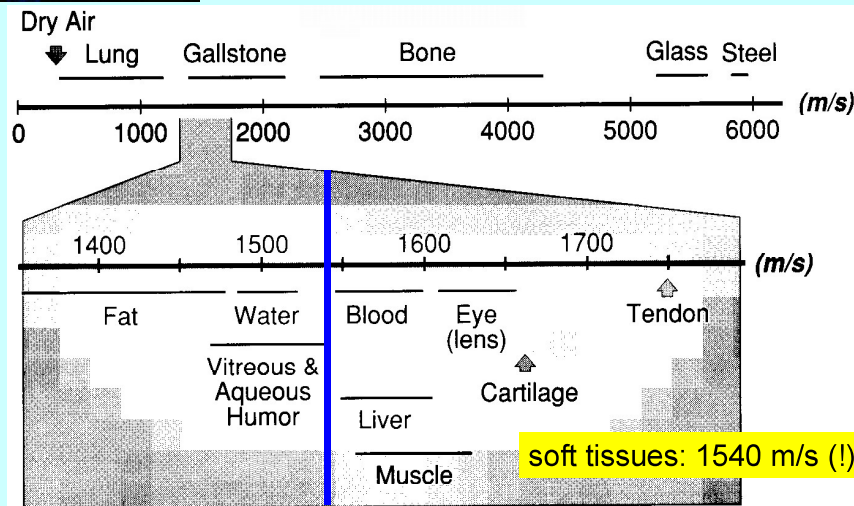
**compressibility**  
relative volume decrease  
over pressure

Speed of propagation is higher in solids than in liquids.





## Speed of sound/US in various media



## Propagation of sound/ultrasound

$$c = \frac{1}{\sqrt{\rho \kappa}}$$



$$\kappa = \frac{-\Delta V / V}{\Delta p} \quad [\text{Pa}^{-1}]$$

$$Z = \frac{p}{v} = \frac{p_{\max}}{v_{\max}}$$

acoustic **impedance**  
(definition)

$$Z_{\text{el}} = \frac{U}{I}$$

$$Z = c\rho = \sqrt{\frac{\rho}{\kappa}}$$

acoustic **impedance**  
(useful form)

$$[\text{kg} / \text{m}^2 \text{ s}]$$

14

## Propagation of sound/ultrasound

$$c = \frac{1}{\sqrt{\rho \kappa}}$$

$$\kappa = \frac{-\Delta V / V}{\Delta p}$$

$$Z = c\rho = \sqrt{\frac{\rho}{\kappa}}$$

| material   | $\rho$<br>[kg/m <sup>3</sup> ] | $\kappa$<br>[1/GPa] | $c$<br>[m/s] | $Z$<br>[kg/(m <sup>2</sup> ·s)] |
|------------|--------------------------------|---------------------|--------------|---------------------------------|
| air        | 1,3                            | 7650                | 331          | $0,00043 \cdot 10^6$            |
| water 20°C | 998                            | 0,45                | 1492         | $1,49 \cdot 10^6$               |
| aluminum   | 2700                           | 0,009               | 6400         | $17,28 \cdot 10^6$              |
| quartz     | 2650                           | 0,011               | 5736         | $15,2 \cdot 10^6$               |

15

### Problem:

A sound beam of 3 MHz frequency and 50 mW/cm<sup>2</sup> intensity propagates in blood.

What is the pressure? What is the maximal displacement and velocity of particles in this beam?

$$Z_{\text{blood}} = 1,66 \times 10^6 \text{ kg/m}^2 \text{ s}$$

### Solution:

Intensity:

$$J = \frac{p_{\max}^2}{2Z}$$

$$p = \sqrt{2JZ} = 40,74 \text{ kPa}$$

Velocity:

$$v = \frac{p}{Z} = \frac{40,74 \cdot 10^3}{1,66 \cdot 10^6} = 0,0245 \text{ m/s} = 24,5 \text{ mm/s}$$

Displacement:

$$A = \frac{v}{\omega} = \frac{24,5}{2 \cdot \pi \cdot 3 \cdot 10^6} = 1,3 \cdot 10^{-6} \text{ mm} = 1,3 \text{ nm}$$

16

## Intensity of US

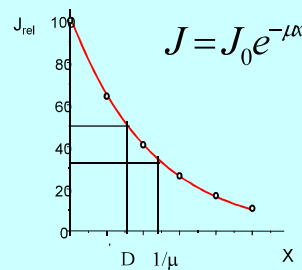
$$J = \frac{1}{Z} \Delta p_{eff}^2$$

intensity = energy/current density

$$P_{el} = \frac{1}{Z_{el}} U_{eff}^2$$

electric analogy

### Loss of energy during propagation (absorption)

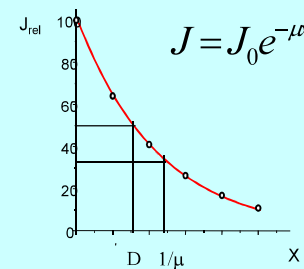


attenuation:  $\alpha = 10 \cdot \lg \frac{J_0}{J} \text{ dB}$

$$\alpha = 10 \cdot \mu \cdot x \cdot \lg e \text{ dB}$$

17

### Loss of energy during propagation (absorption)



attenuation:  $\alpha = 10 \cdot \lg \frac{J_0}{J} \text{ dB}$

$$\alpha = 10 \cdot \mu \cdot x \cdot \lg e \text{ dB}$$

$\mu$  is proportional to frequency in the diagnostic range

At  $f = 1 \text{ MHz}$

$D_{air} \sim 1 \text{ cm}$

$D_{water} \sim 1 \text{ m}$

Specific attenuation:  $\frac{\alpha}{f \cdot x}$

18

$\mu$  is proportional to frequency in the diagnostic range

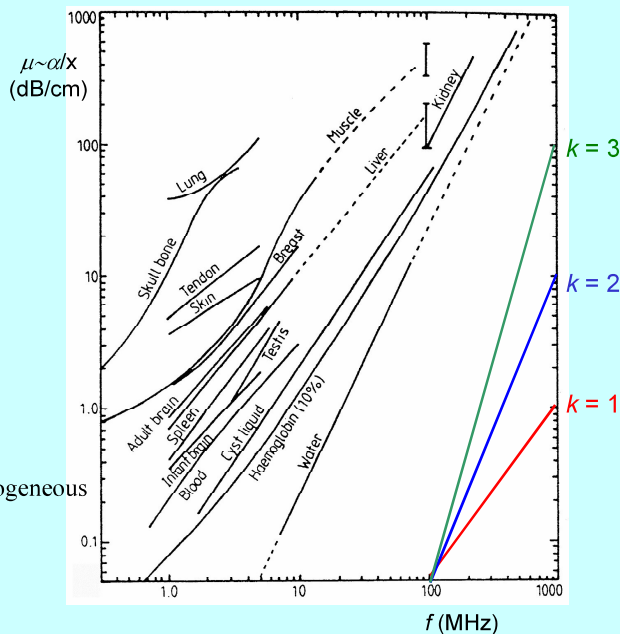
$$\mu \sim f^k, \quad k \sim 1(?)$$

$$\lg \mu \sim k \lg f$$

if the graph is a linear, the power function approximation is valid

specific attenuation for soft tissues (homogeneous tissue model):

$$\frac{\alpha}{f \cdot x} \sim 1 \frac{\text{dB}}{\text{cm MHz}}$$



19

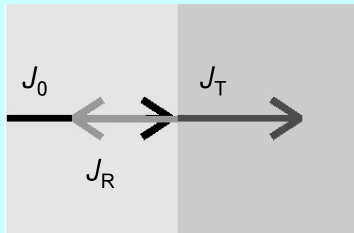
$$\frac{\alpha}{f \cdot x} \sim 1 \frac{\text{dB}}{\text{cm MHz}}$$

| tissue | Specific attenuation |
|--------|----------------------|
| liver  | 0,6 – 0,9            |
| kidney | 0,8 – 1,0            |
| fat    | 1,0 – 2,0            |
| blood  | 0,17 – 0,24          |
| bones  | 16 – 23              |

20

## Phenomena at the boundary of different media

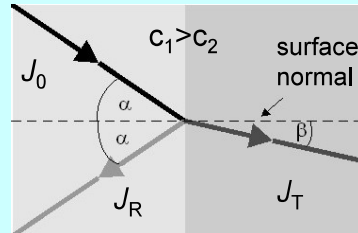
normal/perpendicular incidence



$$J_0 = J_R + J_T$$

reflection and transmission (penetration)

skew incidence

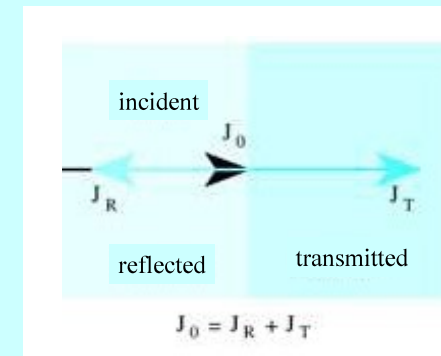


$$\frac{\sin \alpha}{\sin \beta} = \frac{c_1}{c_2}$$

Snellius-Descartes

21

## Reflection of ultrasound



$$R = \frac{J_R}{J_0}$$

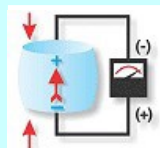
$$R = \left( \frac{Z_1 - Z_2}{Z_1 + Z_2} \right)^2$$

If  $R \approx 1 \longrightarrow$  Total reflection

22

## Detection/Generation of US

*Piezoelectric effect*



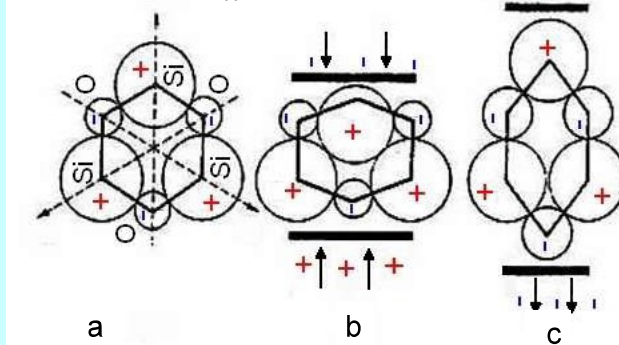
Pressure change

Mechanical deformation of crystal

**Electric potential difference**

23

*Piezoelectric effect*



(a) Center of charge of positive and negative charges coincides.

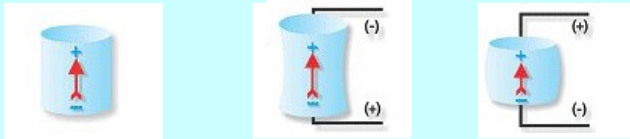
(b) and (c) As a result of pressure, the charge centers are separated, i.e. a potential difference arises

24



## Detection/Generation of US

### Inverse piezoelectric effect



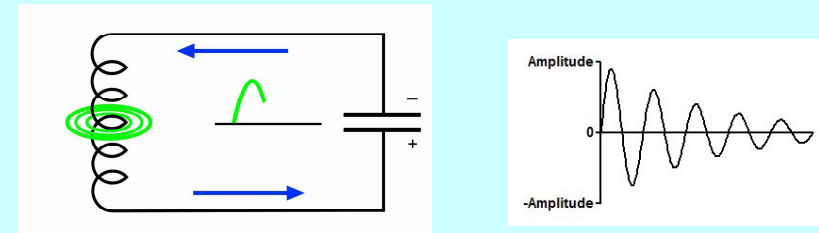
Periodic electric potential difference

The crystal is deformed when voltage is applied

**Mechanical vibration**

25

### Oscillator circuit – LC circuit



L : self inductance [ $\text{s} \cdot \Omega^{-1}$ ]

$$L \sim A N^2$$

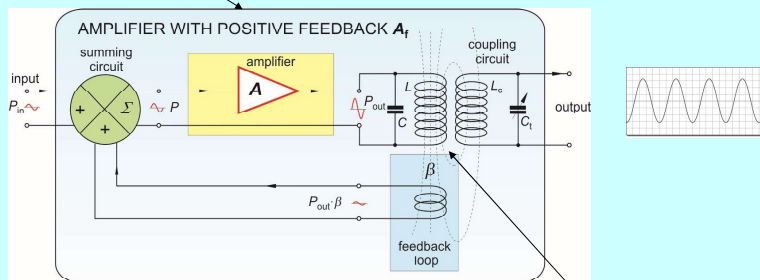
$$f = \frac{1}{2\pi\sqrt{LC}}$$

26

### Source of electric signal: **sine wave oscillator**

$$A_{\text{feedback}} = \frac{U_{\text{out}}}{U_{\text{in}}} = \frac{A}{1 - A \cdot \beta}$$

$A\beta=1$   
amplification = „infinity“  
no input signal, output signal: sine voltage



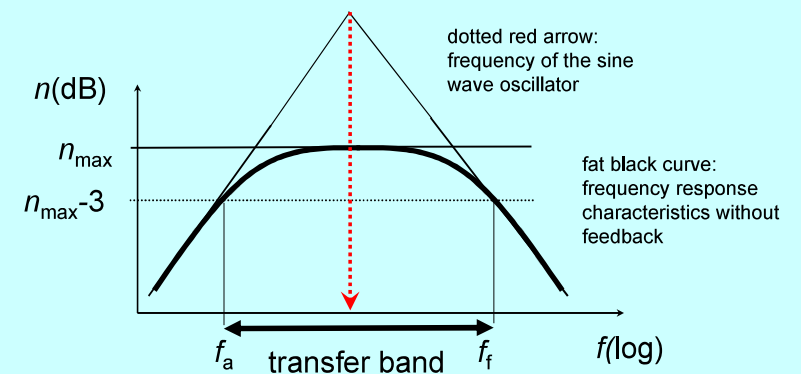
$$f = \frac{1}{2\pi\sqrt{LC}}$$

27

### Source of electric signal: **sine wave oscillator**

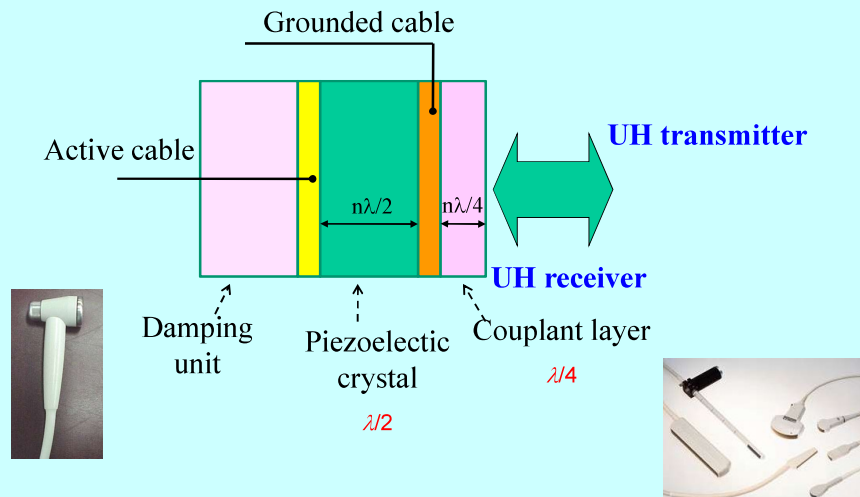
amplifier with positive feedback

$$A_{U, \text{feedback}} = \frac{A_U}{1 - \beta A_U}$$



28

## Detection/Generation of US - Ultrasound transducer



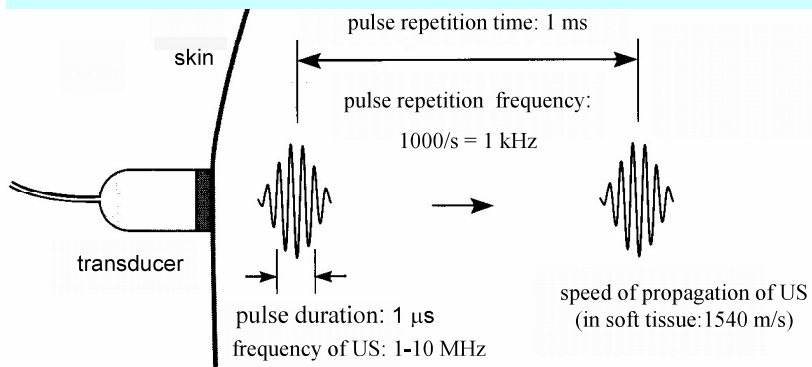
Question of the week

Why is the speed of propagation of US higher in bones than in soft tissues?

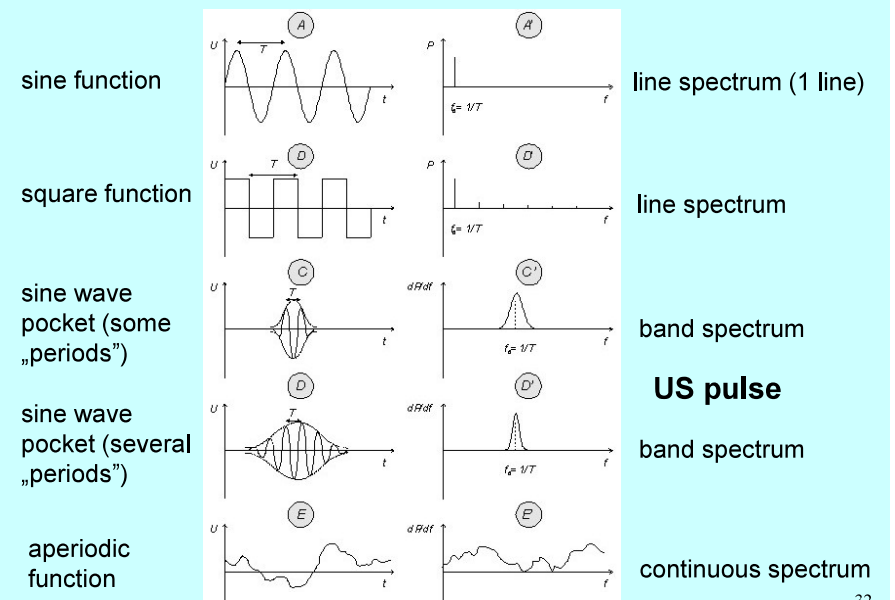
## Characteristic of US pulses

transducer: transmitter and receiver is the same unit

**time sharing** mode: pulses instead of continuous wave US

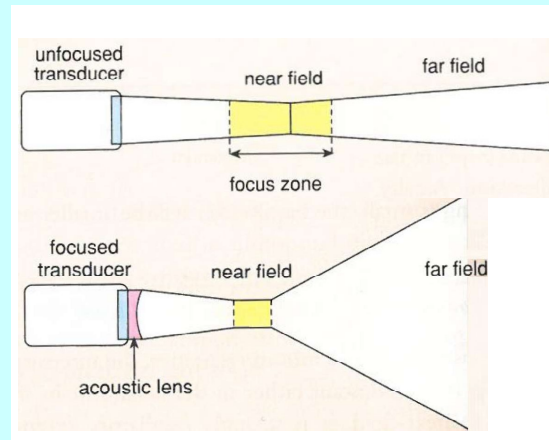


## Time function Spectrum





## *Focusing of the beam*



Focusing increases the divergence of the beam in the far field regime and reduces the depth sharpness.