Medical Biophysics II.

4th lecture: Diffusion, Brownian motion, Osmosis

28th February 2019.

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Diffusion?

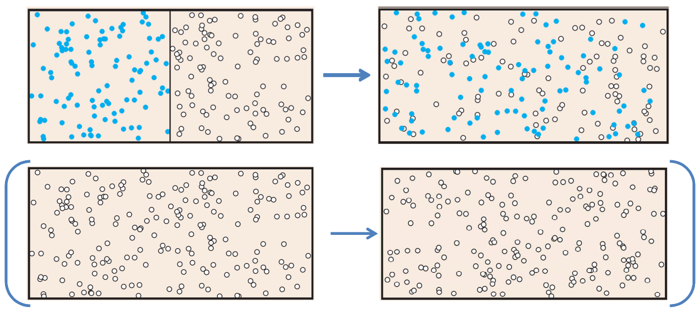
Why?

- physiology: cell function – ion diffusion…
- disorders: fibrosis, oedema, vasculitis, ascites…
- diagnostics: DWI MRI…
- therapy: dialysis, physiological saline….
- drug delivery: transdermal (liposomal), inhaled…

…….

Diffusion?

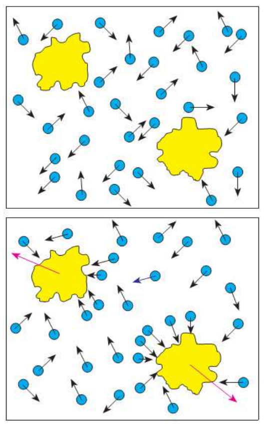
The change in the spatial distribution of particles because of **random thermal motion.**
In microscopic level with **net matter transport**.

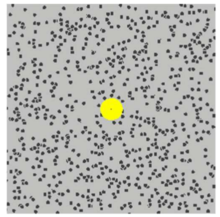


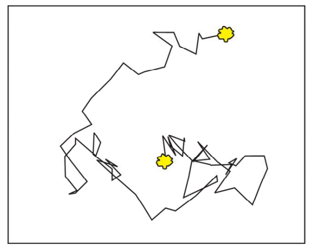
Relevant: NET transport of substance „A” in „B”.

Brownian motion

The „random walk” of a particles resulting from their collision with other particles.

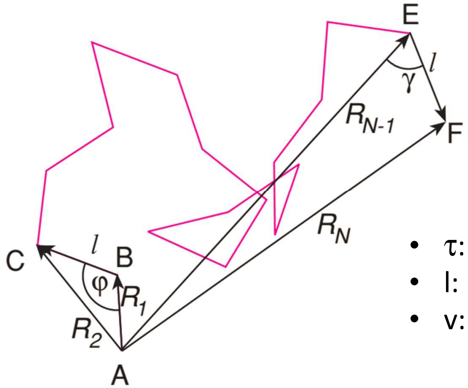






- τ: mean time between collisions
- l: mean free path
- v: mean speed of particles

How far reaches a particle?

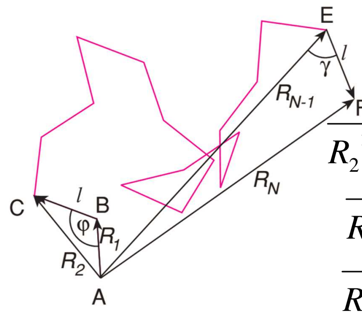


- τ: mean time between collisions
- l: mean free path
- v: mean speed of particles

One particle: $R_2{}^2 = R_1{}^2 + l^2 - 2 \cdot R_1 \cdot l \cdot \cos\varphi$

A „mean" particle:
(mean of *n* particles): $\overline{R_2{}^2} = \frac{1}{n} \cdot \sum_{i=1}^{n} \left(R_1{}^2 + l^2 - 2 \cdot R_1 \cdot l \cdot \cos\varphi_i \right)$

How far reaches a particle?



$$\overline{R_2{}^2} = \frac{1}{n} \cdot \sum_{i=1}^{n} \left(R_1{}^2 + l^2 - 2 \cdot R_1 \cdot l \cdot \cos\varphi_i \right)$$

$$\overline{R_2{}^2} = \frac{1}{n} \cdot \left(n \cdot \left(R_1{}^2 + l^2 \right) - 2 \cdot R_1 \cdot l \cdot \sum_{i=1}^{n} \left(\cos\varphi_i \right) \right)$$

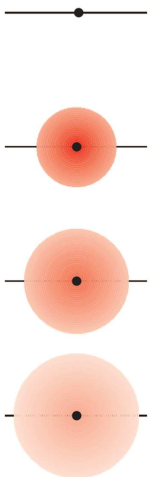
$$\overline{R_2{}^2} = R_1{}^2 + l^2 = l^2 + l^2 = 2 \cdot l^2$$

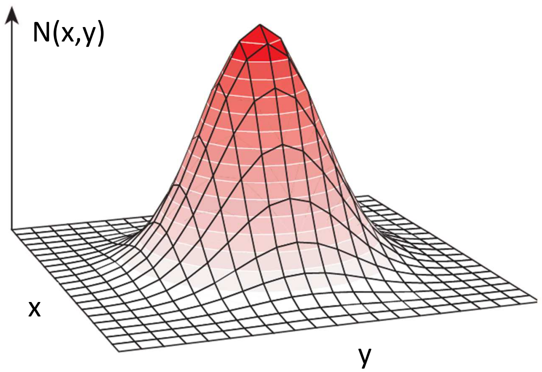
$$\overline{R_N{}^2} = N \cdot l^2$$

$$\overline{R_t} = \sqrt{N \cdot l^2} = \sqrt{\frac{t}{\tau} \cdot l \cdot l} = \sqrt{t \cdot v \cdot l} = \sqrt{3 \cdot D \cdot t}$$

$$\frac{v \cdot l}{3} = D$$

Experiment – 2D distribution





$$\sigma \sim \sqrt{D \cdot t}$$

Matter transport - flow

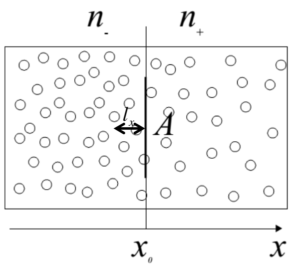
Particle flow rate: $I_N = \frac{\Delta N}{\Delta t}; \left[\frac{1}{s} \right]$

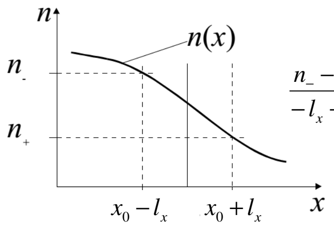
Particle flow density (flux): $J_N = \frac{\Delta I_N}{\Delta A}; \left[\frac{1}{m^2 \cdot s} \right]$

Matter flow rate: $I_\nu = \frac{\Delta \nu}{\Delta t}; \left[\frac{mol}{s} \right]$

Matter flow density (flux): $J_\nu = \frac{\Delta I_\nu}{\Delta A}; \left[\frac{mol}{m^2 \cdot s} \right]$

Fick's first law





$$\frac{n_- - n_+}{-l_x - l_x} = \frac{\Delta n}{\Delta x}$$

$$\Delta N = N_- - N_+ = \frac{1}{2}\cdot V_t \cdot (n_- - n_+) = \frac{1}{2}\cdot \overbrace{v_x \cdot \Delta t}^{l_x} \cdot A \cdot (n_- - n_+)$$

$$\Delta N = \frac{1}{2}\cdot v_x \cdot \Delta t \cdot A \cdot 2 \cdot l_x \cdot -\frac{\Delta n}{\Delta x}$$

$$v_x \cdot l = D$$

$$J_{Nx} = \frac{1}{2}\cdot v_x \cdot 2 \cdot l_x \cdot -\frac{\Delta n}{\Delta x} = -D\cdot\frac{\Delta n}{\Delta x}$$

$$J_v = -D\cdot\frac{\Delta c}{\Delta x}$$

BUT!: Δc is NOT the „driving force"!

Diffusion coefficient

D gives the amount of matter diffused across a unit area in a unit time in a case of unit concentration gradient.

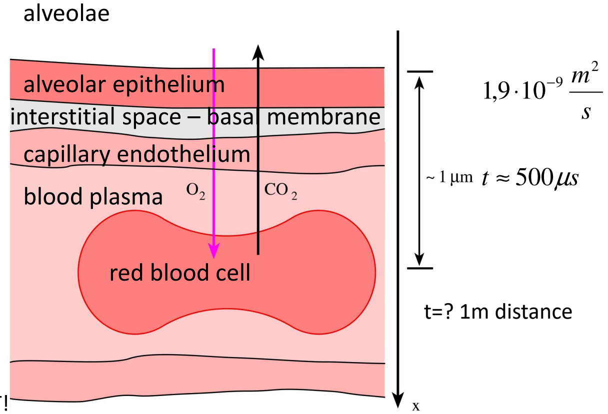
$$D = \frac{v\cdot l}{3}; \left[\frac{m^2}{s}\right]$$

$$D = u\cdot k\cdot T$$

Einstein-Stokes (spheres)

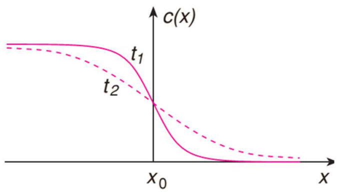
$$D = \frac{k\cdot T}{6\cdot\pi\cdot\eta\cdot r}$$

BUT!
Not directly proportional with T!

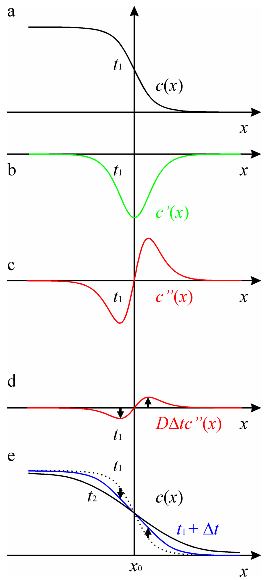


Fick's second law

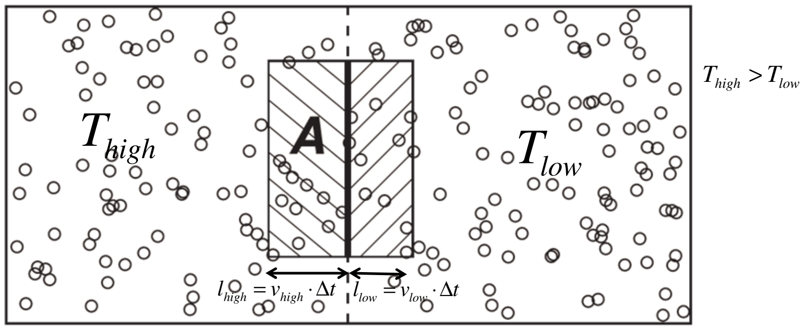
Fick II: change of the concentration gradient in time



$$c(t+\Delta t) = c(t) + D\cdot\Delta t\cdot\frac{\Delta\left(\frac{\Delta c}{\Delta x}\right)}{\Delta x}$$

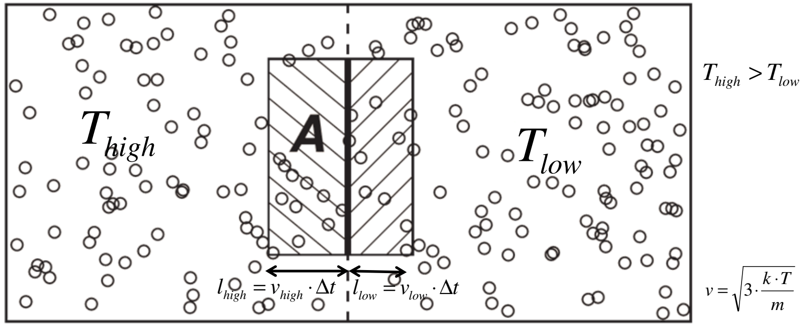


Thermodiffusion



$$\Delta N = N_{high} - N_{low} = \frac{1}{2}\cdot n\cdot\Delta t\cdot A\cdot(v_{high} - v_{low}) = \frac{1}{2}\cdot n\cdot\Delta t\cdot A\cdot(v_{high} - v_{low})\cdot\frac{(v_{high}+v_{low})}{(v_{high}+v_{low})}$$

Thermodiffusion

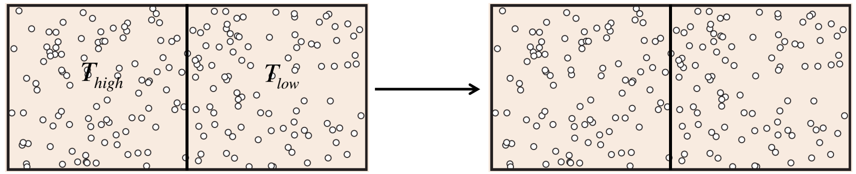


$$\frac{1}{2}\cdot n\cdot\Delta t\cdot A\cdot\frac{(v_{high}^2 - v_{low}^2)}{(v_{high}+v_{low})} = \frac{1}{2}\cdot n\cdot\Delta t\cdot A\cdot\frac{(v_{high}^2 - v_{low}^2)}{(v_{high}+v_{low})} = \frac{1}{2}\cdot n\cdot\Delta t\cdot A\cdot\frac{(\frac{3\cdot k\cdot T_{high}}{m}-\frac{3\cdot k\cdot T_{low}}{m})}{2\cdot v_{mean}}$$

$$\frac{1}{2}\cdot\left(\frac{n\cdot\Delta t\cdot A\cdot 3\cdot k}{m\cdot 2\cdot v_{mean}}\right)\cdot(T_{high}-T_{low})$$

$$J_v = -L_T\cdot\frac{\Delta T}{\Delta x}$$ (Ludwig-Soret effect)

Heat conduction



$T_{high} > T_{low}$

$$\Delta N = N_{high} - N_{low} = 0$$
$$N_{high} = N_{low}$$

$$\bar{\varepsilon} = \frac{3}{2}\cdot k\cdot T$$

Energy flow density $$J_v = \frac{\Delta E}{A\cdot\Delta t} = \frac{N_{high}\cdot\frac{3}{2}\cdot k\cdot\left(T_{high}-T_{low}\right)}{A\cdot\Delta t} = -\lambda\cdot\frac{\Delta T}{\Delta x}$$

(Fourier law for heat conduction)

Generalization

Onsager-relation: $J_{ext.} = L_{cond} * X_{int\_grad}$

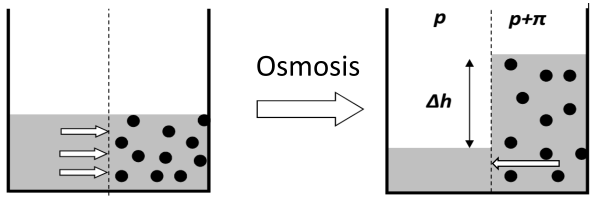
$J_{ext.}$: *flow density of extensive quantity* (eg. J_{matter})

X_{int_grad}: *gradient of intensive quantity* $(eg. \frac{\Delta c}{\Delta x})$

L_{cond}: *conductivity coefficient* $(eg. D)$

Osmosis

One-way diffusion of the SOLVENT.



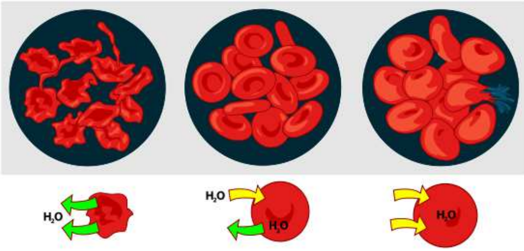
difference in the concentration of the SOLVENT

Hydrostatic pressure difference (osmotic pressure)

$p_{osm} = \pi = c_{solute}\cdot R\cdot T$ (Van 't Hoff law)

Osmotic concentration (equivalent osmotic pressure, „ozmolarity", „ozmolality"):
The concentration of a solution that keeps balance with a heterogeneous solution.
Derived units: *mOsm(/L)*, mmol/L, mmol/kg

Medical practice



Osmotic concentration of the blood: about 300mOsm/L

Physiological („isotonic”) solutions:

Physiological/Normal/Isotonic saline: 0,9% (w/v) NaCl
d5W: 5% (w/v) glucose
Ringer, Ringer’s lactate