Biophysics I.
for dentistry students

Lecture 1st:

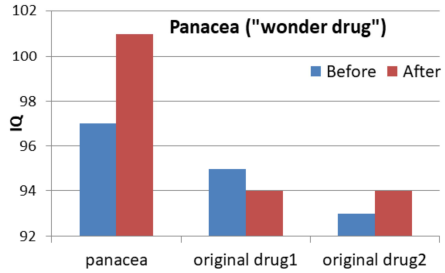
Biostatistics I.

2019. September 9.

Dániel Veres

Biostatistics – why to learn?

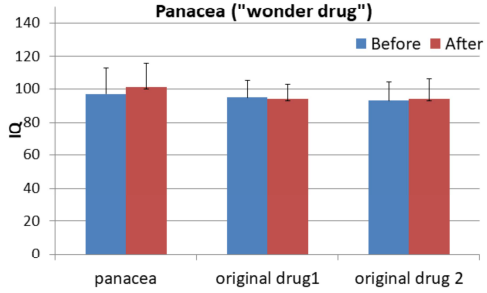
- „ To decide whether we should believe in something we are reading or to see where the mistake is, that is to say, do not fall so easily into statistical „juggling", artifacts and mistakes.



Why to learn Biostatistics?
In general 1. Juggling: eg. With graphs.

Biostatistics – why to learn?

- „ To decide whether we should believe in something we are reading or to see where the mistake is, that is to say, do not fall so easily into statistical „juggling", artifacts and mistakes.



Graphs: 0 as starting point and measures variability.

+Examples

- *Causality?*

(Ananas – tumor frequency, height – sleeping problems)

☺ eg: http://www.fastcodesign.com/3030529/infographic-of-the-day/hilarious-graphs-prove-that-correlation-isnt-causation

- *☺ eg: Chocolate Helps Weight Loss*

https://io9.gizmodo.com/i-fooled-millions-into-thinking-chocolate-helps-weight-1707251800

Other examples are available at:
Correlation:
https://www.fastcompany.com/3030529/hilarious-graphs-prove-that-correlation-isnt-causation
Multiplicity: https://io9.gizmodo.com/i-fooled-millions-into-thinking-chocolate-helps-weight-1707251800

Biostatistics – why to learn?

- „ To decide whether we should believe in something we are reading or to see where the mistake is, that is to say, do not fall so easily into statistical ,,juggling", artifacts and mistakes. (see excel – panacea,…)
- „To judge better whether we were lucky or not – or none of them"
- „ To judge better what is worth , whether it is worth for risking it" (eg. risk of a treatment)

Why to learn Biostatistics?
In general.

Biostatistics – why to learn?

- „ To decide whether we should believe in something we are reading or to see where the mistake is, that is to say, do not fall so easily into statistical ,,juggling", artifacts and mistakes. (see excel – panacea,…)
- „To judge better whether we were lucky or not – or none of them"
- „ To judge better what is worth , whether it is worth for risking it" (eg. risk of a treatment)

- „ So that we can do our best to design and evaluate our own statistics in our work (diploma…)."
- „I got an interested, unexpected result? I just discovered something or just the game of chance I see?"
- „ To make our results more understandable and effective, we can highlight the essence. "
- „ To have a clear understanding of the literature. "

(J. Reiczigel, A. Harnos, N. Solymosi – Biostatisztika nem statisztikusoknak)

Why to learn Biostatistics?
In general and in your studies.
(citated from: J. Reiczigel, A. Harnos, N. Solymosi – Biostatisztika nem statisztikusoknak)

Keywords in Statistics

VARIABILITY

Stochastic

Random

Why do we need statistics? We have an interesting world: things (patients, parameters…) are not the same – it has a variability. We don't know the answer before we ask, measure, observe something – it is stochastic (the starting circumstances do not determine exactly the results). We usually choose random items from all possibilities.

Tastitsticsss? What's that?

Statistics describes **random mass** phenomena.

- ***Data Collecting (Sampling)***
- ***Data Organization***

→ ***Descriptive Statistics***

- ***Data Analysis***
- ***Conclusion***

→ ***Inferential Statistics*** *(Inductive)*

One definition of statistics: statistics describes *random mass* phenomena.

To describe the „random mass" – so (several) variables with several outcomes (see later) - we perform the next actions: *collecting data* (sampling with other words), *organizing data, analyzing data* and *making conclusions*. The first two activity falls within the scope of *descriptive statistics* and the second two belongs to the *inferential statistics* (called inductive statistics also). Although there is no sharp boundary between the two plots.
I have to highlight that descriptive statistics – both data collection and organization – is always needed to create complete statistics and perfect conclusions.

Tastitsticsss? What's that?

Statistics describes **random mass** phanomenons.

- Data Collecting (Sampling)
- Data Organization

Descriptive Statistics

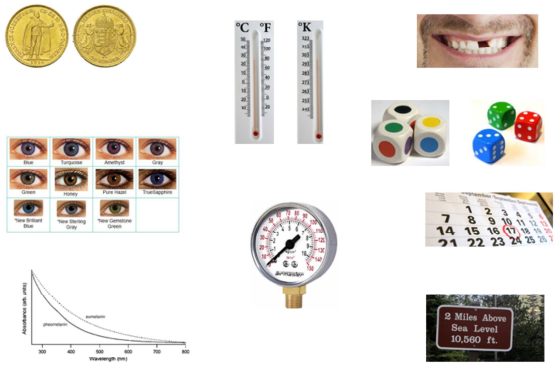
- Data Analysis
- Conclusion

Inferential Statistics (Inductive)

Let's begin with the descriptive statistics. Data organization helps to describe, show or summarize our data in the sample *in a meaningful way* such as, for example, patterns might emerge from the data.

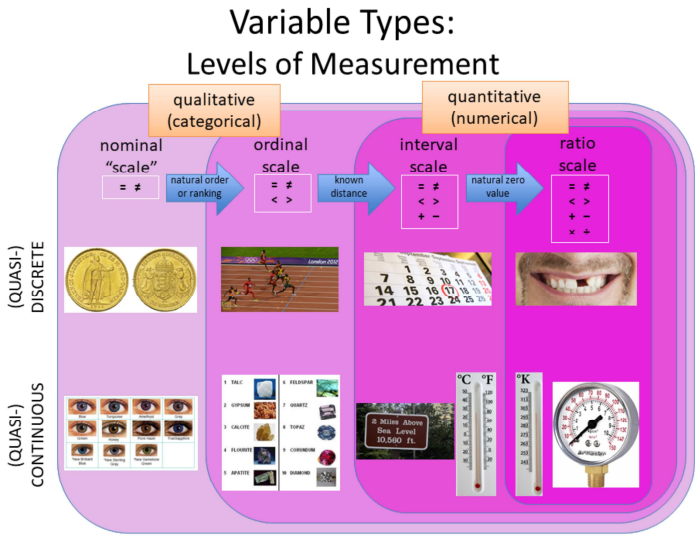
Variables, outcomes

Could be measured or observed



In statistics data are belong to *variable*s.
I give you here a simplified definition for variables: variable could be anything that we could measure or observe. E.g. tossing a coin, hair or eye color, temperature, blood pressure, rolling a die, etc.
When we are measuring or observing variables in a given circumstances it has possible outcomes.
E.g.: the outcomes of tossing a coin could be tail or head, the eye color could be brown, blue ,green, etc. but we can measure and characterize eye color by its spectra.
It is very import to handle the variable based on its outcome in the given situation.

For data organization *the level of the measuring scale* (the type) and the number of the examined variables *is crucial*.



There are many ways to group statistical variables depending on our aim.
In a practical view I would like to show a categorization on variables based on their possible *outcomes* so the *levels of measurement*.

As a first approach, we can group variables into the *qualitative (*also called *categorical)* and quantitative (*numerical*) types.

The most primitive scale s the nominal scale, which is at the bottom of the hierarchy of measurement scales. Examples are personal name, blood group, hair or eye color, citizenship etc. The scale is created by defining categories, these categories can be identified by simple naming (hence, "nominal"). During observations it is possible to determine whether two elements are identical or not. There is no natural order among categories, but there may be practical orders set (e.g. alphabetical order, assigned ordinal number), which are used according to tradition or customs, which help comparison. However, these orders do not have any meaning. Therefore, even the name "scale" is sort of misleading (misnomer), it would be more correct to speak about nominal system, which would not let us expect natural (meaningful) order. The delimitation of nominal categories may either be easier (self evident, like in case of coin tossing) or more difficult (arbitrary, e.g. eye color).

The ordinal scale also uses categories, but there's a natural order among them, examples are school notes, severity grade of diseases or injuries, or the Mohs scale of mineral hardness. Consequently, on a nominal scale not only identity can be defined, but "less than"/"greater than" relationships as well. Scale elements are usually denoted by ordinal numbers, which has to be kept in mind since the usual mathematical operations cannot be carried out on them. The difference or distance between the categories of an ordinal scale are either unequal or cannot be determined.

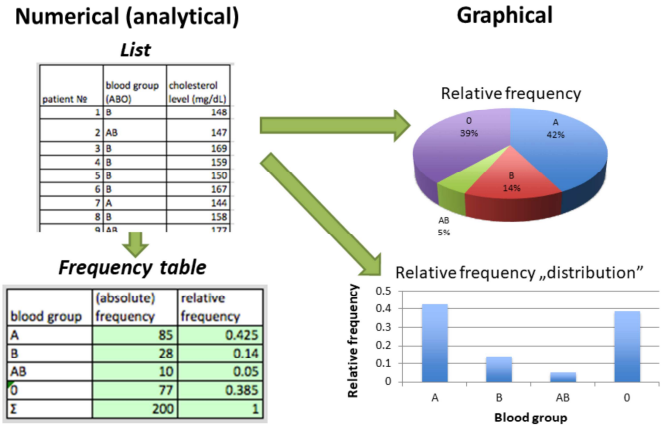
The interval scale is more developed than the ordinal scale because the distance between the possible values is known, so not only the order but the difference and addition can be interpreted. Examples from everyday life are calendar years, temperature in degrees Celsius or Fahrenheit, or the height above see level. It is evident from the examples that the zero value of interval scales is set arbitrarily.

Instead of such arbitrary zero values, ratio scales have natural zero values, actually ratio (and proportion) can be interpreted due to the existence of this natural zero value. So mathematical operations related to proportionality (i.e. multiplication and division) can also be carried out on such scales. Examples are: temperature measured on Kelvin scale, length, blood pressure...

It is possible to distinguish between more or less discrete and continuous variables at all scale levels based on the count of possible different outcomes. In practice usually we say continuous if we have at least 20 different outcomes.

In statistics for data organization and for the further evaluation *the level of the measuring scale* (the type) and the number of the examined variables *is crucial* as you will see in the next slides and the further lectures.

Description of Nominal Variables I.



Let's begin with how to organize a variable in a *nominal scale* using blood group type as an example.
In all statistical descriptions there are basically two options: *numerical* (called analytical also) where numbers are used to interpret the data and *graphical* where data are presented on charts.
After data collection we have a *list* that could be compacted to a *frequency table* or could be visualized on *frequency charts.* It show us the *frequency distribution.*

In the case of nominal variables the shown organizations are *without loosing information* that means we can recreate the original dataset on AB0 blood group if we are interested only on blood type, not taking into account which blood type belongs to which patients – so we made univariate description.

For further analyses or comparisons, it could be too much „information" – so we have to find a typical value – an indicator that could characterize our dataset.

Description of Nominal Variables II.

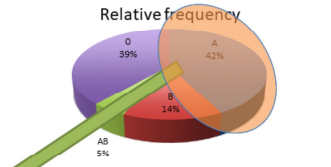
Numerical

Frequency table

blood group	(absolute) frequency	relative frequency
A	85	0.425
B	28	0.14
AB	10	0.05
0	77	0.385
Σ	200	1

Graphical

The brain and the common sense

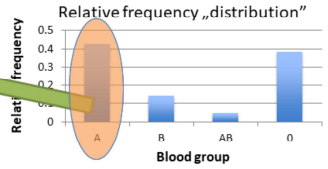


Organization, but **loss of information**

„Typical value" (***indicator***): ~~**Mean?!**~~

Mode: most frequent element(s)

Notation: *Mod*, x_{mod}



Other parameters:

data count (*n*)***, count of categories***

In the case of nominal scale this indicator could be only the *most frequent element, the mode*. (The mean has no meaning here...)

However, this solution has a disadvantage: only knowing the mode we can not restore the original data set – *we lose information*.

Never forget the power of a graphical interpretation: our human brain together with the common sense could easily find meaningful patterns behind the numbers! In the plots we could easily find the mode – that is *A* blood group type in the example. Another benefit of the graphical description that we could also easily see is the „goodness" of the mode in a certain case: in our example we realize that the frequency of *A* and *0* are very close.

There are other important parameters that are essential to the description: the *data count* (count of the data) and the *count of the categories*.

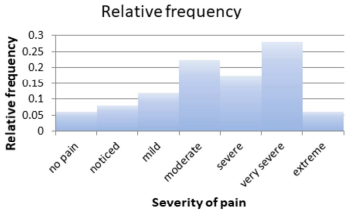
Description of Ordinal Variables I.

Numerical

Frequency table

Severity of pain	Relative frequency	Cumulative relative frequency
no pain	0,06	0,06
noticed	0,08	0,14
mild	0,12	0,26
moderate	0,225	0,485
severe	0,175	0,66
very severe	0,28	0,94
extreme	0,06	1
Σ	1	

Graphical



Indicator:
Mode

Other parameters:

data count (*n*)***, count of categories***

On ordinal scale – for example a severity of pain scale - we can use the same descriptive solutions as in the case of nominal variables: we can use frequency tables, frequency distribution plots and the mode.

But could we give a new indicator that use the advantage of sorting opportunity?

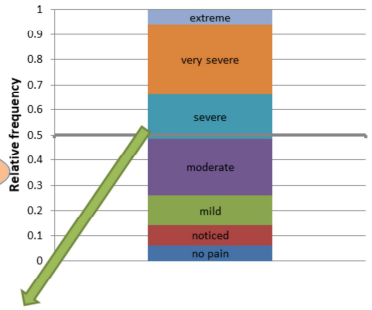
Description of Ordinal Variables II.

Numerical

Frequency table

Severity of pain	Cumulative relative frequency
no pain	0,06
noticed	0,14
mild	0,26
moderate	0,485
severe	0,66
very severe	0,94
extreme	1
Σ	

Graphical



New indicator:
Median: „middle" element(s)
Notation: Me, Med, x_{med}

Based on the ordering ability there is a new indicator that is the *median: the „middle" element(s),* or „middle" point(s) in a sorted dataset. It means that in the sorted dataset 50% of the elements is below of this value and the 50% is over it.
In this example the median value is *severe*.
Why I used plural between quotation marks? Could we use the quarter point like middle point? - Later we returns to it...

+: in this case there is a meaning to calculate *cumulative frequencies* – that means adding the scale frequencies until a limit. In the pain scale the cumulation shows the (relative) frequency of a given maximal pain.

Description of Quantitative Variables I.

Numerical (analytical)

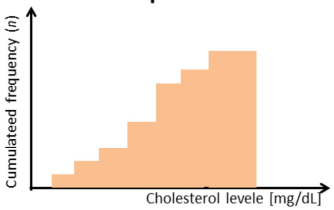
Frequency tables

frequency distributions (differential discrimination functions)				
bins (classes, intervals)	(absolute) frequency (FREQUENCY)	relative frequency	(absolute) frequency density	relative frequency density
x ≤ 100	0			
100 < x ≤ 110	0	0	0	0
110 < x ≤ 120	2	0,01	0,2	0,001
120 < x ≤ 130	5	0,025	0,5	0,0025
130 < x ≤ 140	22	0,11	2,2	0,011
140 < x ≤ 150	31	0,155	3,1	0,0155
150 < x ≤ 160	48	0,24	4,8	0,024

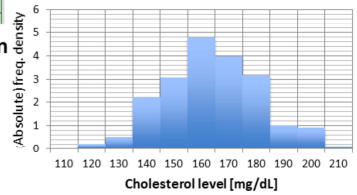
Organizing data – with **loss of information**

Determination of bin width:
- technical and aesthetic concerns
- statistical concerns

Graphical



(absolute)freq.density distribution

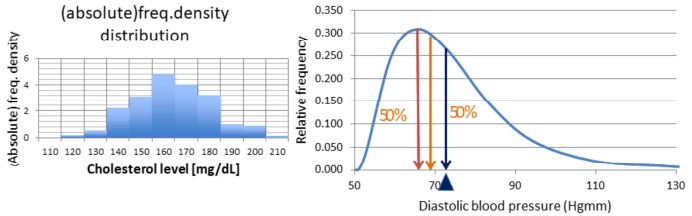


In the next slides we discuss on quantitative (numerical) variables.
In this situation we can preserve all information in case of graphical representation only when we create the cumulated frequency distribution.
Otherwise in the case of quantitative variables to create frequency tables or frequency distribution of the sample usually we have to *define arbitrary categories* called intervals, bins or classes.
Organizing data in this way *resulted loss of information*.
Remark: we don't lose information using cumulated frequency distribution.
How to determine bin width? It is based on *technical and aesthetic concerns together with statistical concerns*.
The statistical determination of the bin width uses often the next formula: bin width = (maximum-minimum)/(square root of data count).
The meaning of the technical and aesthetic concerns are more complicated. For example it has no meaning to use

smaller bin width than the smallest measurable difference or use non integer bin width if our measurand is integer. We like to see series like integers or 0,5,10,15 or 10,20,30...
Summarizing the best way to determine the bin width is first use statistical concerns then round it up based on technical and aesthetic aspects.

If we are using a variable at least an ordinal scale we can cumulate the frequencies: counting outcomes until a given limit (eg. Counting below 150 mg/dl coleseterol level). The frequency distribution made in this way called cumulated frequency distribution.

Description of Quantitative Variables II.



„Typical values" – ***central tendencies*** (special ***measures of location***):

- Mode: most frequent element(s) ?
- Median: „middle" element(s?)
- ***Mean*** (arithmetic mean): „gravity center" , sensitive to „outliers"?
  Notation: x_{mean}, $\bar{x}$

Advantage: compact, ***could be determined from few data***

Formulas: in the formula collection...

To describe a quantitative variable we have the same and several new opportunities as before. To „feel the meaning" of the indicators imagine that I could create a frequency distribution with infinitely small bin width. (The measured variable is the diastolic blood pressure of 4 years old boys.) One kind of „typical values" (indicators) that are very useful called central tendencies that try to describe the center of the distribution. These parameters are special measures of location.
*The mode as the most frequent elemen*t in the dataset belongs to the highest frequency – the peak – in the graph.
The median divide the area under the curve to two same area – 50% of the boys has smaller and 50% of them has higher blood pressure.
The mean is the center of gravity – so if I crop this curve from a paper I could balance this (like a teeter) in the point of the mean value.
The relative position of central tendencies are visible in the graph: in a non symmetric distribution the median and a

mean shifts to the tail respectively.
The advantage of the central tendencies against frequency distributions that these parameters could be determined from few data too.
I will return to the question (?) marks later.

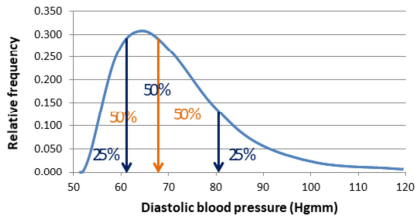
Remark

Average ≠ Mean

In statistics the average could mean:
- mode,
- median,
- means – arithmetic, geometric, harmonic... mean

Remark 1. In statistics average has different meaning than mean. The average could mean mode, median, means – arithmetic, geometric, harmonic... mean. Use the term *mean* in statistics not the *average*!

Quantiles I.



Other measures of location:

- Median: 50-50% (Q_2)
- ***Quartile***: lower quartile(Q_1): 25-75%; upper quartile (Q_3): 75-25%

General

p-quantile(s): is the number to which the count of data are smaller is maximum *n*p* and to which the count of data are larger is maximum *n*(1 – p)*,

where p is between 0 and 1, and *n* is the count of data

Now we definine other measures of location. Like the median as a midpoint we can determine „quadrant points" that divide the area in 25-75% proportion . This point (value) called *quartile* (from latin quartus that means ¼), more precisely *lower or upper quartile*.
We could generalize it and give a general dividing point (value) called quantile. *p-quantile(s)*: is the number to which the count of data are smaller is maximum n*p and to which the count of data are larger is maximum n*(1 – p), where p is between 0 and 1, and n is the count of data.
Using this general definition we could say that the median is the 0.5-quantile. The lower quartile is the 0.25-quantile, because ¼ = 0.25. It is called first quartile (Q1) also because 1 is divided by four. The upper quartile is the 0.75-quantile, or third quartile, because ¾ of all data is smaller than its value. With the same terminology we could call the median to second quartile (2/4).

Outliers…

Day	Waiting time (min)		
1	1,27	median	8,48
2	3,3	lower quartile	3,59
3	3,44	mean	7,72
4	3,64		
5	6,33		
6	7,72		
7	9,23		
8	9,87		
9	10,31		
10	12,29		
11	12,3		
12	12,98		

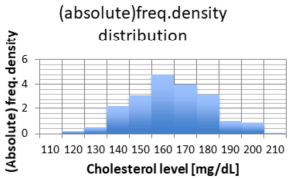
Day	Waiting time (min)		
1	1,27	median	8,48
2	3,3	lower quartile	3,59
3	3,44	mean	8,31
4	3,64		
5	6,33		
6	7,72		
7	9,23		
8	9,87		
9	10,31		
10	12,29		
11	12,3		
12	20		

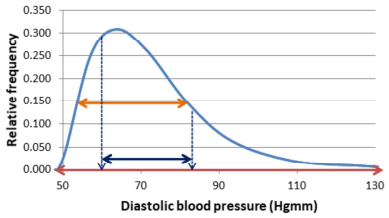
Median, quantiles could differ in theory and practice.
Mean is sensitive to the outliers, but quantiles not (...).
Mode?

Remark 2. In this slide I try to explain some of the points that mentioned before with ?,(s),"" marks.
Our example dataset is the waiting time in the public transport. In the slide I show you the sorted dataset.
At first about why I used plural for medians, quartiles and quantiles. For the median: 50% (p=0.5) of 12 is 6. It means 6 data is below the median and 6 is over the median. Based on the definition (theory) the median is the value between 7.72 and 9.23 – so all the numbers between! We have the same situation for the quartiles and any kind of quantiles. In practice excel calculate only one number using inverse proportions between the two numbers „in the border". For example in our dataset the lower quartile (25-75% smaller and larger) is theoretically between 3.44 and 3.65. The difference of this two numbers (the range between them) is 0.2. In practice the quartile value will be 3.44+(0.75*0.2) = 3.59.
Secondly, observe what will happen with the median and the mean if we have an „outlier" – a value that is far form

the others (we define it later). In our example I changed the highest value, the 12.98, to 20. We could realize that there is no change in the median while the mean changed greatly. In statistics we say that the mean is sensitive and the median is not sensitive to outliers.
Finally: what about the mode? Is there any, or all of them in our dataset? In the case of a continuous variable it is hard and meaningless to define a mode in the sample. (We may give a range if it is necessary.)

Description of Quantitative Variables III.





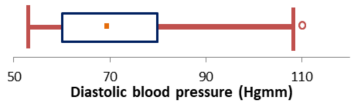
Measures of spread:

- ***Range***: the difference between the maximum and the minimum
- ***Variance (s^2):*** the average of the squared distance from the mean (corrected - sample, uncorrected - population)
- ***Standard deviation (s, sd, SD)***: the square root of the variance the width of the curve
- ***Interquartile range (IQR)*** : the difference between the upper and the lower quartile – not sensitive to the „outliers"

An other kind of indicators are that try to describe the width of the distribution – so the variations within the dataset. These parameters called measures of spread.
One of them is the *range*: the difference between the maximal and minimal values.
The *variance* is the average of the squared difference from the mean. We use the (Bessel) corrected variance if we describe the sample and without correction if we describe our population. The formulas for them are available in the formula collection.
The *standard deviation* is the square root of the variance.
The *interquartile range* is the difference between the upper and lower quartiles.
The variance and the standard deviation is sensitive for outliers while the interquartile range is not.

Description of Quantitative Variables IV.

Graphical: Box plot



Middle point: mean, or *median*
Box: 2*standard deviation, or *interquartile range,* p-quantile range
Whisker: 3*SD, minimum and maximum, 0.05 and 0.95 quantiles, p-quantiles, *1.5*IQR*...
out of whiskers: ***outliers***

Trimmed mean: mean calculated without outliers

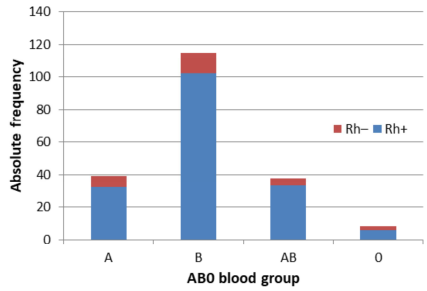
There is a very effective graphical representation of a quantitative variables using the mentioned indicators. That is the *Box plot*, also called Whisker plot.
It consists of a *middle point* that is typically the median, but sometimes the mean. (Now I represented the median.) We may use mean if we have a symmetrical distribution without outliers.
It has a *box* that represent typically the interquartile range, but it could show the standard deviation, standard error too. We use standard deviation or standard error (if we have few data) if we use the mean as a middle point. We use the interquartile range if we have the median as middle (as in the example).
And it has *whiskers*. If the dataset doesn't contains values that are „very different" we could use the minimum and maximum for whiskers. Otherwise we use the multiple times of the SD (usually 2 times) or IQR (1.5 times typically, as in our example) for the mean or median respectively. The 1.5*IQR is commonly called non outlier range.
The outliers are the values that are out of the outlier range.
As you see there is a lot of possibility how we can construct our box plot, I only gave a recommendation that you have to know.
It is important to show what we use in the certain case.
There is an other variant of mean called trimmed mean that is the mean calculated without outliers.

Qualitative Bivariate Description

Numerical: ***contingency*** table

	A	B	AB	0	Σ
Rh+	32	102	33	6	173
Rh-	7	13	5	2	27
Σ	39	115	38	8	200

Graphical: ***stacked bar chart***

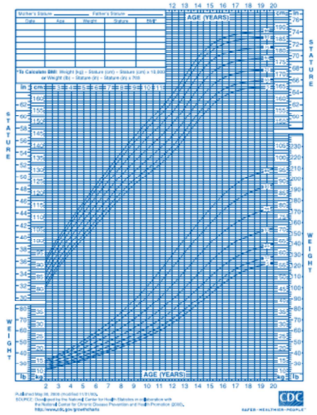


Describing more than one variable together is more difficult.
Now I give only some example for them. To organize two qualitative variables we usually use *contingency tables* – that is a 2 way frequency table. For graphical representation we can use *stacked bar charts*. Here the power of graphs for our mind is very obvious.

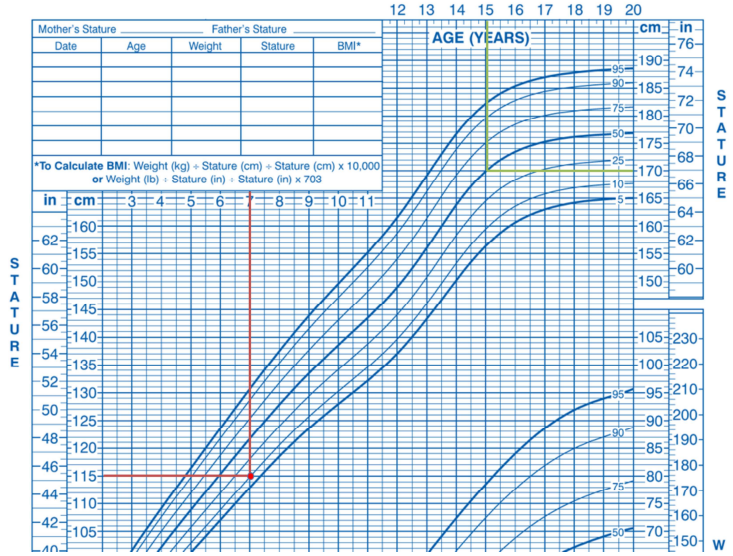
Quantitative Bivariate Description

Graphical: ***percentile curves***

Percentile: quantile expressed as percentage



For two quantitative variable we usually use *scatterplots*. But in medical and dental practice there is an other representation for two quantitative variables called *percentile curves*. These are very common in pediatrics. The percentile is a quantile expressed as percentage.



How to use this curves? What is the meaning of them?
What is the meaning of the red point?
In our example the red point – that is in the 10th percentile curve – means that 10% of the 7 years old boys are less than 115 cm.

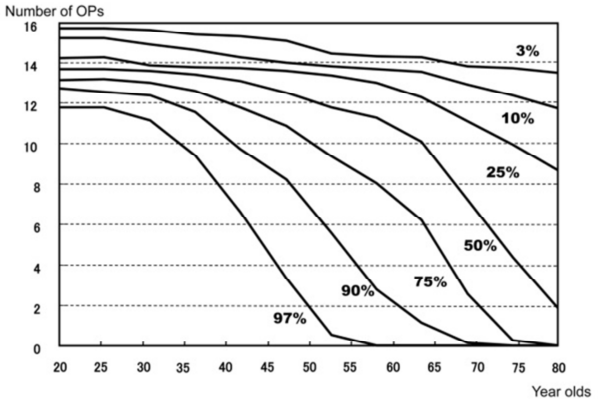
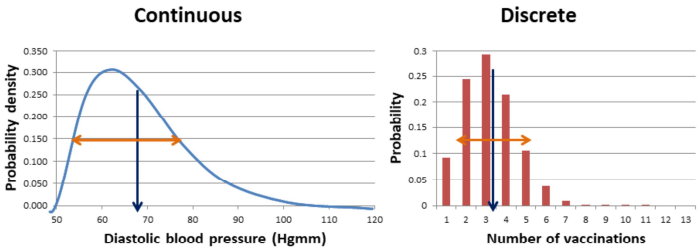


Fig. 1 Percentile curves of occluding pairs in males (n = 1,535)

Dental example.

Parameters of Theoretical Distributions



- ***Expected value(E) (location parameter)***

$$E(\xi) = \int_{-\infty}^{\infty} p_i \cdot x_i \qquad E(\xi) = \sum_{i=1}^{m} p_i \cdot x_i$$

- Theoretical variance (Var, D^2) (scale parameter)

$$Var(\xi) = E\left[\left(\xi - E(\xi)\right)^2\right]$$

27

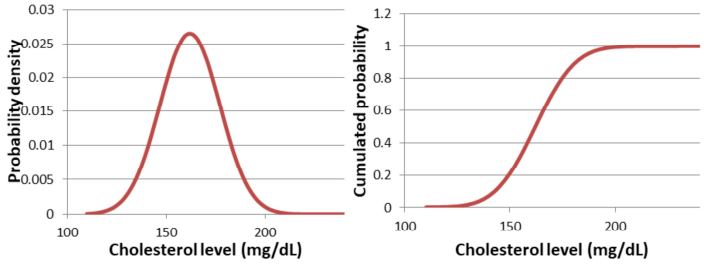
Theoretical means here : imagine that our dataset is infinitely large – we measured everything/everybody. Theoretical distributions have similar parameters as mentioned before.
There is a parameter that describe the center of the distribution and an other one that describe the width of the distribution.
The first one called *expected value* (abbreviated with E), the second is the *theoretical variance* (Var). In the equation *x* is the given value and *p* is the probability of that value. The expected value calculated slightly differently for continuous and discrete variables.

The **expected value is equal with the *mean of the infinite dataset***. For continuous variable we use infinite small binwidth for summarization – that is the integral (∫).

This two indicator (the expected value and the variance) defines exactly the distribution that means knowing this indicators we could calculate the probability for all value.

Calculation of an expected value and theoretical variance has been shown in the lecture. (see attached excel file).

Normal (Gaussian) Distribution I.



Cholesterol level, glucose level.....
Height, BMI...
Diastolic blood pressure of adults
.....

$$E(\xi) = \mu$$

$$Var(\xi) = \sigma^2$$

$$P = \frac{1}{\sigma \cdot \sqrt{2} \cdot \pi} \cdot e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

Normal (reference) range: 95% of data is in this range: $\sim \mu \pm 2 * \sigma$

The normal or Gaussian distribution is the most common theoretical distribution in medical and dental practice.
In this slide I plotted both the relative distribution and cumulative frequency functions, because this is the most important distribution for us.

If we know the mean and standard deviation (SD) we know everything! Eg. ~95% of data is not more fram from the mean than 2*SD – called normal or reference range.

The most of the variables in medical practice follows normal (Gaussian) distribution – e.g.. enzyme levels, height, body mass index (BMI), blood pressures...
Why?

Gaussian Distribution II.

Central limit theorem (on variables): for given conditions, adding a large number of independent variables yields a normally distributed variable.

Central limit theorem (on sampling): for given conditions, sampling with large sample size (n) the distribution of the sample means is normal with:

$$\mathrm{Var}_{\mathrm{normal}} = \frac{\mathrm{Var}_{(\mathrm{original})}}{\mathrm{n}}$$

The reason of why we have normal distribution in most of the variables in medical and dental practice described by the central limit theorem. It says that summarizing large number of independent variables resulted a normally distributed variable. In medical practice most of the measure values affected by several factor: gens from father, gens from mother, nutrition, way of life...

An other important wording of this law: if you take a sample with a sample size n and n is large the distribution of the sample means is normal and its variance is Var(original)/n

(Example given in the attached excel file.)

Tastitsticsss? What's that?

Statistics describes **random mass** phanomenons.

- Data Collecting (Sampling)
- Data Organization

Descriptive Statistics

- Data Analysis - estimations
- Conclusion

Inferential Statistics (Inductive)

Let's continue with the second part : inferential statistics. Here we execute data analyses („estimations") and we word conclusions.

Population and Sample

Population

The size of the **population** usually does not allow the examination of all of its elements.

RANDOMNESS!

Sample

Therefore, only a subset of the population is examined. That is what we call a **sample**.

31

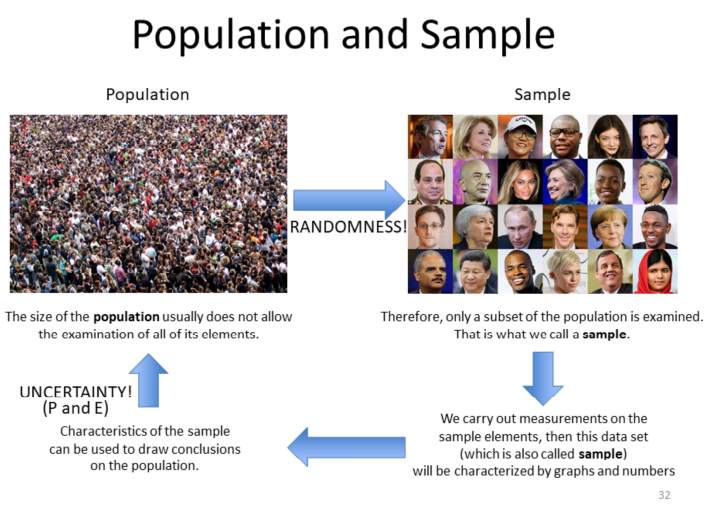
Here I try to describe the basic problematics of statistical inference.

As we mentioned before, statistics examines random mass phenomena. This means that during examination of a phenomenon many, if not infinitely many measurements would be possible. The set containing the outcomes of all these theoretically possible measurements is called **population**. Theoretically, the complete understanding of a variable would require the execution of all the possible measurement, but of course it is not possible.

Consequently, we only observe a subset of the population, which is called **sample**. The most evident way of generating this subset is **random selection**.

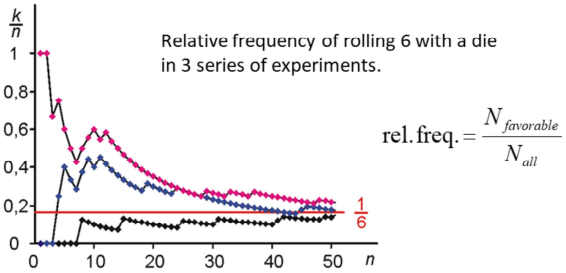
We carry out measurements on the sample, the set of measurement results is also called **sample**. (That is: in less precise way the sample may be a group of students

[individuals, objects] of the university as a population. In more precise way, the population is the height of all people at the university, the sample is the set of height values for a group that was actually measured.)



The sample might be characterized graphically or numerically as we learned in the last lecture, then the properties learned that way may be extrapolated to the population. E.g. if 25% of people in a group have blood type “A”, we may expect the same from the whole population. Since the sample is chosen randomly, it will not necessary represent the population, the frequency of occurrence of different values within the population perfectly. As a result, every conclusion drawn from a sample carries a burden of **uncertainty**.
What is the quantity of the uncertainty? How to define?
We are using 2 term for it: probability (P) and error E.

Probability I.



Relative frequency of rolling 6 with a die in 3 series of experiments.

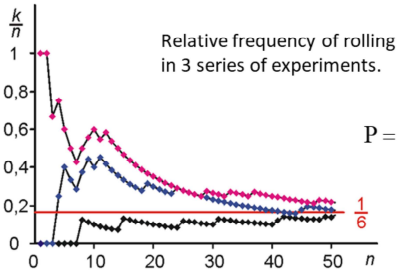
$$\text{rel.freq.} = \frac{N_{favorable}}{N_{all}}$$

We experience that **relative frequencies** – although with fluctuations – **tend to a certain value** independetly from the actual series of experiments if we **increase the number of the experiments**.

Let's imagine we roll a die 50 times and we count the relative frequency of rolling 6. We repeat this experiment 3 times.
We experience that **relative frequencies** – although with fluctuations – **tend to a certain value** independently from the actual series of experiments if we **increase the number of the experiments**.

In an other experiment we roll a die 50 times and we count the relative frequency of rolling 6. We repeat this experiment 3 times. We experience that the *relative frequencies* (frequency of favorable/all measurement) *tend to a certain value independently from the actual series of experiments if we increase the number of the rolls.*

Probability as a Quantity



Relative frequency of rolling 6 with a die in 3 series of experiments.

$$\text{P} = \text{rel.freq.} = \frac{N_{favorable}}{N_{all}}$$

$$N_{all} \to \infty$$

Law of large numbers (on relative frequencies): the relative frequency in an infinite sequence tends to a certain value.
We assign that **certain value** to an **event**: ***1/6*** to ***rolling 6*** with a die.
This value is called the ***probability of an event***.
This is an *empirical law – cannot be proven* by logical sequence.

Law of large numbers (on relative frequencies): the relative frequency in an infinite sequence tends to a certain value.
We assign that certain value to an event: 1/6 to rolling 6 with a die.
This value is called *probability of an event*. (The probability of an outcome of a variable in a given situation.)
The relative frequency is equal to the probability if the sequence is infinite.
This is an empirical law – can not be proven by logical sequence.

Probability of Events I.

Axioms on probability of events (Kolmogorov):
1. **0 ≤ P(A) ≤ 1**

2. **P(*sure*) = 1** *(The patient will die sooner or later)*
P(*impossible*) = 0 *(I'm 310 cm tall)*

3. Mutually exclusive events (i.e. P(AandB)=0)
P(AorB)=P(A)+P(B)
(probability of being pregnant or male)

And a theorem:
+4. *Independent events*: **P(AandB)=P(A)*P(B)**
(probability that our first patient is male and the second one is female)

To describe the probability of events we have axioms. Now we show the Kolmogorov axioms. (In a simplified way.)
1. The probability of an event is between 0 and 1.

2. The probability of a sure event (*the patient will die sooner or later* – we know that life is a sexually transmitted lethal disease☺) is 1. The probability of an impossible event is 0 (*I'm 310 cm tall*).

3. The probability of A or B events occur if A and B are mutually exclusive (they could not happen in the same time) events is the sum of the probability of A and the probability of B events. (*The probability that being pregnant or male is the probability that being pregnant + the probability that being male.*)

A theorem based on the axioms:
+4. The probability of A and B events occur if A and B are independent events (the occurance of an event has no effect on the occurance of the another) is the multiplication of the probability of A and the probability of B.
(*Probability that our first patient is male and the second one is female is the probability that our first patient is male * the probability that our second patient is female.*)

These mentioned statements are true from other way round. For example if P(A)*(P(B)=P(AandB) then A and B are independent events.

Probability of Events II.

Conditional events calculation:
general form: **P(A|B)=P(AandB)/P(B)**
Special cases:
I. Independent events:
Probability that our second patient is male
***if** the first one is female*

P(A|B)=P(AandB)/P(B)
P(A|B)=P(A)*P(B)/P(B)
P(A|B)=P(A)

Probability that our second patient is male
***if** the first one is female = Probability that our second patient is male*

There is an other important calculation that you have to know on conditional events. I showed here a simplified form of Bayes' law. The general form of the „multiplication role" is P(A|B)=P(AandB)/P(B). First examine 2 special case.
Case1: check the conditional probability in case of independent event. E.g. *probability that our first patient is male **if** the second one is female.* As we see the result the independent condition has no effect on the probability of the event; e.g. *probability that our second patient is male **if** the first one is female = probability that our second patient is male.*
We have the same with tossing a coin, rolling a die, etc.: the previous results has no effect to the next one.

Probability of Events II.

II. event A is a subset of event B
Probability that a patient has a flu
if suffering from a viral infection

P(A|B)=P(AandB)/P(B)
P(A|B)=P(A)/P(B)

Calculation:
The probability that a patient coming to our office has viral infection is 8% =P(B)
The probability of occurrence of flu infections at our office is 2% = P(A)
The probability that a patient suffering from a viral infection has actually flu is: P(A|B) = 2% / 8 % = 25%.

Case2: event A is a subset of event B (all A is a B, but not all B is A).
An example:
The probability that a patient coming to our office has viral infection is 8% =P(B) – that is the probability of the condition.
The probability of occurrence of flu infections at our office – that is the probability of our event in the given sample. *P(A)* = 2%
The probability that a patient suffering from a viral infection has actually flu - P(A|B) – is 25%.

Probability Calculus

Permutations,
Variations,
Combinations

Probability calculation and statistics are based on the permutations, variations and combinations. But in this course we won't go into the details of math.

Probability Calculus Example

During last year's flu epidemic 402 out of the total 2989 patients who turned up at a doctor's office required vaccination. Based on last year's data what is the probability that 4 vaccines will be sufficient (exactly, i.e. no vaccines will be left), if we are expecting a total number of 25 patients?

$$P = \binom{n}{k} \cdot (p)^k \cdot (1-p)^{(n-k)} = \binom{25}{4} \cdot \left(\frac{402}{2989}\right)^4 \cdot \left(1-\frac{402}{2989}\right)^{(25-4)} \approx 0{,}2$$

How to calculate (in excel)? How to read out from a graph, table? Which equation, table, excel function should we use?

An example why and how we use the probability calculus. During last year's flu epidemic 402 out of the total 2989 patients who turned up at a doctor's office required vaccination. Based on last year's data what is the probability that 4 vaccines will be sufficient (exactly, i.e. no vaccines left) in a certain day, if we are expecting a total number of 25 patients?

To answer question like that, our main questions will be: How to calculate (in excel)? How do we know the equations? Which equation, table, excel function we should use?

Human thinking and probability...

Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

Which is more probable?

a) Linda is a teacher in a secondary school
b) Linda works in bookstore and participates in yoga courses
c) Linda is a member of the league of women voters
d) Linda is a bank teller.
e) Linda is an insurance agent
f) Linda is a bank teller and is active in the feminist movement.

I'd like to highlight two statement: *d* and *f*. I hope everybody found out that co-occurrence is less probable than occurrence of a given event. The intersection of sets is always equal or smaller than the sets. So it is less probable that Linda is a bank teller and active feminist than she is a bank teller.

Test Questions #1

- Name some ordinal variables, scales.
- Name some discrete numerical variables, scales.
- Name some continuous numerical variables, scales.
- What is the substantial difference between a nominal and an ordinal scale?
- Give example for interval scale.
- What is the substantial difference between an ordinal and an interval scale?
- Give examples for ratio scale.
- What is the substantial difference between an interval and a ratio scale?
- Why is it important to define a statistical variable properly?
- What are the indicators that we can use to describe a nominal variable?
- What are the indicators that we can use to describe an ordinal variable?
- What are the indicators that we can use to describe a numerical variable?
- Define the mode(s) of a dataset.
- Define the median(s) of a dataset.
- What are the central tendencies in case of a numerical variable?
- What is the „meaning" of the mode in a diagram?
- What is the „meaning" of the median in a diagram?
- What is the „meaning" of the mean in a diagram?
- Define the mean of a dataset.
- Which central tendency sensitive to outliers?
- What is the advantage of indicators versus distribution functions?
- What are the measures of location?
- Define the p-quantile.
- Define the lower quartile.
- Give the definition of probability based on relative frequences.
- What is the law of large numbers?
- How tends the relative frequencies to the probability? [fluctuations, infinite seuence]
- How we can prove the law of large numbers?
- How to calculate P(A) if P(A|B) and P(B) is given?
- What are the Kolmogorov's axioms?
- What is the central limit theorem?

The following questions may be answered using lecture material, consultation with practice teacher, or your own investigation (on the library or the internet). These test questions are examples for multiple choice items that may occur in the midterm and exam tests.