

# Power Function: Example

## Mathematical and Physical Basis of Medical Biophysics

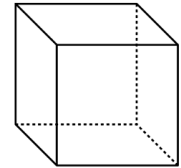
### Lecture 1

Mathematics Necessary for Understanding Physics  
Physical Quantities and Units

10<sup>th</sup> September 2018

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mass  $\propto$  volume  $\propto$  [body]length<sup>3</sup>  
surface area  $\propto$  [body]length<sup>2</sup>



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## Power Function

**INTEGRAL FORM**

**VARIABLES:** dependent variable  $y$ , independent variable  $x$

$y = b \cdot x^a$

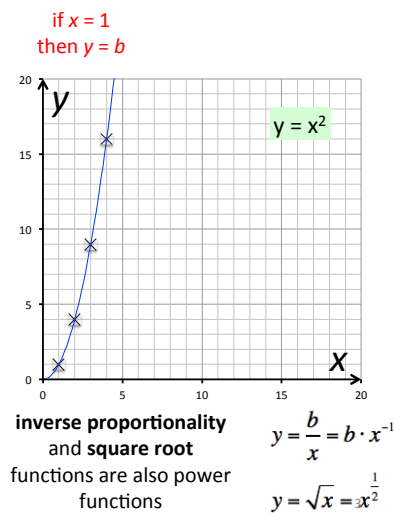
**PARAMETERS:** pre-exponential coefficient  $b$ , exponent  $a$

explicit for  $y$ :  $y = b \cdot x^a$   
explicit for  $x$ :  $x = (y/b)^{1/a}$

**"DIFFERENTIAL" FORM**

$\Delta y/y \propto \Delta x/x$

The **relative change** of the dependent variable is proportional to the **relative change** of the independent variable



## Power Function: Linearization

**graphical linearization**  
plot both  $y$  and  $x$  on log scales:  
the relationship **looks** linear but it **is** still power function

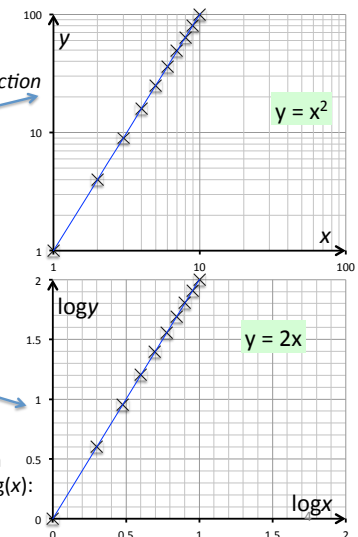
**INTEGRAL FORM**

$y = b \cdot x^a$

$\log y = \log(b \cdot x^a)$   
 $\log y = \log b + \log(x^a)$   
 $\log y = \log b + a \cdot \log x$   
 $\log y = a \cdot \log x + \log b$

intercept =  $\log b$   
 $\log 1 = 0$   
slope =  $a$   
 $a = 2$

**arithmetical linearization**  
plot  $\log(y)$  as a function of  $\log(x)$ :  
the relationship **is** linear

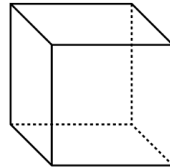
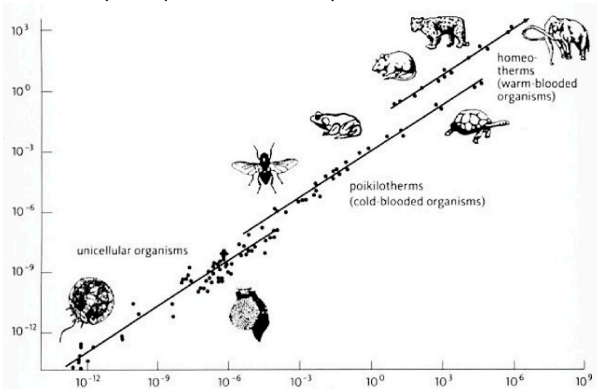


## Power Function: Example

Allometric scaling  
(E.g. Kleiber's law)

mass  $\propto$  volume  $\propto$  [body]length<sup>3</sup>  
surface area  $\propto$  [body]length<sup>2</sup>

hourly heat production  $\propto$  body mass<sup>3/4</sup>



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## Power Function: Some Examples from the Biophysics Formula Collection

#1: The de Broglie wavelength  
(I.3)

$$\lambda = h/p$$

$$\lambda = h \cdot p^{-1}$$

$$y = b \cdot x^a$$

#2: Stefan-Boltzmann law  
(II.41)

$$M_{\text{black}} = \sigma \cdot T^4$$

$$y = b \cdot x^a$$

#3: Duane-Hunt law  
(II.80)

$$\lambda_{\text{min}} = \frac{hc}{eU_{\text{anode}}}$$

$$\lambda_{\text{min}} = hc/e \cdot U^{-1}$$

$$y = b \cdot x^a$$

#4: Mass dependence of eigenfrequency  
(Resonance 6)

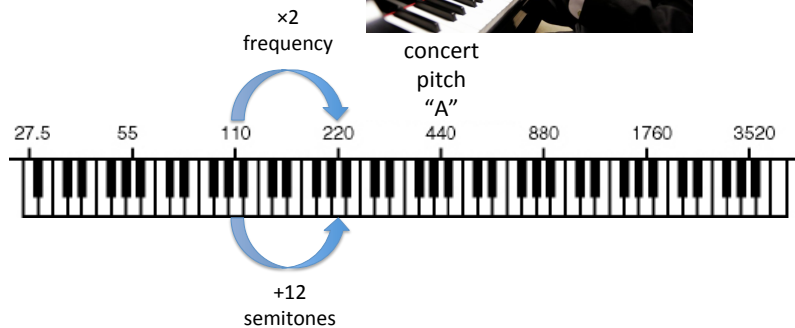
$$f_0 = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

$$f_0 = k^{1/2}/(2\pi) \cdot m^{-1/2}$$

$$y = b \cdot x^a$$

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## Logarithmic Function: Example



## Logarithmic Function

INTEGRAL FORM

$$y = b \cdot \log_a(x)$$

PRACTICAL CONSIDERATIONS:

- base is 10 (sometimes  $e$  or 2)
- if the base is fixed this will modify the factor parameter according to the following identity:

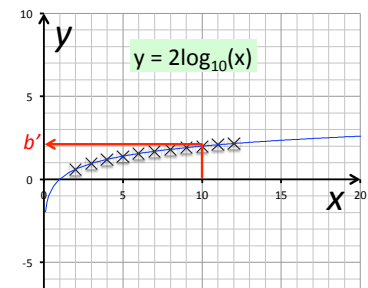
$$b \cdot \log_a(x) = b/\log_{10}(a) \cdot \log_{10}(x) = b' \cdot \log_{10}(x)$$

VARIABLES: dependent variable independent variable

$$y = b' \cdot \log_{10}(x)$$

PARAMETERS: factor parameter

if  $x = 10$   
then  $y = b'$



„DIFFERENTIAL“ FORM

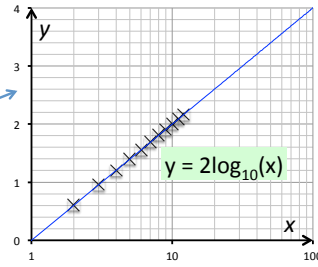
$$\Delta y \sim \Delta x/x$$

The **absolute change** of the dependent variable is proportional to the **relative change** of the independent variable

# Logarithmic Function: Linearization

## graphical linearization

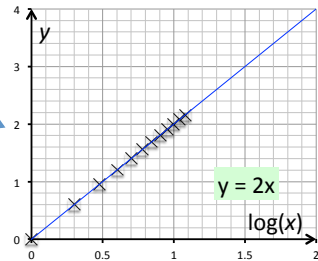
plot  $y$  on lin and  $x$  on log scales:  
the relationship **looks** linear but it **is** still a log function



## INTEGRAL FORM

$$y = b' \cdot \log_{10}(x)$$

arithmetical linearization  
plot  $y$  as a function of  $\log(x)$ :  
the relationship **is** linear



# Logarithmic Function: Some Examples from the Biophysics Formula Collection ...and elsewhere

#1: The statistical definition of entropy

(III.72)

$$S = k \ln \Omega$$

$$S = k \cdot \log_e(\Omega)$$

$$y = b \cdot \log_a(x)$$

#2: The decibel (dB) scale

(VII.10)

$$n = 10 \log A_p$$

$$n = 10 \cdot \log_{10}(A_p)$$

$$y = b \cdot \log_a(x)$$

#3: The definition of absorbance

(VI.34)

$$A = \lg(J_0/J)$$

$$A = 1 \cdot \log_{10}(J_0/J)$$

$$y = b \cdot \log_a(x)$$

#4: The pH scale

$$\text{pH} = -\log[\text{H}^+]$$

$$\text{pH} = -1 \cdot \log_{10}([\text{H}^+]/(1 \text{ M}))$$

$$y = b \cdot \log_a(x)$$

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# Derivative and Integral: Example #1

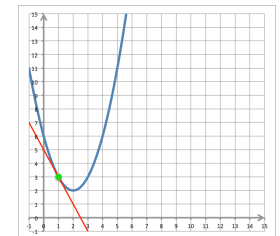
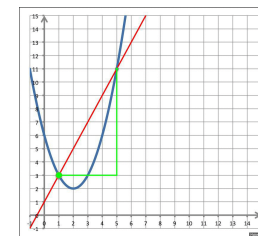
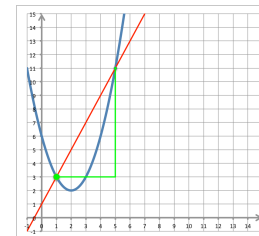
$x$	$y = x^2$	$y' = \Delta y / \Delta x$	$y'' = \Delta(\Delta y / \Delta x) / \Delta x$
0	0		
1	1	1	
2	4	3	2
3	9	5	2
4	16	7	2
5	25	9	2
6	36	11	2
7	49	13	2
8	64	15	2
9	81	17	2
10	100	19	2

# Derivative: slope of tangent line

difference quotient:  
 $\Delta y / \Delta x$   
slope of **secant** line

$$\Delta \rightarrow d$$

derivative:  
 $dy/dx$   
slope of **tangent** line

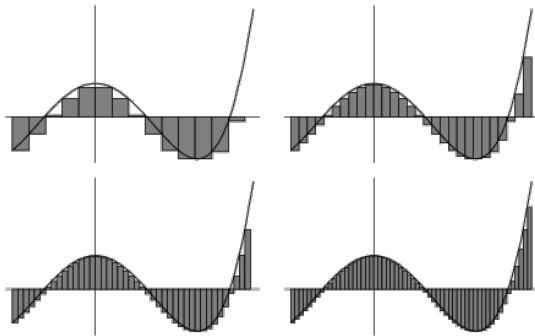


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## Integral: Area Under the Curve (AUC)

$$\Sigma \rightarrow \int$$



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## Rectilinear Motions

Quantities, Units, and Equations

displacement:  $\Delta s = s_2 - s_1$      $[\Delta s] = \text{m}$   
 velocity:  $v = \Delta s / \Delta t$      $[v] = \text{m/s}$   
 acceleration:  $a = \Delta v / \Delta t$      $[a] = \text{m/s}^2$

Uniform rectilinear motion

$s_t = s_0 + v \cdot t$   
 $v = \text{parameter}$   
 $a = 0$

Uniform rectilinear acceleration

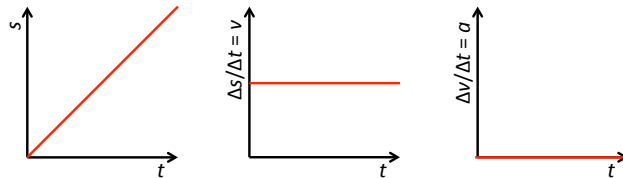
$s_t = s_0 + v_0 \cdot t + a/2 \cdot t^2$   
 $v_t = v_0 + a \cdot t$   
 $a = \text{parameter}$

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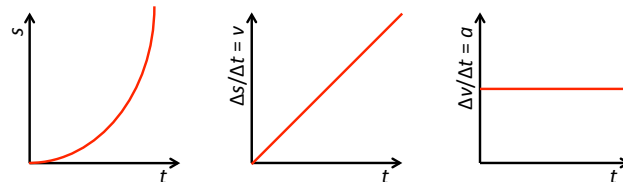
## Derivative and Integral: Application

Rectilinear Motion

uniform rectilinear motion:



uniform rectilinear acceleration:



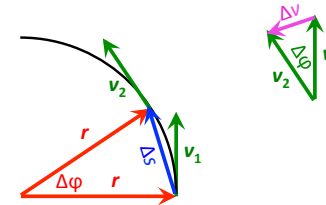
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## Circular Motion

Quantities, Units, and Equation

angular displacement:  $\Delta \varphi = \varphi_2 - \varphi_1$      $[\Delta \varphi] = \text{rad}$   
 angular velocity, angular frequency:  $\omega = \Delta \varphi / \Delta t$      $[\omega] = \text{rad/s}$   
 tangential velocity:  $v = r \cdot \Delta \varphi / \Delta t = r \cdot \omega$      $[v] = \text{m/s}$

centripetal acceleration:  $a_{cp} = v^2 / r = r \cdot \omega^2$      $[a] = \text{m/s}^2$



(1) approximation in case of small angles:  
 displacement = arc length =  $v \cdot \Delta t \approx \Delta s$

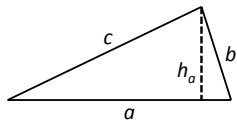
(2) due to similarity:  
 $\Delta v / v = \Delta s / r$

(1) + (2):  
 $\Delta v / v = v \cdot \Delta t / r$

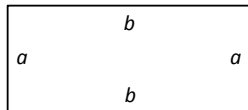
$a_{cp} = v^2 / r$

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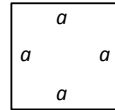
## Perimeter & Area



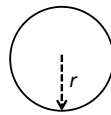
TRIANGLE  
perimeter:  $a+b+c$   
area:  $a \cdot h_a / 2$



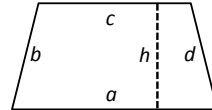
RECTANGLE  
perimeter:  $2 \cdot (a+b)$   
area:  $a \cdot b$



SQUARE  
perimeter:  $4a$   
area:  $a \cdot a = a^2$



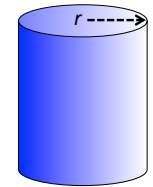
CIRCLE  
perimeter:  $2\pi r$   
area:  $r^2\pi$



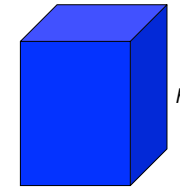
TRAPEZOID  
perimeter:  $a+b+c+d$   
area:  $(a+c)/2 \cdot h$

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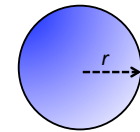
## Surface & Volume



CYLINDER (open)  
surface (wall only):  $2\pi r \cdot h$   
volume:  $r^2\pi \cdot h$



PRISM (open)  
surface (wall only):  
(perimeter of base)  $\cdot h$   
volume: (area of base)  $\cdot h$



SPHERE  
surface:  $4r^2\pi$   
volume:  $4r^3\pi/3$

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## Units – SI Base & Derived Units

physical quantity	symbol	unit	symbol
length	$l, x, s, d$	meter	m
mass	$m$	kilogram	kg
time	$t$	second	s
temperature	$T$	kelvin	K
electric current	$I$	ampere	A
amount of substance	$n, N, \nu$ [nu]	mole	mol
luminous intensity	$I_v$	candela	cd

The SI base units

physical quantity	symbol	unit	symbol	derivation
speed	$v, c$	–	–	$\text{m} \cdot \text{s}^{-1}$
acceleration	$a$	–	–	$\text{m} \cdot \text{s}^{-2}$
force	$F$	newton	N	$\text{kg} \cdot \text{m} \cdot \text{s}^{-2}$
energy	$E$	joule	J	$\text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2}$
power	$P$	watt	W	$\text{kg} \cdot \text{m}^2 \cdot \text{s}^{-3}$
intensity	$I$	–	–	$\text{kg} \cdot \text{s}^{-3}$
pressure	$p$	pascal	Pa	$\text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-2}$

Some SI derived units

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## Units – SI Prefixes

prefix	symbol	meaning	etymology
exa	E	$\times 10^{18} = \times 1000^6$	Greek 6 (ἕξ = hex)
peta	P	$\times 10^{15} = \times 1000^5$	Greek 5 (πέντε = pente)
tera	T	$\times 10^{12} = \times 1000^4$	Greek 4 (τέτταρες = tettares), originally: monster (τέρας = teras)
giga	G	$\times 10^9 = \times 1000^3$	Greek giant (γίγας = gigas)
mega	M	$\times 10^6 = \times 1000^2$	Greek great (μέγας = megas)
kilo	k	$\times 10^3 = \times 1000^1$	Greek 1000 (χίλιοι = khilioi)
hekto	h	$\times 10^2$	Greek 100 (ἑκατόν = hekaton)
deca	da (dk)	$\times 10^1$	Greek 10 (δέκα = deka)
deci	d	$\times 10^{-1}$	Latin 10 (decem)
centi	c	$\times 10^{-2}$	Latin 100 (centum)
milli	m	$\times 10^{-3} = \times 1000^{-1}$	Latin 1000 (mille, pl. milia)
micro	$\mu$	$\times 10^{-6} = \times 1000^{-2}$	Greek small (μικρός = mikros)
nano	n	$\times 10^{-9} = \times 1000^{-3}$	Greek dwarf (νῆνος = nanos)
pico	p	$\times 10^{-12} = \times 1000^{-4}$	Spanish small, bit (pico)
femto	f	$\times 10^{-15} = \times 1000^{-5}$	Danish 15 (femten)
atto	a	$\times 10^{-18} = \times 1000^{-6}$	Danish 18 (atten)

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# Units – Conversion

## from “with prefix” to “no prefix”:

$$15 \text{ km} = 15 \cdot 10^3 \text{ m}$$

$$15 \text{ cg} = 15 \cdot 10^{-2} \text{ g}$$

## from “no prefix” to “with prefix”:

$$15 \text{ m} = 15 / 10^3 \text{ km}$$

$$15 \text{ g} = 15 / 10^{-2} \text{ cg}$$

## from “with prefix” to “with prefix”:

$$15 \text{ km} = 15 \cdot 10^3 \text{ m} = 15 \cdot 10^3 / 10^{-2} \text{ cm}$$

## when the unit has an exponent:

$$15 \text{ km}^3 = 15 \cdot (10^3 \text{ m})^3 = 15 \cdot (10^3)^3 \text{ m}^3$$

$$15 \text{ m}^3 = 15 / (10^3)^3 \text{ km}^3$$

## liters to and from cubic meters:

$$1 \text{ m}^3 = 10 \text{ hL} = 1000 \text{ L}$$

$$1 \text{ dm}^3 = 1 \text{ L}$$

$$1 \text{ cm}^3 = 1 \text{ mL}$$

$$1 \text{ mm}^3 = 1 \text{ }\mu\text{L}$$

## time to seconds:

$$2 \text{ days } 3 \text{ h } 12 \text{ min } 30 \text{ s} = ((2 \cdot 24 + 3) \cdot 60 + 12) \cdot 60 + 30 \text{ s}$$

## degrees, minutes of arc, seconds of arc:

$$45^\circ 40' 30'' = (45 + 40/60 + 30/60^2)^\circ$$

## degrees to and from radians:

$$1 \text{ rad} = (360/2\pi)^\circ$$

$$1^\circ = (2\pi/360) \text{ rad}$$

## compound units:

$$15 \text{ kg/m}^3 = 15 \cdot 10^3 / (1/(10^{-2})^3) \text{ g/cm}^3$$

$$45 \text{ km/h} = 45 \cdot 10^3 / 3600 \text{ m/s}$$

## degrees Celsius to and from kelvins:

$$T = 15^\circ\text{C} = (15 + 273) \text{ K}$$

$$T = 15 \text{ K} = (15 - 273)^\circ\text{C}$$

$$\Delta T = 15^\circ\text{C} = 15 \text{ K}$$

$$\Delta T = 15 \text{ K} = 15^\circ\text{C}$$