Physical Basis of Medical Biophysics

Mechanics – Work and Energy

Tímea Feller 2019.09.20.
feller.timea@med.semmelweis-univ.hu

Energy

Ability of a system to:

- move or deform bodies
- exert heat
- emit radiation

E ***[J]***
or ***[cal]*** *1 cal = 4.19 J*
or ***[eV]*** *1 eV = 1.6·10–19 J*

Scalar quantity, that describes the **state** of an **isolated** system

Isolated system: no matter or energy exchange between the system and the environment

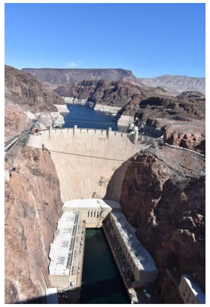
describes the **state**, so:

- can be transferred (from one system to another)
- can be stored
- can be converted in different types

Energy

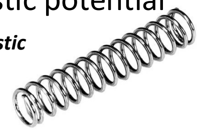
Types (mechanical):

Gravitational potential E_{pot}



Kinetic E_{kin}

Elastic potential $E_{elastic}$



Rotational E_{rot}

Energy:
-kinetic of water
-rotational
-electricity

→ Device:
-turbine
-generator
-lamp

→ Energy:
-rotational
-electricity
-light

Conservation law of energy

Julius Robert MAYER, James Prescott JOULE:

$$E_{total} = E_1 + E_2 + E_3 + \ \dots = \sum_{i=1}^{n} E_i \qquad \Delta E_{total} = 0$$

The total amount of energy in an isolated system remains constant

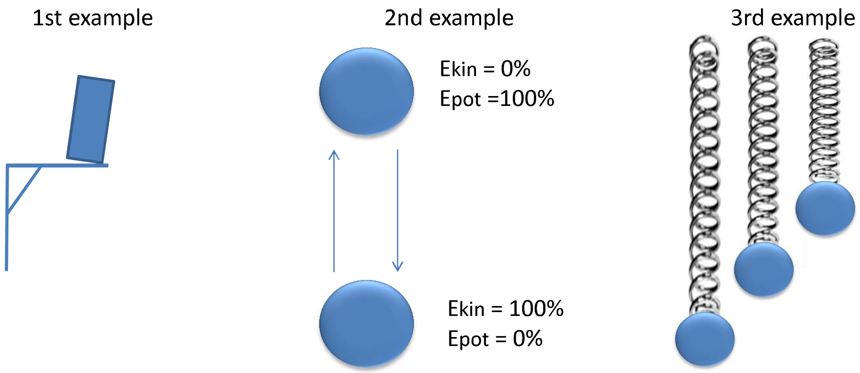
Hermann von HELMHOLTZ:

Energy can not be created or destroyed (it just changes forms).

$$E = m \cdot c^2 \ ??$$

Conservation law of energy mechanics

$E_{total} = E_{kin} + E_{pot} + E_{elastic} + E_{rotational}$ =constant

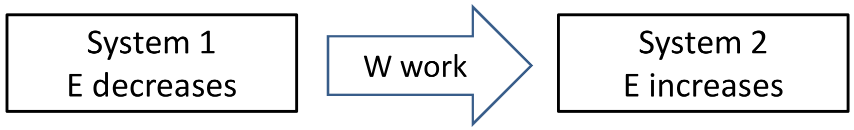


A ball with 100 g mass is dropped from 10 m height and collides with the ground in a completely elastic way. The collision lasts for 0,2 s. Estimate the change in the momentum and in the energy of the ball due to the collision. Estimate the average force acting on the ball during the collision. Δv=2v=2*$\sqrt{2 * g * h}$=28,3 m/s ΔI= 2,83 kg*m/s ΔE=0 (elastic collision!) F= ΔI/t=14,15 N

Work (mechanical)

Mechanical work is exerted, if a body or a system is displaced (moved) of deformed

Mechanical work: energy transported by a given force



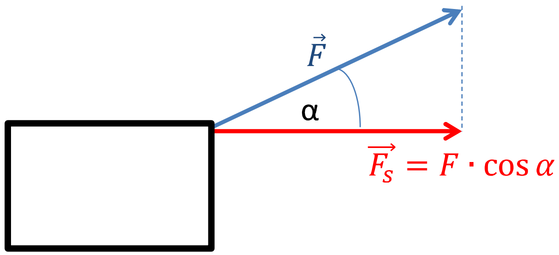
If the force is constant and acts in the same direction as the displacement:

$$\boldsymbol{W = F \cdot s}$$

W: work $[N \cdot m]$ *or [J]*
F: force [N]
s: distance [m]

Work is a **scalar quantity**. But…

If the force acts in a different direction than the displacement:



A sled is pulled in an angle of 60° with 50 N force at a constant velocity of 10 m/s. Calculate the work done in 6 minutes. W=F*cosα*s=90 kJ

Work and gravitational potential energy

$$F = m \cdot g$$

During lifting:

$$W = F \cdot s = m \cdot g \cdot h \qquad E_{pot} = m \cdot g \cdot h$$

Calculate the gravitational potential energy of a climber (mass 80 kg) on the top of the 8848 m high Mount Everest, compared with a) the sea level b) the base camp in 6850 m height. a) 6,94 MJ b)1,57 MJ c) 6,86 MJ

9. A ball (m=0.8 kg) falls to the floor from a height of 2 m and bounces back to a height of 1.2 m. Calculate the amount of energy lost due to air drag and collision with the ground. m*g*h_1-m*g*h_2=6,28 J

Work and kinetic energy

$$F = m \cdot a$$

During motion:

$$W = F \cdot s = m \cdot a \cdot s = \frac{1}{2} m \cdot v^2 \qquad E_{kin} = \frac{1}{2} m \cdot v^2$$

Work-energy principle

$$W_{gy} = \sum_{i=1}^{n} W_i = \Delta E_m$$

Calculate the kinetic energy
a) of a runner with 80 kg mass at 10 m/s velocity 4 kJ
b) of a bullet with 2 g mass and 800 m/s velocity 640 J

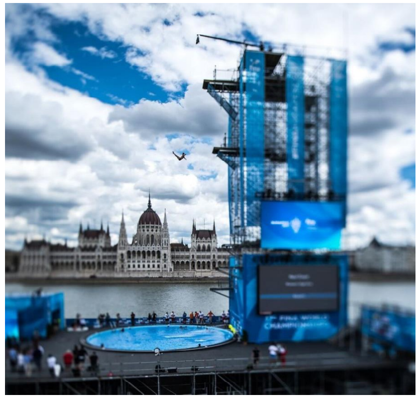
A train (mass is 380 t) moves with 100 km/h velocity. At which speed would be the kinetic energy doubled? $E \sim v^2 so\ 2E \sim (2v)^2\ so\ the\ speed\ increases\ \sqrt{2}\ - times$

2. A car (m=1.2 t) is uniformly accelerating from rest for 12 seconds to reach a velocity of 100 km/h.
a) Calculate the work done by the accelerating force.
b) Calculate the kinetic energy of the car at the end of the acceleration.

A father is pulling a sled (mass is 10 kg) at an angle of 45^0 with a constant force and accelerates it from 5 m/s to 15 m/s velocity in 20 s against a 20 N friction force. Calculate the work done by the father. $F_{accelerating}$=m*a=m*Δv/Δt=5N
Force in *s* direction=$F_{friction}$+$F_{accelerating}$=25N, ~~F_{father}=25 N/ cos(α)=35,4 N~~ because we only need F in the direction of the displacement!!
W=F*s where s=0,5*a*t²+v_0*t=200m so W=25N*200m=5 kJ

A high diver (mass is 60 kg) jumps from the 27 m high FINA tower. By neglecting drag, calculate the kinetic, potential and total energy of the diver
a) at start and E_{pot}=m*g*h=15,9 kJ E_{kin}=0
b) at the moment of impact. E_{pot}=0 E_{kin}=15,9 kJ
c) What is the final velocity of the diver when he/she reaches the pool?

$$v = \sqrt{\frac{2 \cdot E_{kin}}{m}} = 23{,}3 \frac{m}{s} = 83{,}6\ km/h$$



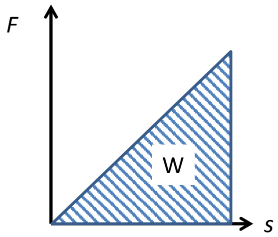
Work and elastic (potential) energy

$$F = D \cdot s$$

Extended/compressed:

$$W = \frac{1}{2} F \cdot s = \frac{1}{2} D \cdot s \cdot s = \frac{1}{2} D \cdot s^2$$

$$E_{rug} = \frac{1}{2} D \cdot s^2$$



7. Calculate the amount of energy stored in the Achilles tendon with a spring constant of 3·10⁵ N/m that is extended by 2 mm.

A spring is extended by 2 cm. How will the elastic energy change if I further extend the spring by 4 cm? $W \sim s^2\ now\ instead\ of\ s\ we\ have\ 3s\ so{:}\ x \cdot W \sim (3s)^2$
$x = 9, the\ elastic\ enery\ increased\ 9 *$

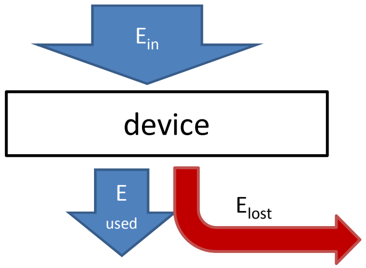
Power

The rate of doing work

$$P = W/t$$
$$[W] = [J/s]$$

A car is moving on the highway with constant, 130 km/h velocity. The power of the engine is 30 kW. Calculate the force needed to maintain this speed.
P=W/t=(F*s)/t=(F*v*t)/t=F*v, so
F=P/v= 831 N

Efficiency



$$\eta = \frac{E_{used}}{E_{in}}$$

$$\eta = \frac{W_{used}}{W_{in}}$$

$$\eta = \frac{P_{used}}{P_{in}}$$

A power plant is operating at 45% efficiency. 90% of the produced energy gets to the household, where a lamp, that uses this enegy, operates at 20 % efficiency. Calculate the efficiency of the whole process.
100%*0,45*0,9*0,2=8,1%

What is the intensity of the X-ray at 1 m from the focus of the X-ray tube, if the power is 250 W and the efficiency of the tube is 0.37 %? Assume that the radiation emerges from a point-like focus, and it is distributed uniformly in a 2π spherical angle (in a hemisphere) P_{used}=250W*0,0037=0,925 W
$A_{hemisphere}=2\pi r^2$=6,28 m^2
I=P/A=0,147 W/m^2

Mass-energy equivalence

$$E = m \cdot c^2$$

10. Calculate the rest energy of an electron (me = 9.11·10–31 kg)! Convert your result to eV! 512 keV

Calculate the rest energy of your own mass!