

Mathematical and Physical Basis of Medical Biophysics

Lecture 3
Mechanics:
Statics and Dynamics
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Kinematics vs. Dynamics

- Kinematics: deals with movements of bodies without investigating the cause of the movement
- Dynamics: handles the causes of the different forms of movement
 - study of forces acting between bodies
 - study of the effect of forces on motion

Medical Relevance of Dynamics

- Sports medicine
- Orthopedics
- Physiotherapy
- Audiology
- Dentistry



Different Interactions between Bodies

Interaction: bodies exert forces on each other

Macroscopic interactions (we encounter them in everyday life):

- Gravitation
- Electromagnetic interaction
 - electric attraction and repulsion
 - magnetic attraction and repulsion
 - friction, adhesion, etc.:all everyday phenomena besides gravitation

Microscopic interactions (between subatomic particles):

NOT PART OF BASIC PHYSICS

- Strong nuclear interaction
- Weak nuclear interaction

Force

The 'force' describes the strength of interaction

Effect of force:

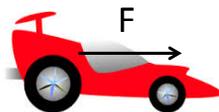
- change in motion (more conventional)
- or change in shape

We can measure (and define) the force by these changes.

Problem IV/2

A racing car ($m = 1500$ kg) accelerates from rest with uniform acceleration. It reaches the 100 km/h velocity in a time interval of 3,1 s.

- Calculate the force that is accelerating the car.
- Calculate the distance in which the car reaches the 100 km/h velocity.



Force

$$F = m \cdot a$$

F: force [N] = [newton] = [kg*m / s²]

m: mass [kg]

a: acceleration [m / s²]

Direction of F equals the direction of a , but not necessarily that of the speed (vector quantities)

Newton's laws (axioms) of motion

- I. The law of inertia
- II. Fundamental law of dynamics
- III. Law of equal action and reaction

Newton's first law



- an object will remain at rest or in uniform motion in a straight line unless acted upon by an external force
- objects will remain in their state of motion unless a force acts to change the motion: inertia

Only in an 'inertial reference frame'

It is a special case of the Second law.

Newton's second law



- The acceleration (a) of an object and the force (F) acting on it are proportional to each other

$$F = m \cdot a$$

- If several forces are acting on the object at the same time:

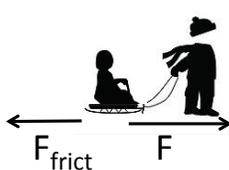
$$\Sigma F = m \cdot a$$

where ΣF is the vectorial sum of all forces (a.k.

Problem IV/5

A father is pulling a sled for 5 seconds starting from rest with a constant force of 105 N. The mass of the sled together with the child on it is 25 kg. The friction force acting on the sled is 15 N.

- Calculate the acceleration of the sled.
- What will be the final velocity of the sled after 5s?
- How far can dad pull the sled during this time?



Problem IV/6.

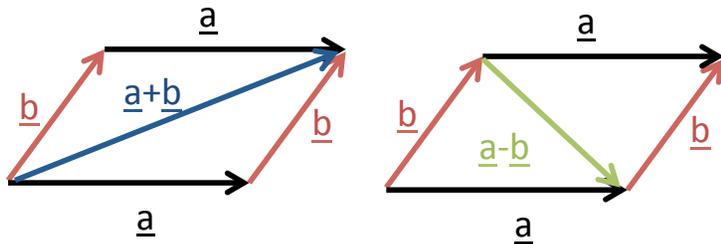
A man is pulling a sled with constant velocity ($m = 20$ kg). The rope suddenly breaks. The sled slides on for 6,1 seconds with uniform deceleration and stops after 9,2 m.

- Calculate the velocity of the sled at the moment when the rope breaks.
- Calculate the acceleration of the sled (in fact it is deceleration).
- Calculate the force that slows the sled.



Resultant force, net force

The parallelogram law gives the rule for vector addition of vectors \underline{a} and \underline{b} . The sum $\underline{a+b}$ of the vectors is obtained by placing them head to tail and drawing the vector from the free tail to the free head.

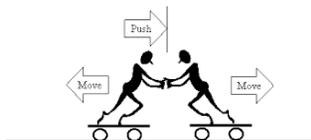


Newton's third law

- When one body exerts a force on a second body, the second body simultaneously exerts a force on the first body equal in magnitude and opposite in direction

$$F_{1 \rightarrow 2} = -F_{2 \rightarrow 1}$$

- Forces always appear in pairs
- Action is always followed by reaction (symmetry of interactions)



Newton I. – Newton II.

- The first law is a special case of the second:
In equilibrium: $\Sigma F = 0 \rightarrow a=0$
If $a=0$
- velocity remains constant (,uniform motion')
- velocity is zero (,remains at rest') \rightarrow **STATICS**



Newton's third law



A truck driving down the road runs into a bug flying the other way.

- a) Which object has greater force exerted on it?

Both objects have the same small force exerted on them.

- b) Which object will change its motion the most?

The bug changes its motion significantly more because of its smaller mass.

$$F_{1 \rightarrow 2} = -F_{2 \rightarrow 1}$$

$$m \mathbf{a} = m \mathbf{a}$$

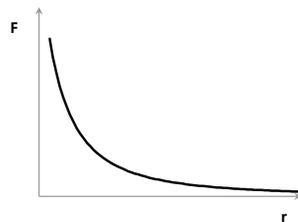
Limits of Newtonian mechanics

Newtonian mechanics is not applicable in all situations:

- 1) velocities near speed of light → special relativity
- 2) atomic-sized bodies → quantum mechanics
- 3) non-inertial reference frame (e.g. accelerating aircraft)

Law of universal gravitation

$$F = G * m_1 * m_2 / r^2$$



F: gravitational force [N]

G: univ. gravitational constant = $6.67 * 10^{-11} \text{ m}^3 * \text{kg}^{-1} * \text{s}^{-2}$

m_1 : mass of first object [kg]

m_2 : mass of second object [kg]

r: distance between the objects [m]

Inertial reference frame

In a straight flying aircraft a tennis ball is placed on the folding table, which remains at rest relative to the aircraft.



(a)

If the pilot accelerates the aircraft the ball rolls suddenly towards the sitting person.



(b)

Accelerating (or rotating) aircraft: non-inertial system
 Ground or aircraft flying with constant speed: in good approximation inertial systems

Problem IV/8.

Calculate the gravitational force between two asteroids (masses are 200 000 and 300 000 tons) when they pass by each other at a distance of 2 km.



Gravity

- a special case of the gravitational law
- accelerates an object falling freely towards the Earth

$$F_{\text{gravity}} = \gamma \cdot m_{\text{Earth}} \cdot m_{\text{object}} / r^2 = m \cdot g$$

gravitational acceleration (g):

$$g = \gamma \cdot m_{\text{Earth}} / r^2 = 9.81 \text{ m/s}^2$$

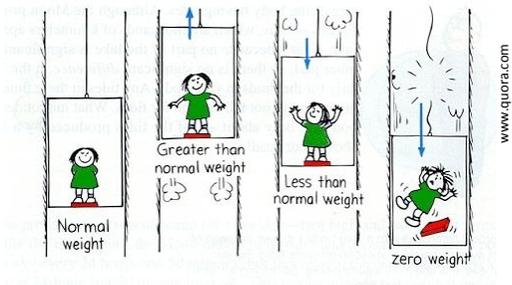
Earth is not spherical, g is greater near the Equator

Weight – F_{grav} – mass

- Weight (W): force with which an object pressing or pulling the surface that holds it

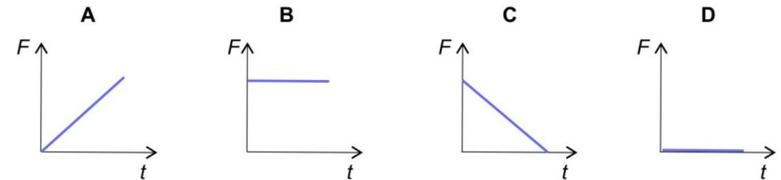
In equilibrium: $F_{\text{grav}} = W = m \cdot g$

Free falling: $F_{\text{grav}} = m \cdot g$, but $W=0$ (weightlessness)



Problem IV/13 a

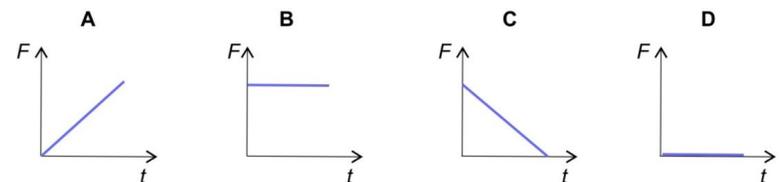
The figures show the change in force as a function of time:



We throw a ball straight up. Which figure shows correctly the change of gravity acting on the ball?

Problem IV/13 c

The figures show the change in force as a function of time:

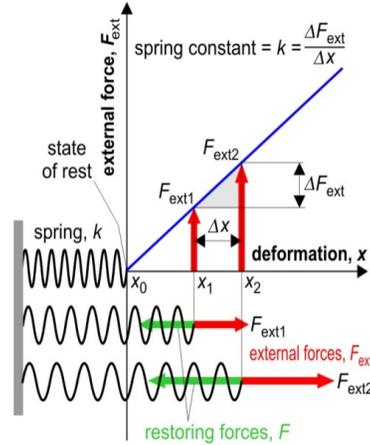


A ball falls freely towards the ground. Which figure shows correctly the change of the ball's weight?

Hooke's law

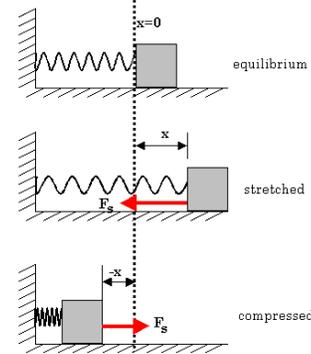
$$F = -k \cdot x$$

- F: restoring force [N]
- (-): opposite direction to the extension
- k: spring constant [N/m]
- depends on the material and dimensions of the spring
- x: extension, deformation [m]



Hooke's law

- The same formula can be applied for compression.



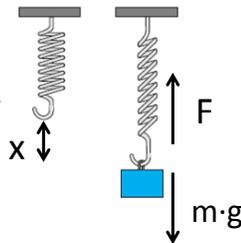
$$F = -k \cdot x$$

Applies approximately for tendons and ligaments in human body if they are not too heavily loaded.



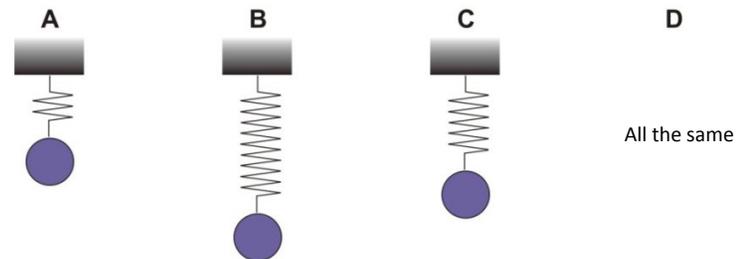
Problem IV/11

We hang a mass of 2 kg on a spring. Its extension after equilibrium is 25 cm. Calculate the spring constant.



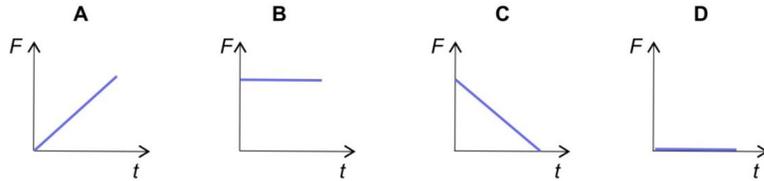
Problem IV/12

All the spring shown in the figures are extended by 10 %, when we hang the same mass on them. Which spring has the highest spring constant? Or do all have the same?



Problem IV/13 b

The figures show the change in force as a function of time:



We slowly compress a spring uniformly. Which figure shows correctly the change of spring force during compression?

Problem IV/3

In a particle accelerator an ion with the mass of $2,5 \cdot 10^{-25}$ kg is accelerated by a constant force of $1,6 \cdot 10^{-12}$ N.

- Calculate the acceleration of the ion.
- Calculate the increase in velocity during 10 ns.

Problem IV/1

A bullet of mass 10 g moving at 200 m/s strikes the wall and stops after 0,002 s. Assuming the deceleration of the bullet is uniform, calculate:

- The braking force acting on the bullet.
- How deep did the bullet penetrate the wall?

Problem IV/4

The acceleration of a paratrooper ($m = 70$ kg) at a given moment during its fall is $0,5$ m/s². What kind of forces are acting on the paratrooper at this moment? Calculate the value of these forces!

Problem IV/7

Calculate the gravitational force between the nucleus (one proton) and the electron of a hydrogen atom, if their distance is 100 pm. (the mass of proton: $1,67 \cdot 10^{-27}$ kg; the mass of electron: $9,11 \cdot 10^{-31}$ kg.)

Problem IV/9

A sand bag of 40 kg hangs from a rope.

- a) Calculate the gravity pulling the bag.
- b) Calculate the force with which the bag pulls the rope (its weight).
- c) Calculate the gravity and the weight if we hang the bag in an elevator that is accelerating with 2 m/s^2 downwards.

Problem IV/10

Let us consider the Achilles tendon as a simple spring with a spring constant of $3 \cdot 10^5 \text{ N/m}$. Calculate the force that is needed to stretch the tendon by 2 mm.