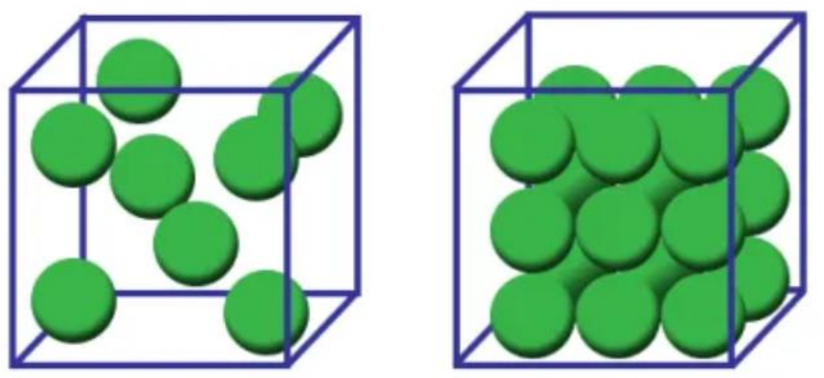
Fluid mechanics

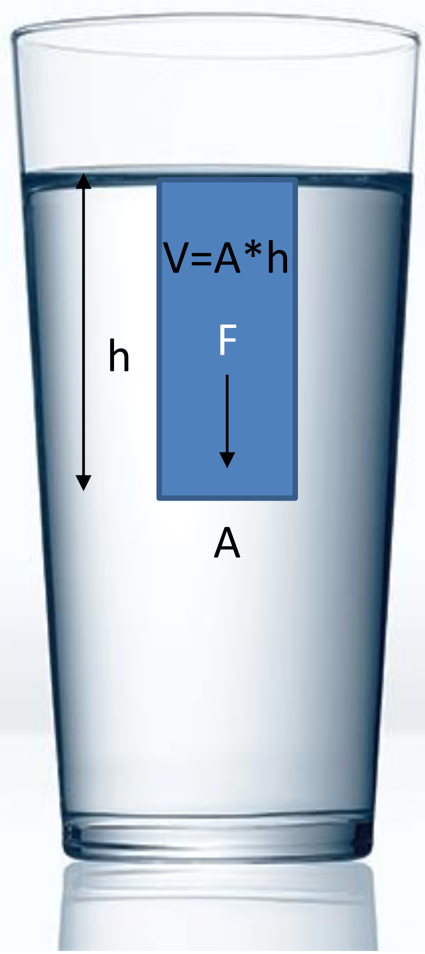
Ádám Orosz

Density of materials



material	ρ (g/cm^3)
air (0°C and 101 kPa)	0,00129
water (4°C és 101 kPa mellett)	1
water (100°C és 101 kPa mellett)	0,958
ice	0,917
aluminum	2,7
mercury	13,6
gold	19,3
human body (average)	1,04

Hydrostatic pressure



$$F = m \cdot g$$

$$F = \rho \cdot V \cdot g$$

$$F = \rho \cdot A \cdot h \cdot g$$

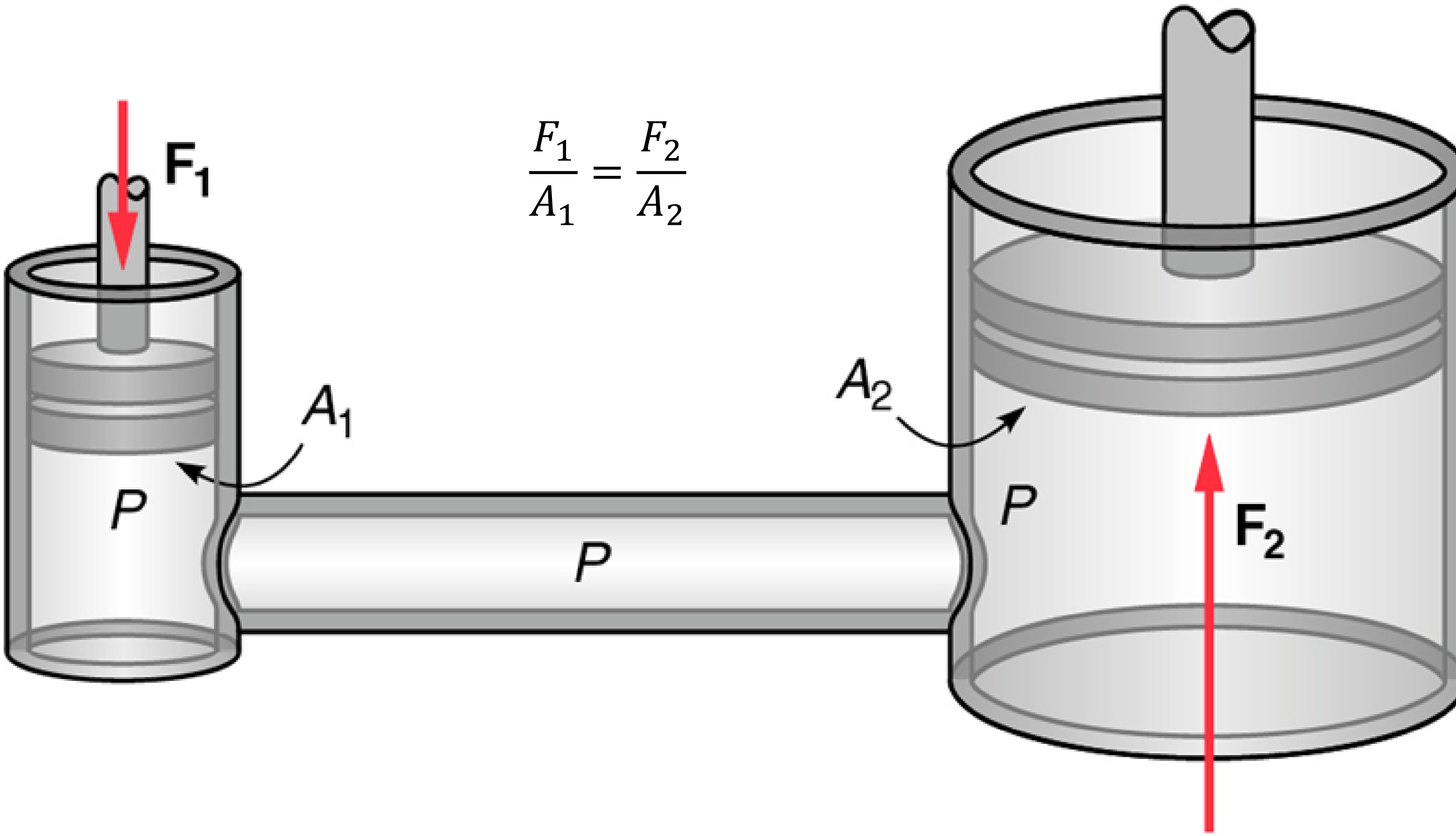
$$p = \frac{F}{A}$$

$$p = \rho \cdot g \cdot h$$

Hydrostatic paradox

Pascal’s law

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$



Problem 1.

Calculate the hydrostatic pressure generated by blood in the foot of a standing man.

Density of blood is 1.05 g/cm^3 and the height of the man is 170 cm.

$p = ?$

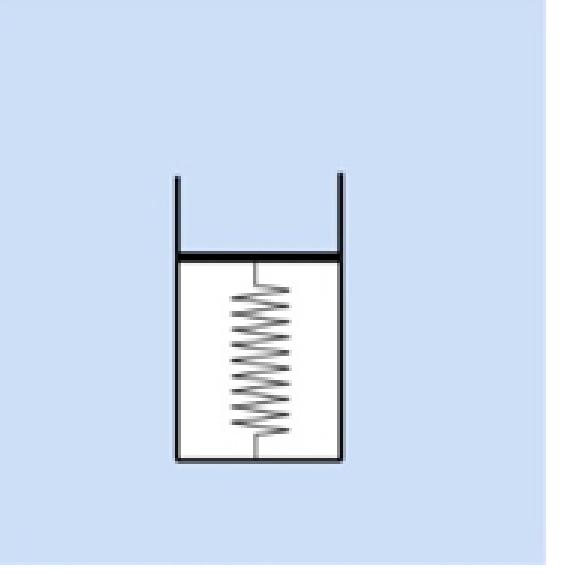
$\rho_B = 1,05\ g/cm^3 = 1050\ kg/m^3$

$h = 170\ cm = 1,7\ m$

$p = 1050 \cdot 9,81 \cdot 1,7 = 17510,85\ Pa = 17,5\ kPa$

Problem 2.

The figure shows a device for a simple pressure measurement. The small cylinder has vacuum inside and its top is sealed with a light piston. The piston is connected to the bottom of the cylinder with a spring. If we place this device in vacuum, then the spring will be uncompressed. The cross-sectional area of the piston is 2 cm^2 , and the spring constant is $4 \cdot 10^3$ N/m.



a) *When this device is placed in the atmosphere, the compression of the spring is 5.1 mm. Calculate the atmospheric pressure!*

b) *Calculate the compression of the spring if we place the device to the bottom of a 10-m-deep pond, that has a temperature of 4 °C! Assume that the atmospheric pressure is the same as in part „a"!*

$A = 2\,cm^2 = 0{,}0002\ m^2$

$F = -k \cdot s \qquad k = 4 \cdot 10^3 \frac{N}{m}$

a.) $s = 5{,}1\,mm = 0{,}0051\,m$

$F = 20{,}4\,N$

$\frac{F}{A} = \rho \cdot g \cdot h$

$\frac{20{,}4\,N}{0{,}0002\,m^2} = 102\,kPa$

b.) $p = \rho \cdot g \cdot h = 1000 \frac{kg}{m^3} \cdot 9{,}81 \cdot 10 = 98{,}1\,kPa$

$98{,}1\,kPa + 102\,kPa \approx (199{,}1\,kPa)$ 200 kPa

199100 Pa

$199{,}1 \cdot 0{,}0002\,m^2 = 39{,}8\,N = -k \cdot s$

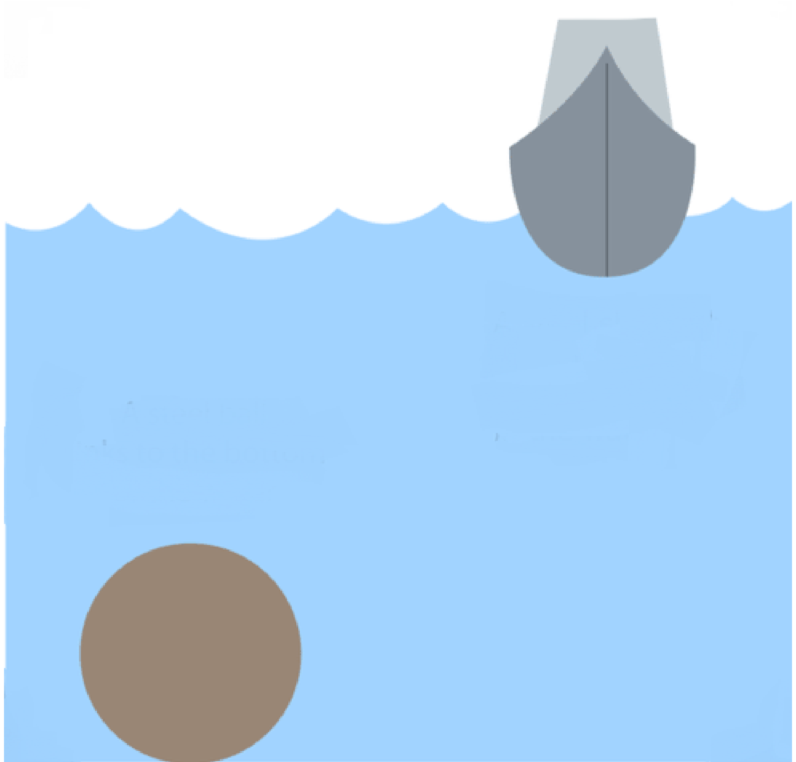
$200\,000\,Pa \cdot 0{,}0002\,m^2 = 40\,N$

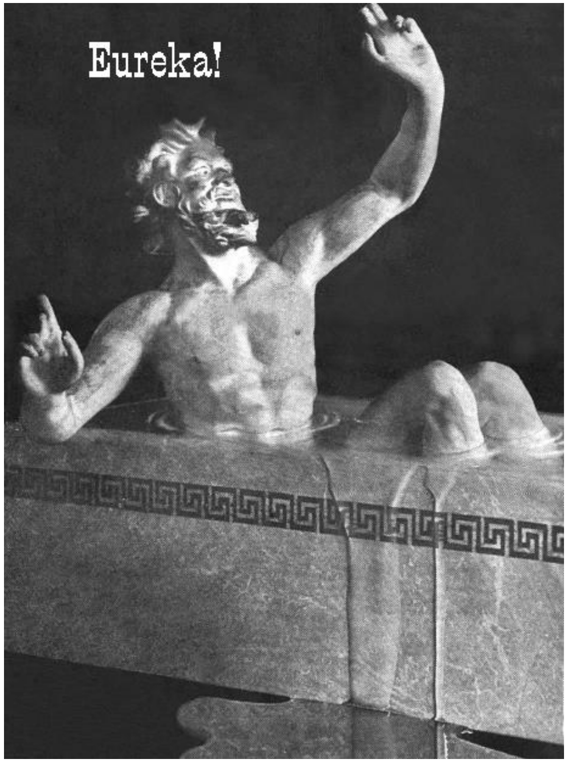
$\left(\frac{F}{k} = s \right.$

$\frac{40}{4 \cdot 10^3} = 0{,}01\,m$ (10 mm)

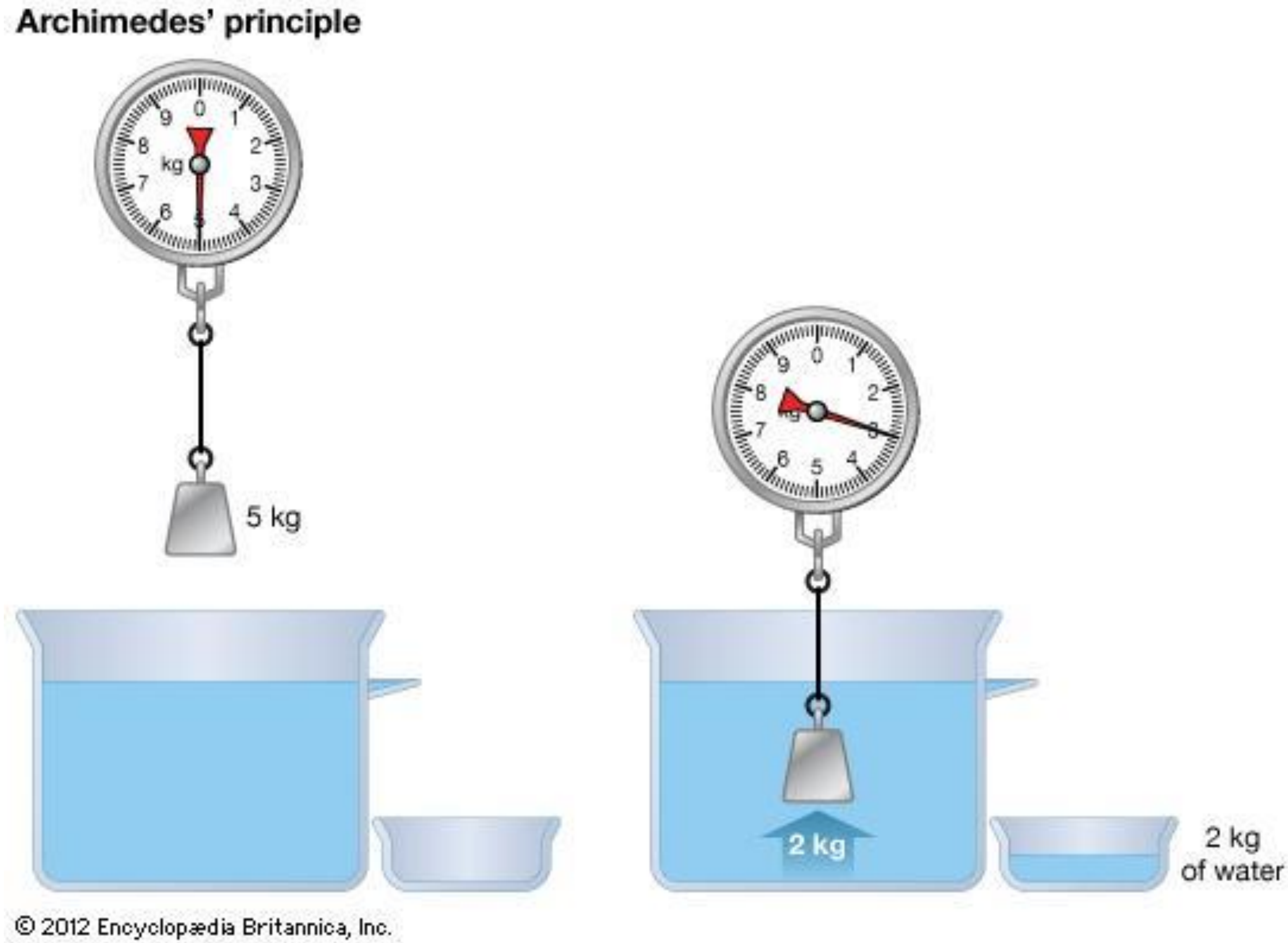
$s = 9{,}955\,mm$

Archimedes’ principle, buoyancy

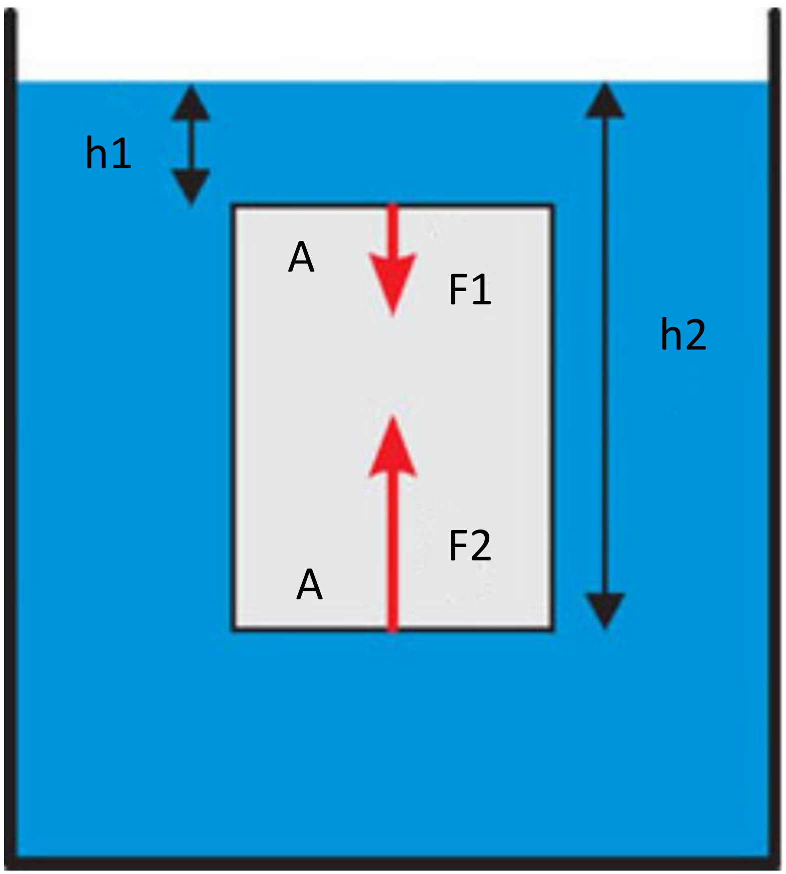




Archimedes’ principle



Archimedes' principle, buoyancy



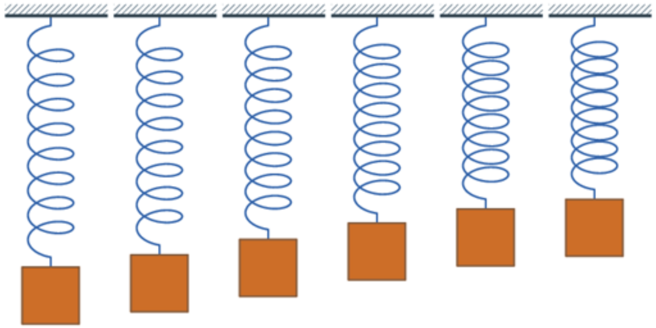
$$F_1 = \rho \cdot A \cdot h_1 \cdot g$$

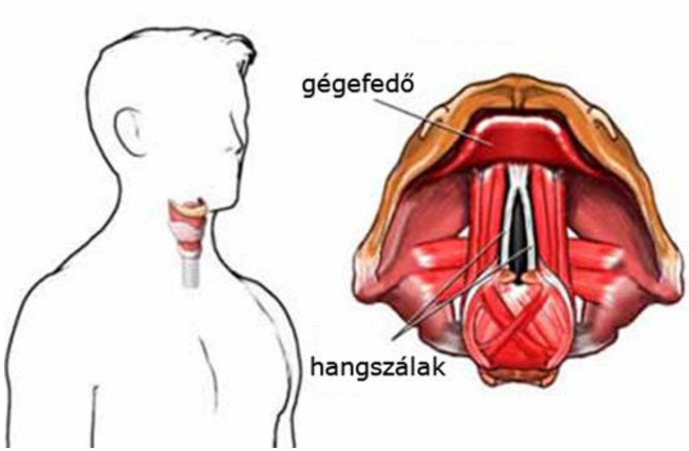
$$F_2 - F_1 = F_b = \rho \cdot A \cdot (h_2 - h_1) \cdot g$$

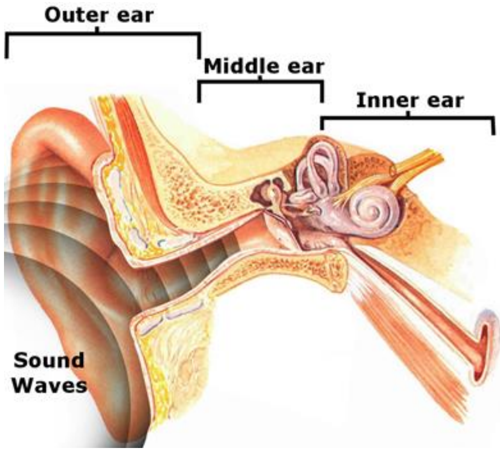
$$F_b = \rho \cdot V_b \cdot g$$

$$F_2 = \rho \cdot A \cdot h_2 \cdot g$$

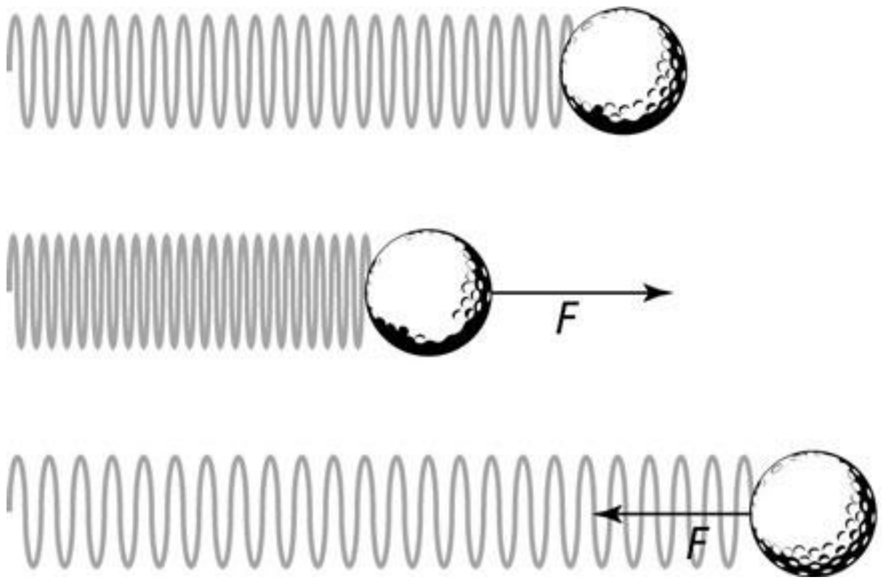
Oscillations







Parameters



A

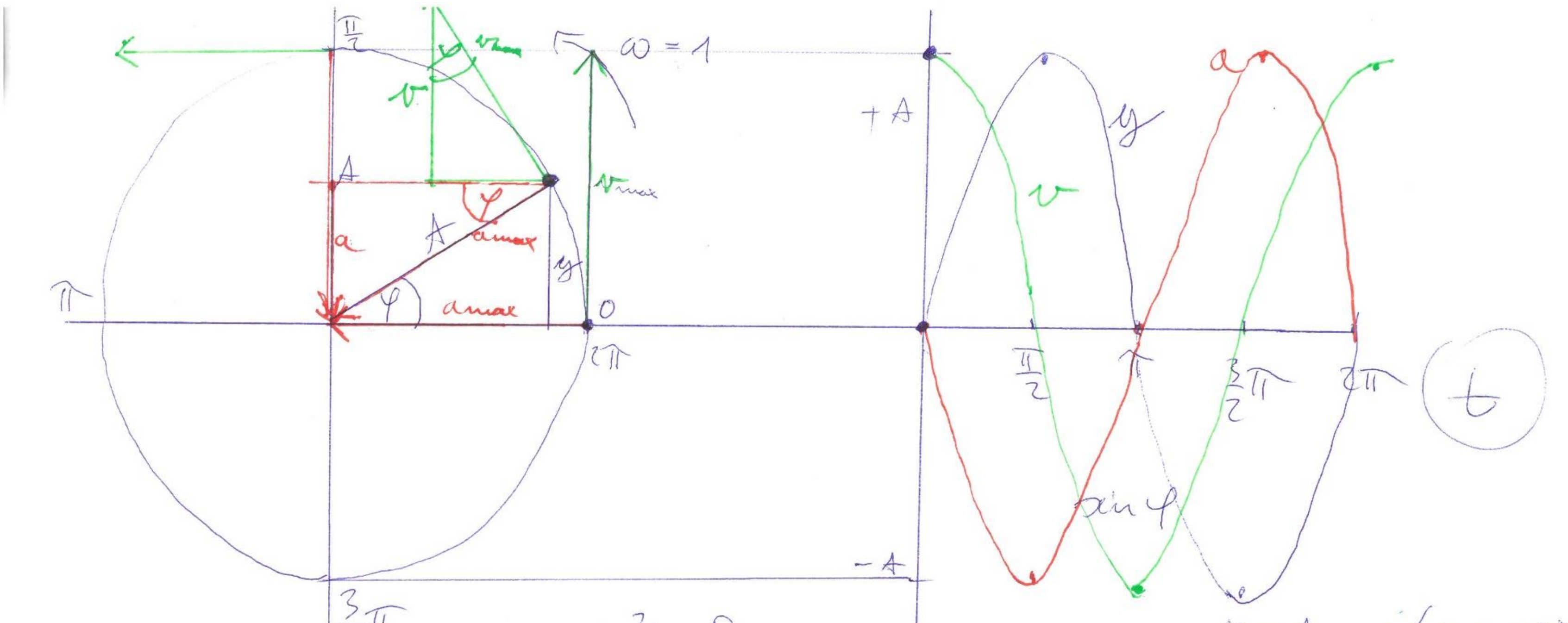
B

C

Types of oscillations

harmonic	non-harmonic
undamped	damped
free	driven

Displacement, velocity, acceleration, force



$$y = A \cdot \sin(\omega \cdot t + \varphi_0)$$

$$\omega^2 = \frac{D}{m}$$

$$\omega = \sqrt{\frac{D}{m}}$$

$$2\pi \cdot f$$

$$f = \frac{1}{2\pi}\sqrt{\frac{D}{m}}$$

$$v_{max} = r \cdot \omega = A \cdot \omega$$

$$a_{max} = \frac{v^2}{r} = r \cdot \omega^2 = A \cdot \omega^2$$

$$F = m \cdot a = m \cdot A \cdot \omega^2 \cdot \sin\varphi$$

$$= -m \cdot \omega^2 \cdot y$$

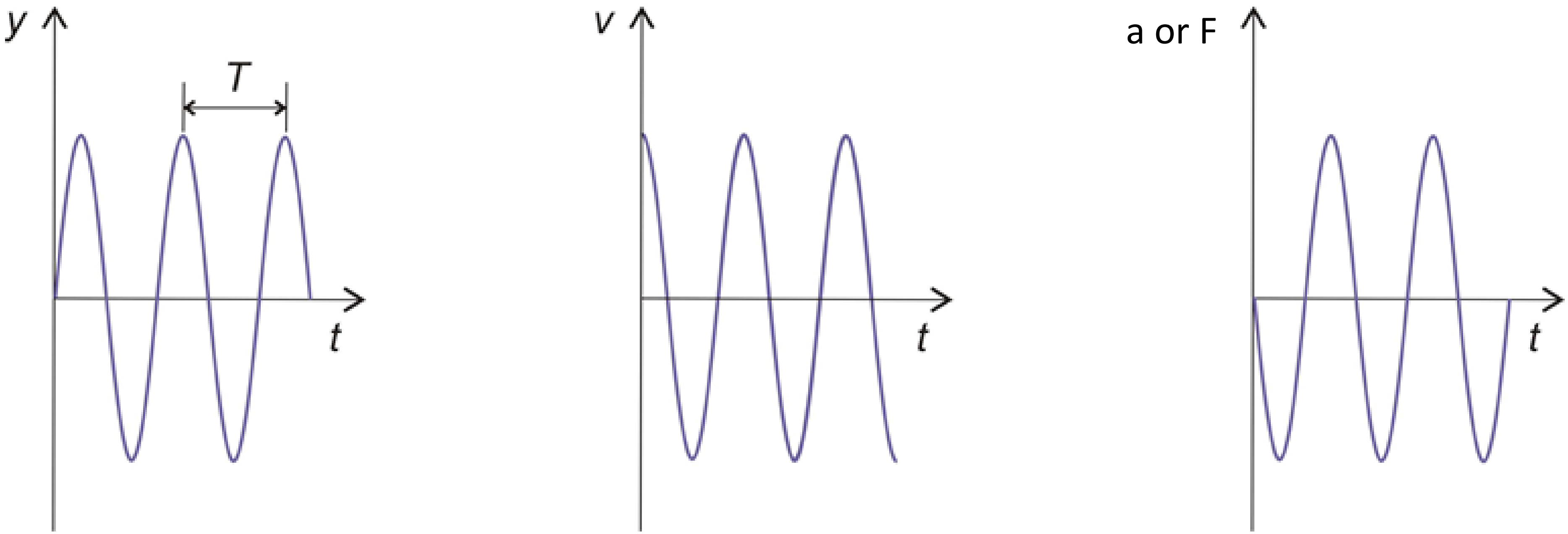
$$F = -D \cdot y \qquad -D \cdot y = -m \cdot \omega^2 \cdot y$$

$$\sin\varphi = \frac{y}{A} \Rightarrow y = A \cdot \sin\varphi$$

$$\cos\varphi = \frac{v}{v_{max}} = \frac{v}{A \cdot \omega}$$

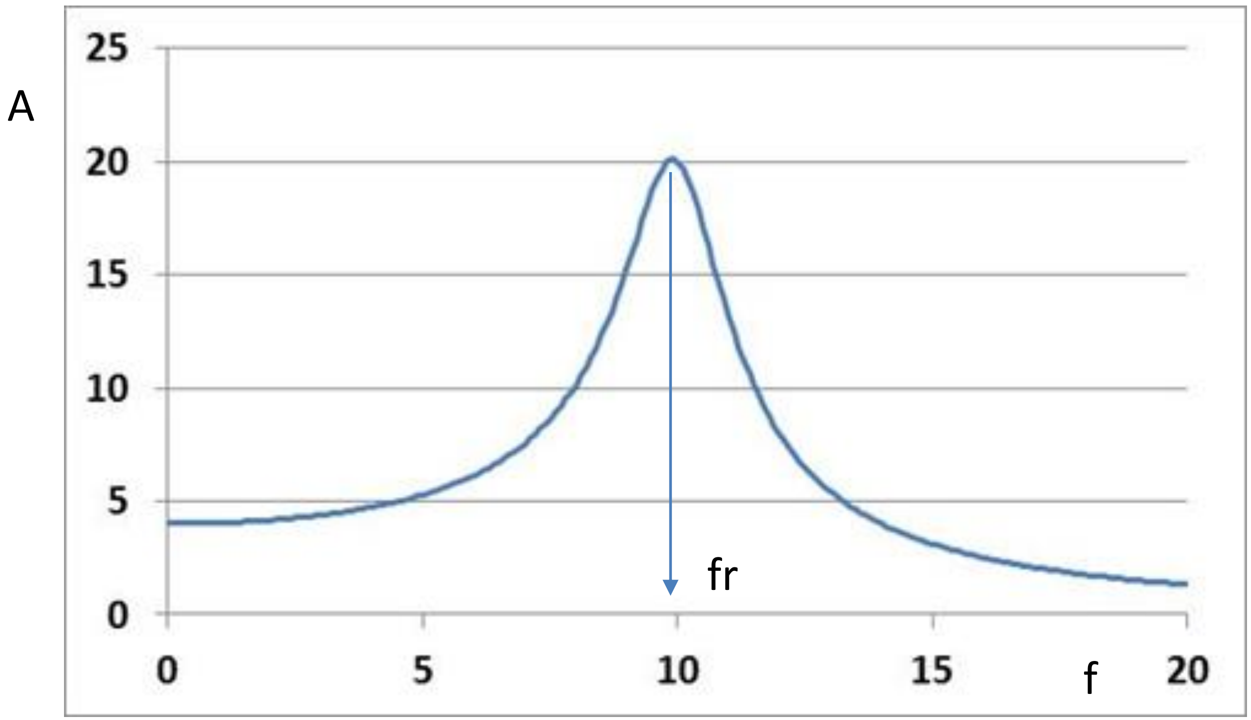
$$\Rightarrow v = A \cdot \omega \cdot \cos\varphi$$

$$\sin\varphi = \frac{a}{a_{max}} = \frac{a}{A \cdot \omega^2} \Rightarrow a = A \cdot \omega^2 \cdot \sin\varphi$$



y
T
t
v
t
a or F
t

Natural frequency (eigenfrequency), resonance



Problem

We suspend a ball of 0.4 kg on a vertically positioned spring with a spring constant of 60 N/m. Upon releasing the ball the system undergoes harmonic oscillation.

a) Calculate the amplitude of the oscillation!

b) Calculate the period of the oscillation!

$m = 0{,}4\,kg$ $\qquad k = 60\,\frac{N}{m}$

$$f = \frac{1}{2\pi}\sqrt{\frac{60}{0{,}4}} \qquad f = 1{,}95\,Hz$$

$$\frac{1}{f} = T = 0{,}513\,s$$

$$F = -m\omega^2 y$$

$$\omega = 2\pi \cdot f = 12{,}245\,\frac{1}{s}$$

$$mg = m\omega^2 y$$

$$\frac{g}{\omega^2} = y = A \qquad \downarrow 12{,}245^2$$

$$\frac{9{,}81}{149{,}96} = 0{,}065\,m \rightarrow 6{,}5\,cm$$