Diffusion, polymers, reptation

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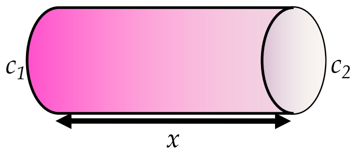
Thermodynamic currents

Differences in the intensive variable at different points in the system induce currents of the conjugate extensive variable.

Thermodynamic current	Intensive quantity	Current density	Law
Heat	Temperature (T)	$J_E = -\lambda \frac{\Delta T}{\Delta x}$	Fourier
Volume	Pressure (p)	$J_V = -\frac{R^2}{8\eta}\frac{\Delta p}{\Delta x}$	Hagen-Poiseuille
Electric current	Electric potential (φ)	$J_Q = -\frac{1}{\rho}\frac{\Delta \varphi}{\Delta x}$	Ohm
Diffusion	Concentration (c)	$J_n = -D\frac{\Delta c}{\Delta x}$	Fick

Fick's first law

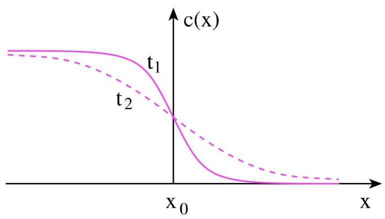


$$\frac{m}{tA} = J_n = -D\frac{\Delta c}{\Delta x}$$

m = amount of substance (number of moles)
t = time
x = length
$\Delta c/\Delta x$ = concentration gradient
A = cross section area
J_n = diffusion flux
D = diffusion constant

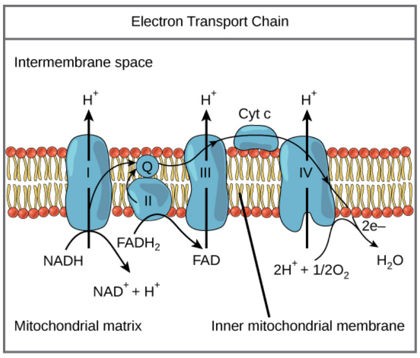
Fick's second law

$$D\frac{\Delta\left(\frac{\Delta c}{\Delta x}\right)}{\Delta x} = \frac{\Delta c}{\Delta t}$$



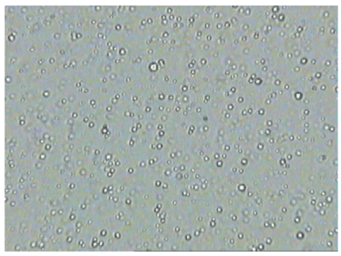
D = diffusion constant
c = concentration
x = distance
t = time

Directed transport by diffusion



Microscopic background: Brownian motion

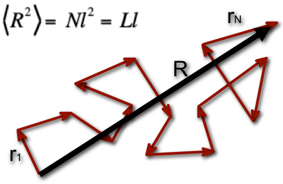
Robert Brown (1773-1858)



Fat droplets in milk (droplet size 0.5 - 3 μm)

Brownian motion – random walk

$\langle R^2 \rangle = Nl^2 = Ll$



R = displacement
N = number of steps
$l = |\vec{r}_i|$ = average free path
r_i = one step
Nl = *L* = total path

Diffusion constant: $D = \frac{1}{3} v l$

Average particle velocity: $v = \frac{l}{\tau}$

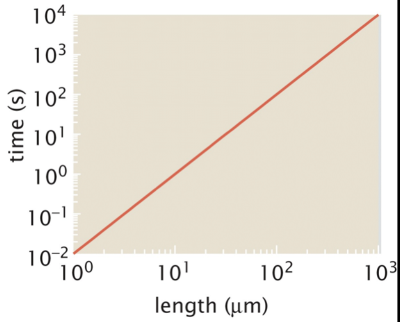
$$\langle R \rangle = \sqrt{Nl^2} = \sqrt{\frac{t}{\tau} l^2} = \sqrt{tvl} = \sqrt{3Dt}$$

The speed of the diffusion

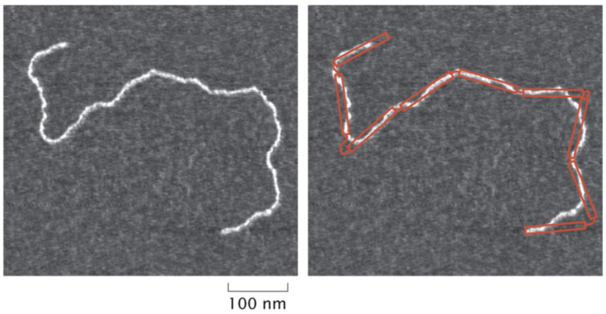
Since the departure of the particles is proportional to the square root of the time, diffusion is

fast on short distances and

slow on long distances.

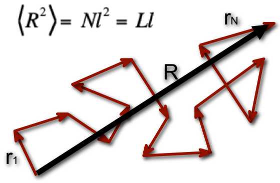


Polymers as random walks

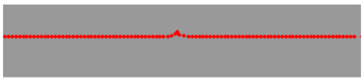


Polymer random walk

$\langle R^2 \rangle = Nl^2 = Ll$

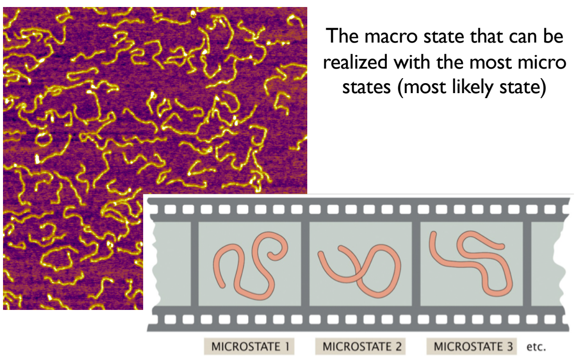


The tendency for elementary vectors to be orientationally disordered results in **elasticity** of the polymer.

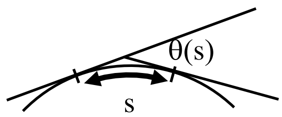


The “equilibrium” state of polymer molecules

The macro state that can be realized with the most micro states (most likely state)



Wormlike chain



$$\langle \cos\theta(s) \rangle = \exp\left(-\frac{s}{l_p}\right)$$

$$l_p = \frac{EI}{k_B T}$$

l_p = persistence length
E = Young's modulus
I = second order moment of the cross section (shape dependent)
$k_B T$ = thermal energy

Connection between the shape and persistence length of the polymer

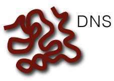
Rigid chain

$L_p >> L_c$ Microtubule

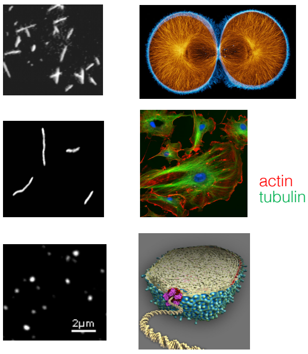
Semiflexible chain

$L_p \approx L_c$ Microfilament (actin)

Flexible chain

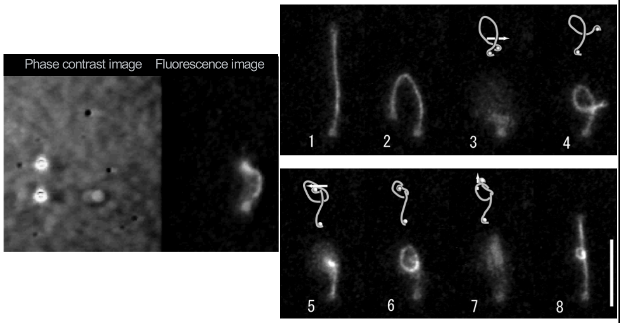


$L_p << L_c$



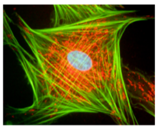
L_p = persistence length, L_c = contour length

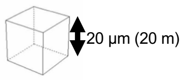
Tying a knot on a DNA molecule using optical trap

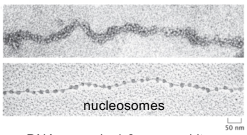


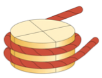
Physical size of the human genome

	Idealized cell: 20 µm edge cube	**House:** 20 m edge cube
DNA thickness	2 nm	2 mm
DNA total length	2 m	2000 km
Persistence length	50 nm	5 cm
Average end-to-end distance	320 µm	320 m
Gyration radius (R_G)	130 µm	130 m







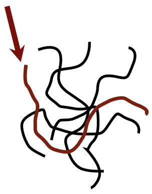


Single nucleosome particle: histone protein octamer + DNA wound ~1.6 x around it

Reptation - a special case of diffusion

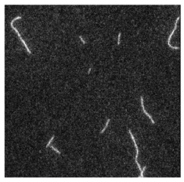
Snake-like diffusion of a polymer in a polymer matrix

Filament in a reptation channel



Indiana Jones and Raiders of the Lost Ark

Actin filaments in metilcelulose matrix



$$\tau_r = \frac{L^2 \cdot N}{\mu \cdot k \cdot T}$$

L = contour length
N = number of monomeric units
μ = chain mobility
kT = thermal energy