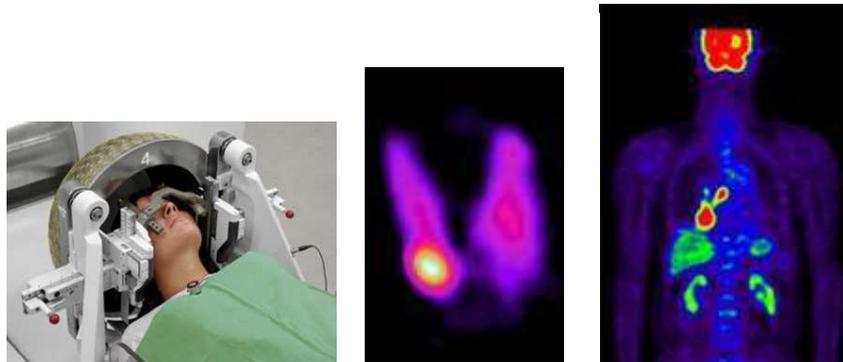


# The atomic nucleus. Radioactivity. Nuclear radiations

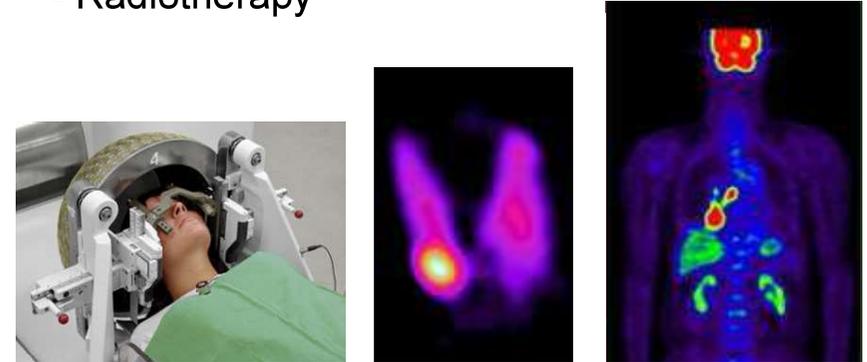
László Smeller



# Why?

Medical applications of the nuclear radiation:

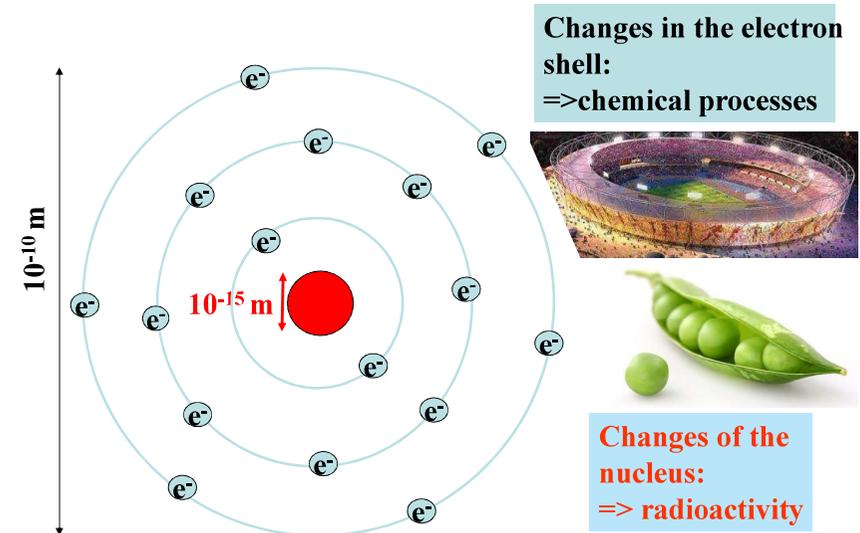
- Nuclear imaging
- Radiotherapy



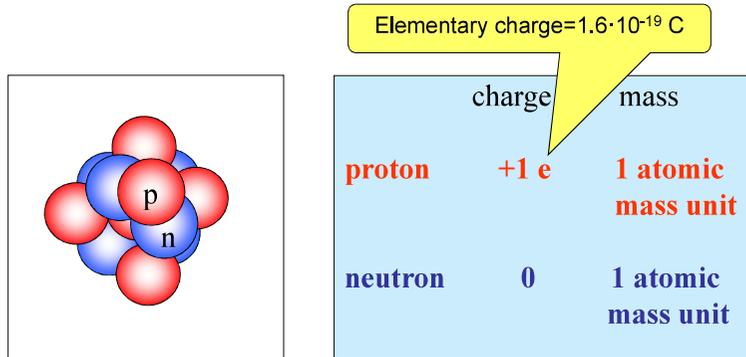
## Length scale of the nature

m			
$10^0$	meter	men	
$10^{-3}$	millimeter	size of letters you can read	
$10^{-6}$	micrometer	size of a cell (e.g. erythrocyte)	
			$\varnothing 7\mu\text{m}$
$10^{-9}$	nanometer	protein	
$10^{-10}$	- angstrom	diameter of an atom, bond length	
		H atom $\varnothing \approx 1$ angstrom (Å)	
$10^{-12}$	picometer	wavelength of the X-ray	
$10^{-15}$	femtometer	size of the nucleus	

## The electrons and the nucleus



# Structure of the nucleus



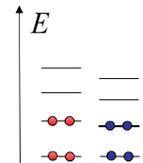
$A$  (mass number) = number of protons + number of neutrons  
 $Z$  (atomic number) = number of protons

→ 99  
 → 43 **Tc**

99 nucleon, 43 proton and 56 neutron

# Stability of the nucleus

- Coulomb force: destabilization (electrostatic repulsion between the protons)
- Nuclear force: very strong attractive force acts only on short range (~fm) independent on the charge
- Quantized energy levels for the nucleus.
- Typical binding energy is in the MeV range  $eV=1.6 \cdot 10^{-19} \text{ J}$



# Isotopes

Variants of an element, where the

- numbers of protons are the same

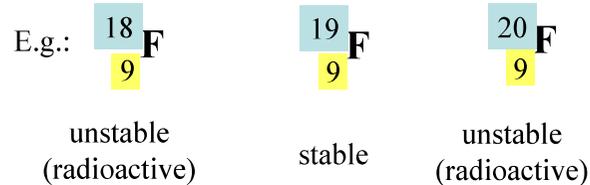
⇒ the chemical properties are identical.

- numbers of neutrons are different

⇒ the mass numbers are different

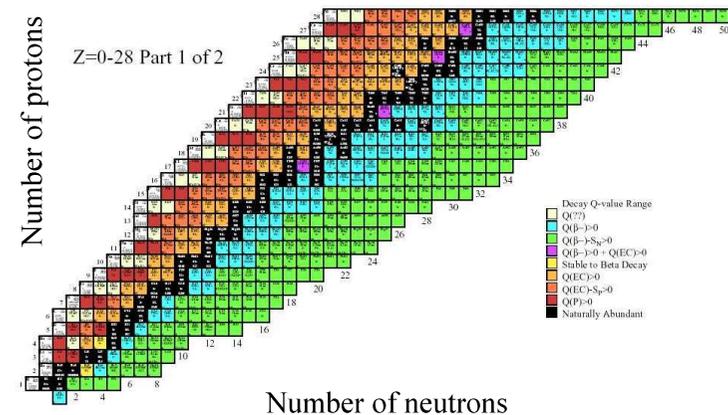


György Hevesy  
Nobel prize 1943

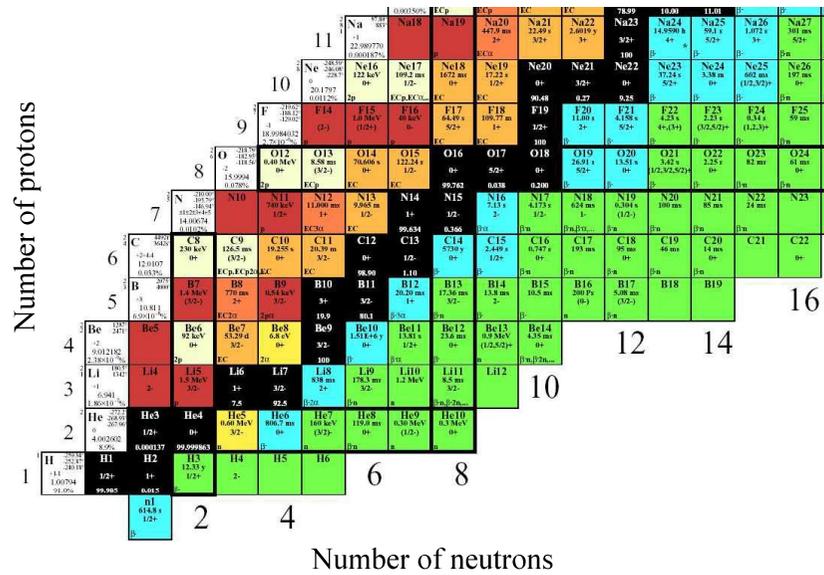


isotope  $\leftrightarrow$  radioactive isotope

# Table of isotopes



# Radioactive decays and particles



$\alpha$  - decay

$\alpha$  - particle =  ${}^4_2\text{He}$  nucleus

$\beta$  - decay :  $\beta^-$   
 $\beta^+$

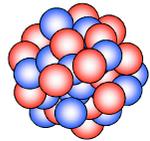
$\beta^-$  particle = electron  
 $\beta^+$  particle = positron

Isomeric transition

$\gamma$ -ray

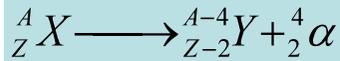
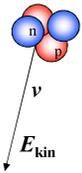
K-electron capture

characteristic x-ray photon

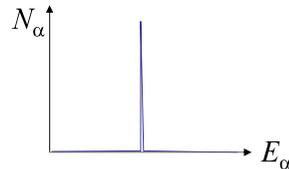


## $\alpha$ - decay

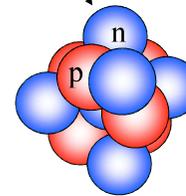
an  $\alpha$  particle ( ${}^4\text{He}$  nucleus) will be emitted  
typical for the heavy atoms



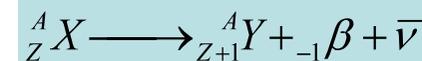
Line spectrum  
 $E_\alpha \sim \text{MeV}$



neutron surplus



## $\beta^-$ - decay

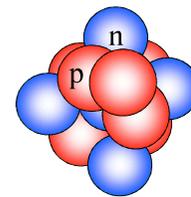


${}^1_0 n \rightarrow {}^1_1 p + {}_{-1} \beta + \bar{\nu}$

↑ remains in the nucleus  
↑ leave the nucleus

e.g.:

- ${}^{20}_9\text{F}$
- ${}^{32}_{15}\text{P}$
- ${}^{59}_{26}\text{Fe}$
- ${}^{131}_{53}\text{I}$



$\beta^-$ -ray

continuous energy spectrum

$$\beta^- = {}_{-1} \beta = e^-$$



proton surplus

## $\beta^+$ - decay

e.g.:  ${}^{11}_6\text{C}$   
 ${}^{15}_8\text{O}$   
 ${}^{18}_9\text{F}$   
 ${}^{52}_{26}\text{Fe}$

$${}^A_Z X \longrightarrow {}^A_{Z-1} Y + {}^0_{+1} \beta + \nu$$

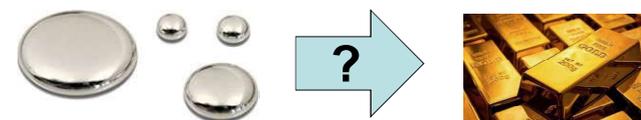
$${}^1_1 p \longrightarrow {}^1_0 n + {}^0_{+1} \beta + \nu$$

remains in the nucleus      leave the nucleus

e.g.:  ${}^{30}_{15}\text{P} \longrightarrow {}^{30}_{14}\text{Si} + {}^0_{+1} \beta + \nu$

$\beta^+$ -ray  $\rightarrow$  continuous energy spectrum  
 These isotopes must be produced artificially (e.g. in cyclotron)

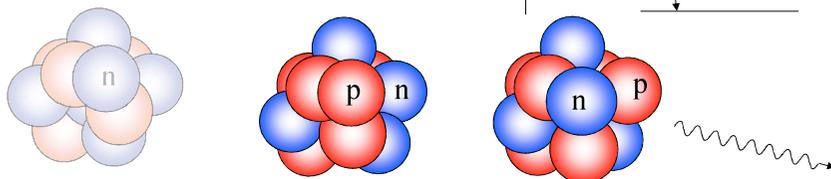
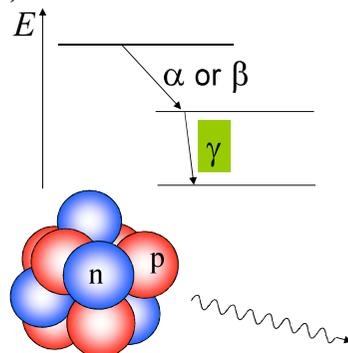
Medical application: PET



## Prompt $\gamma$ -radiation

The daughter nucleus might have an **energetically unfavoured** arrangement of nucleons. (excited state)

The surplus energy will normally be emitted immediately ( $< \text{ps}$ ) in form of the  $\gamma$  radiation



Atomic number, mass number are unchanged.

## Isomeric transition

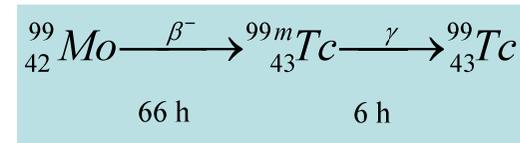
In some rare cases the excited state of the daughter nucleus is metastable, the  $\gamma$ -radiation will be emitted later.

The parent and daughter atoms can be separated: the daughter atom emits **only  $\gamma$ -radiation!**

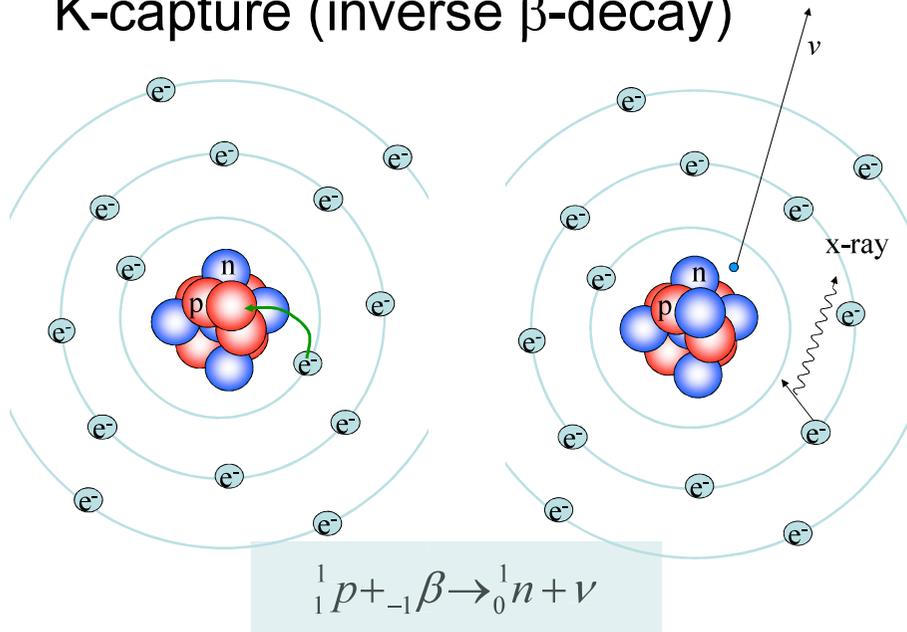
$\Rightarrow$  Isotope diagnostics (nuclear imaging)



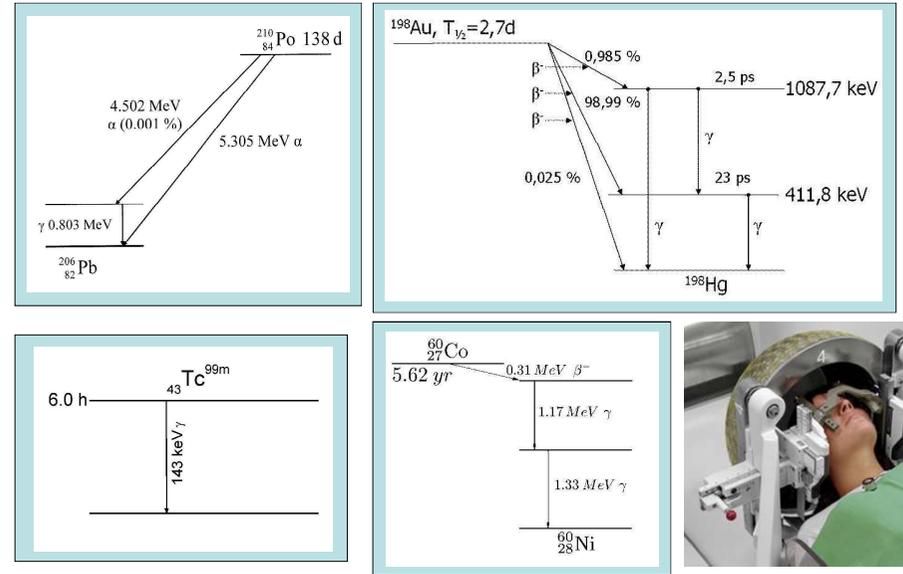
E.g.:  ${}^{99m}\text{Tc}$



# K-capture (inverse $\beta$ -decay)



# Some examples of the decay paths



## Characteristics of radioactive decays in general

- activity characterizes the source
- half life time characterizes the speed of the decay\*
- particle energy characterizes the radiation\*

\*depends on the type of the isotope

## Activity ( $\Lambda$ )

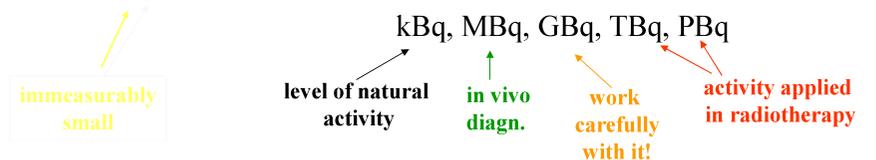
$$\Lambda = \left| \frac{\Delta N}{\Delta t} \right|$$

$N$  = Number of undecayed atoms  
 $t$  = time  
 $-\Delta N$  = Number of decays during  $\Delta t$  time

Activity = number of decays in a unit time

unit: becquerel Bq  
 1 Bq = 1 decay/sec

old unit: Ci = 37GBq



## Law of radioactive decay

$$\Delta N = -\lambda N \Delta t$$

$N$ : Number of undecayed nuclei  
( $-\Delta N$  = Number of decays during  $\Delta t$  time)

$$\frac{\Delta N}{\Delta t} = -\lambda N$$

$\lambda$ : decay constant (probability of the decay [1/s])  
 $1/\lambda = \tau$  average lifetime

Differential equation

solution:

$$N(t) = N_0 e^{-\lambda t} \quad \text{Exponential decrease}$$

number of undecayed atoms at  $t=0$

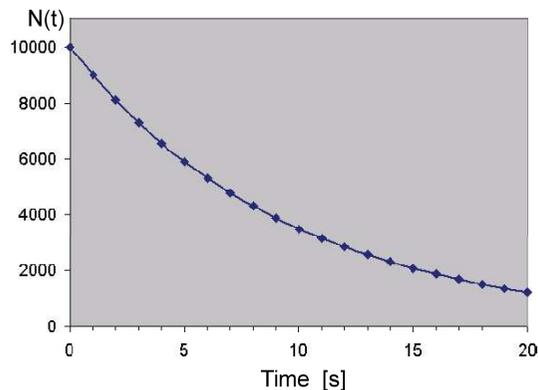
## An example

- At start:  $N_0=10000$  (number of atoms)  
 $\lambda=0.1$  1/s (decay probability)  
 $\Delta N = -\lambda N \Delta t$
- after 1 sec: 9000 (10000x0.1=1000 decayed)
- after 2 sec: 8100 (9000x0.1=900 decayed)
- after 3 sec: 7290 (8100x0.1=810 decayed)
- after 4 sec: 6561 (7290x0.1=729 decayed)
- ....

## An example

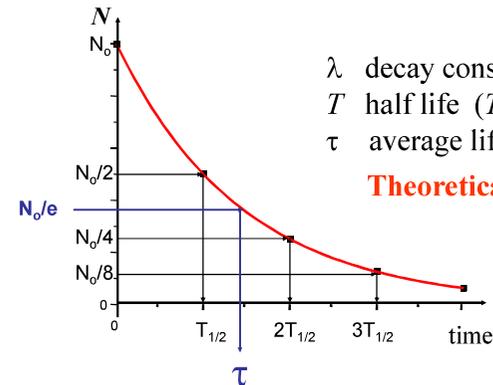
- At start:  $N_0=10000$  (number of atoms)  
 $\lambda=0.1$  1/s (decay probability)

- after 1 s: 9000
- after 2 s: 8100
- after 3 s: 7290
- after 4 s: 6561
- ....



## Law of radioactive decay

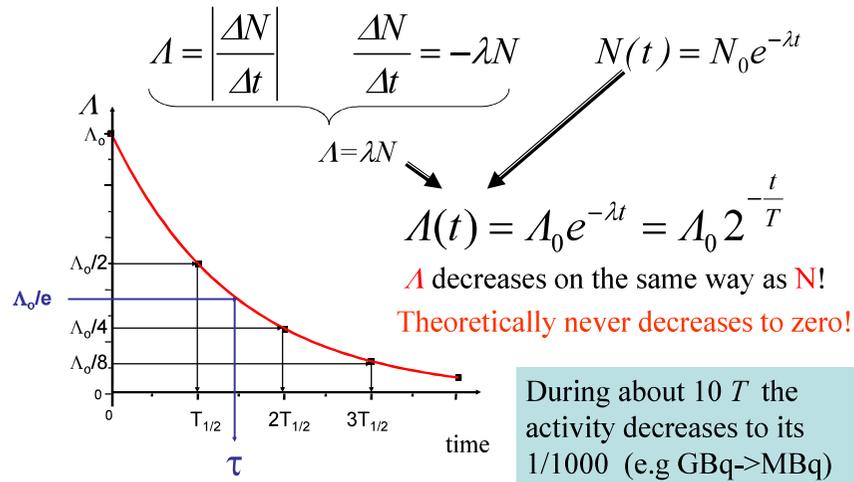
$$N(t) = N_0 e^{-\lambda t} = N_0 2^{-\frac{t}{T}} \quad T = \frac{\ln 2}{\lambda} = \frac{0,693}{\lambda}$$



$\lambda$  decay constant (probability of the decay)  
 $T$  half life ( $T_{1/2}$ )  
 $\tau$  average lifetime

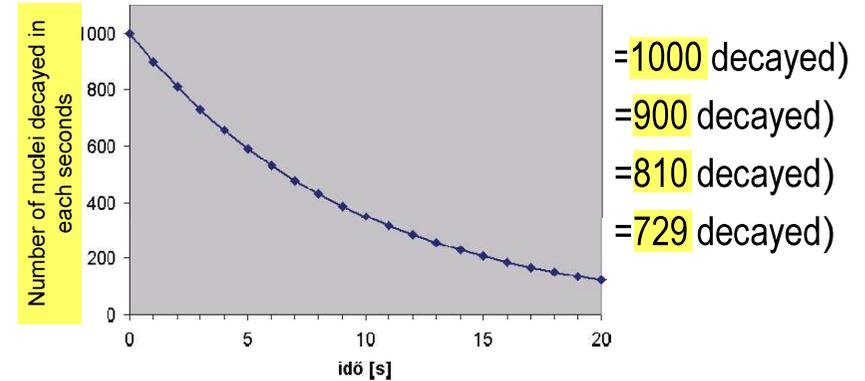
**Theoretically never decreases to zero !**

## Decrease of the activity as a function of time



## An example

- At start:  $N_0=10000$  (number of atoms)  
 $\lambda=0.1 \text{ 1/s}$  (decay probability)



## Few examples for half life

$^{232}\text{Th}$	$1.4 \cdot 10^{10} \text{ y}$	$^{60}\text{Co}$	5.3 y
$^{238}\text{U}$	$4.5 \cdot 10^9 \text{ y}$	$^{59}\text{Fe}$	1.5 m
$^{40}\text{K}$	$1.3 \cdot 10^9 \text{ y}$	$^{56}\text{Cr}$	1 m (28 d)
$^{14}\text{C}$	5736 y	$^{131}\text{I}$	8 d
$^{137}\text{Cs}$	30 y	$^{99\text{m}}\text{Tc}$	6 h
$^3\text{H}$	12.3 y	$^{18}\text{F}$	110 min
		$^{11}\text{C}$	20 min
		$^{15}\text{O}$	2 min
		$^{222}\text{Th}$	2.8 ms

Don't learn these numbers!

## Typical energy levels in the microworld

Excitation of the outer electrons

eV (aJ)

light



Electron transition between inner electrons

keV (fJ)

X-ray



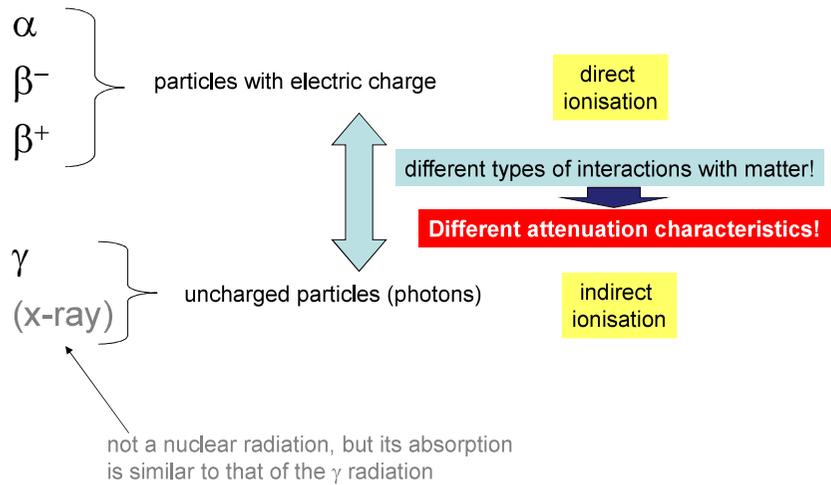
Transformation of the nucleus (decay)

MeV (pJ)

Nuclear radiation  
 $\alpha, \beta, \gamma$



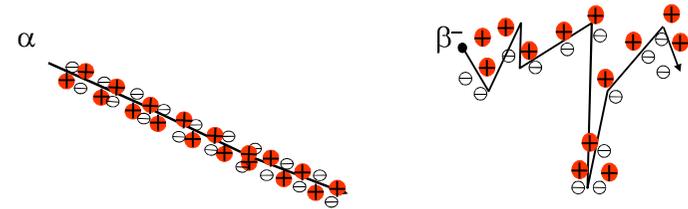
# Absorption of the nuclear radiation



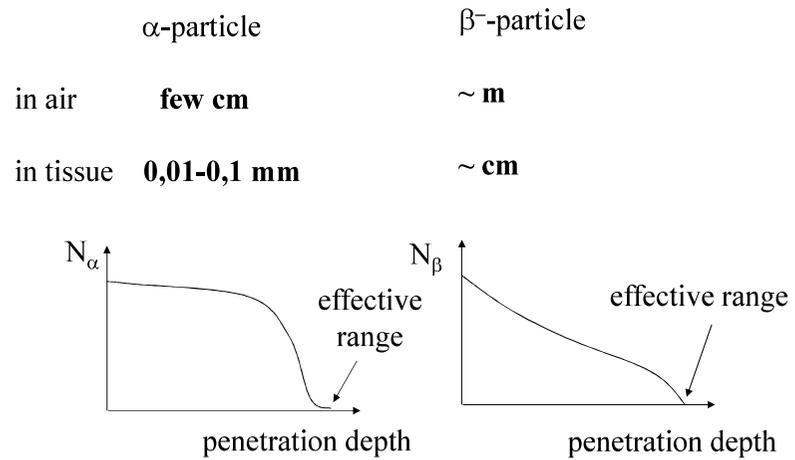
# Absorption of the charged particles

Ionizing during the path => continuous decrease of the particle energy  
 The energy after a given path length decreases to the thermal value

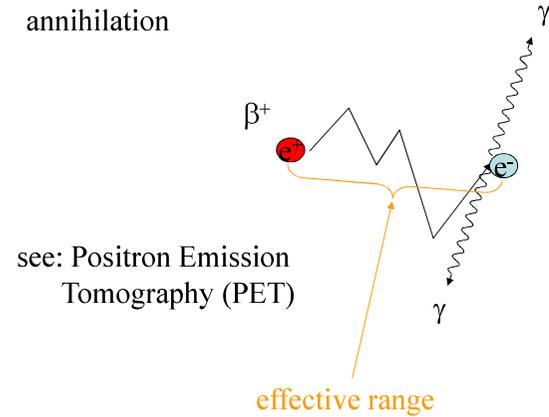
effective range



# Effective range

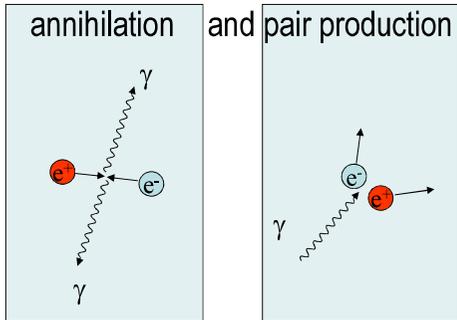


# $\beta^+$ -radiation



# Electron and positron

- particle - antiparticle
- same mass,
- charge: same value, but different sign

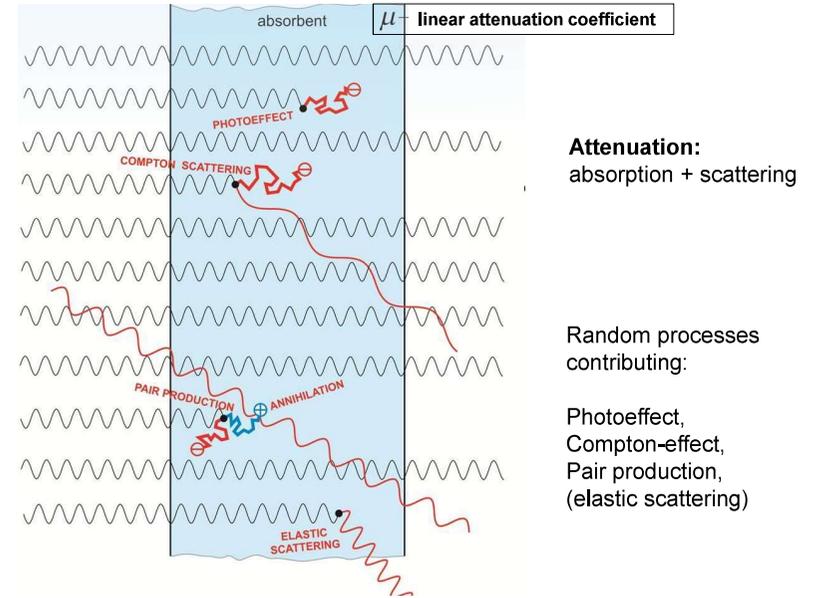


Einstein:  
mass-energy  
ekvivalence

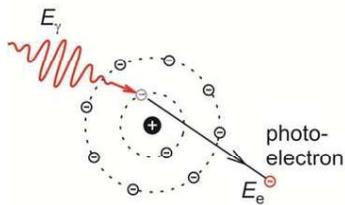
$$E=mc^2$$

$$m_e c^2 = 511 \text{ keV} \approx 0.5 \text{ MeV}$$

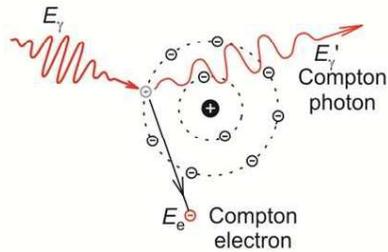
# Absorption of the $\gamma$ -radiation (and x-ray)



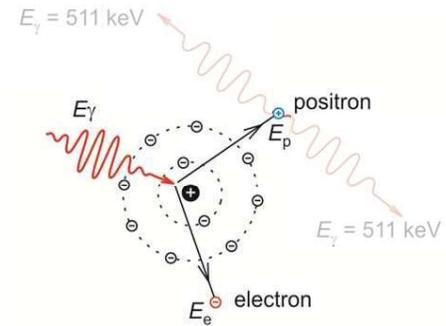
Photoeffect



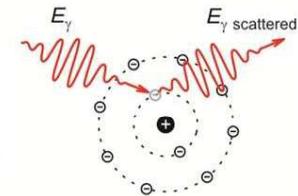
Compton effect  
Compton-scattering



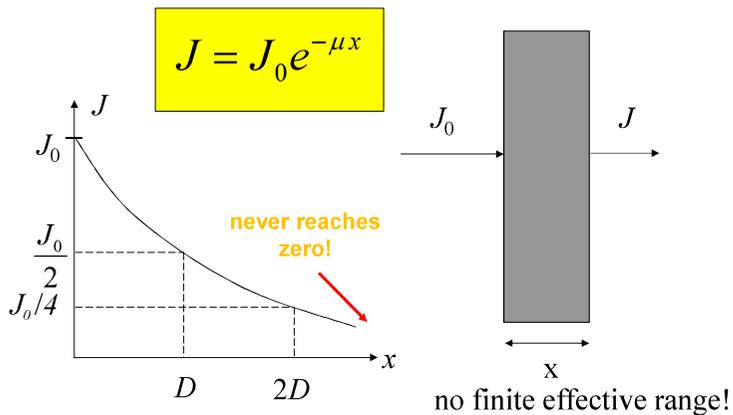
Pair production



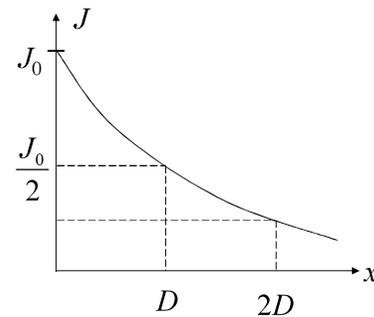
elastic scattering



# Attenuation of the $\gamma$ -radiation and x-ray



few rules of thumb:  $x_{1/10} = 3,32 D$      $x_{1/1000} = 10 D$



$$J = J_0 e^{-\mu x}$$

$\mu$ : (linear) attenuation coefficient  
its units are: 1/m, 1/cm

$\delta = \frac{1}{\mu}$  „penetration depth”  
Intensity decreases to the e-th part (c.a. 37%)

$\mu$ (material, number of absorbing centers, energy of the radiation)  
 $= \mu(\text{material}, \rho, E_{\text{photon}}) \sim \rho$

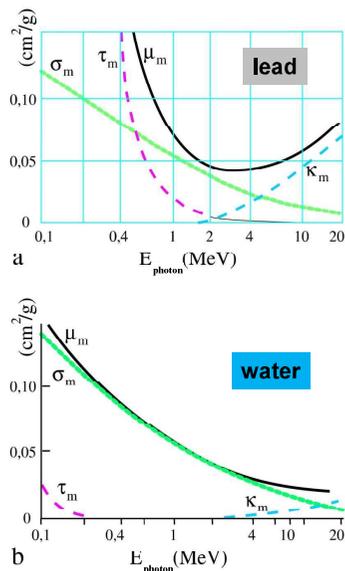
$$\mu_m = \frac{\mu}{\rho} \quad \text{mass attenuation coefficient}$$

mass attenuation coeff.

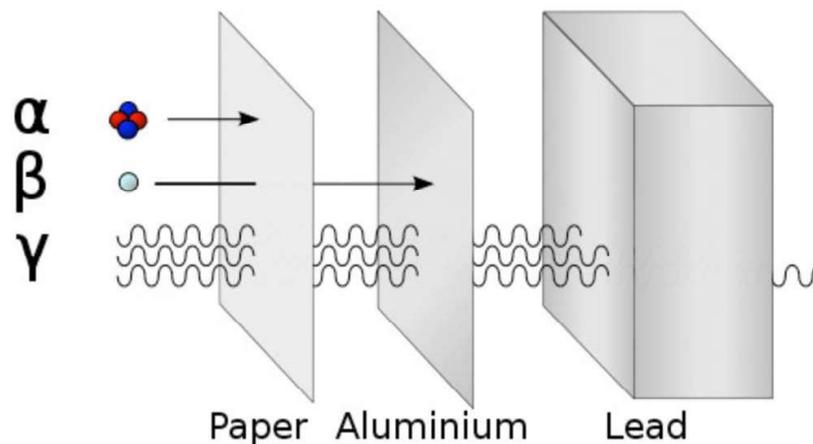
$$\mu_m = \frac{\mu}{\rho}$$

$$\mu_m = \tau_m + \sigma_m + \kappa_m$$

$$\tau_m = c \lambda^3 Z^3$$



Summary of the absorption of  $\alpha$ ,  $\beta$  and  $\gamma$  radiation



# Applications (attenuation)



← lead glass

→ protective apron



# Applications: isotopes and nuclear radiation

