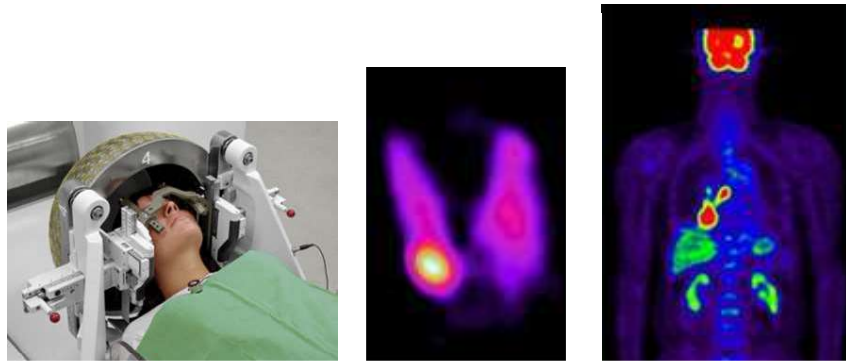


The atomic nucleus. Radioactivity. Nuclear radiations

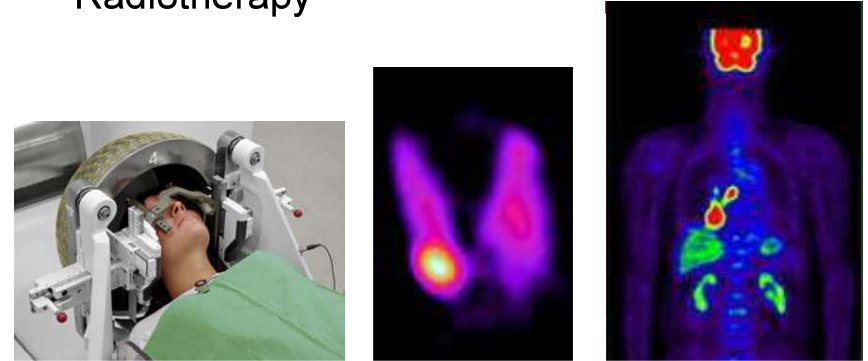
László Smeller



Why?

Medical applications of the nuclear radiation:

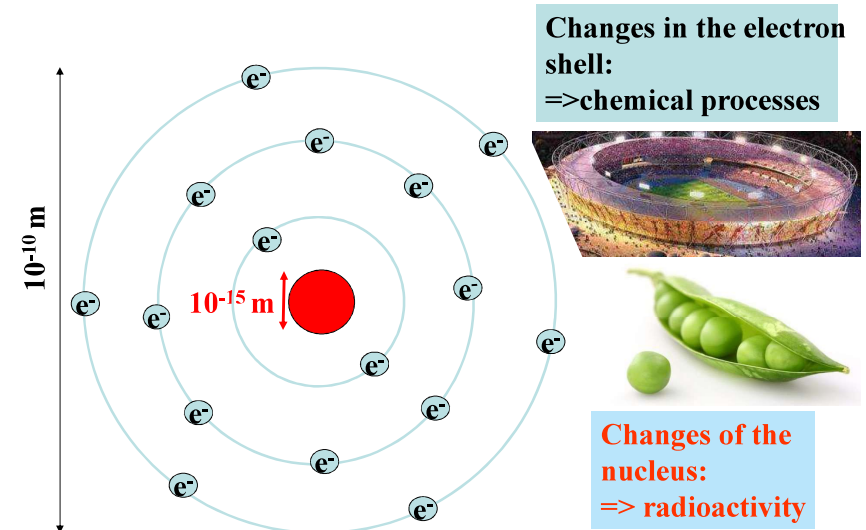
- Nuclear imaging
- Radiotherapy



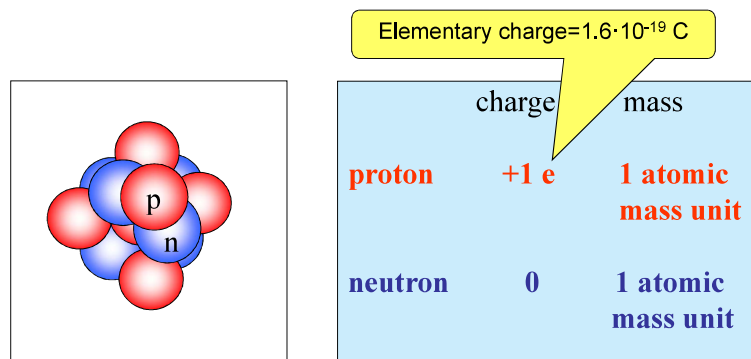
Length scale of the nature

m		
10^0	meter	men
10^{-3}	millimeter	size of letters you can read
10^{-6}	micrometer	size of a cell (e.g. erythrocyte) $\varnothing 7\mu\text{m}$
10^{-9}	nanometer	protein
10^{-10}	– angstrom	diameter of an atom, bond length H atom $\varnothing \approx 1$ angstrom (Å)
10^{-12}	picometer	wavelength of the X-ray
10^{-15}	femtometer	size of the nucleus

The electrons and the nucleus



Structure of the nucleus



A (mass number) = number of protons
+ number of neutrons

Z (atomic number) = number of protons

99 nucleon, 43 proton and 56 neutron

99
43 **Tc**

Isotopes

Variants of an element, where the

- numbers of protons are the same

=> the chemical properties are identical.

- numbers of neutrons are different

=> the mass numbers are different

E.g.: $^{18}_9\text{F}$ $^{19}_9\text{F}$ $^{20}_9\text{F}$

unstable
(radioactive)

stable

unstable
(radioactive)

isotope <-> radioactive isotope



György Hevesy
Nobel prize 1943

Stability of the nucleus

- Coulomb force: destabilization
(electrostatic repulsion between the protons)

- Nuclear force: very strong
attractive force
acts only on short range (~fm)
independent on the charge



- Quantized energy levels for the nucleus.
- Typical binding energy is in the MeV range
 $eV = 1.6 \cdot 10^{-19} \text{ J}$

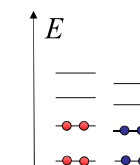
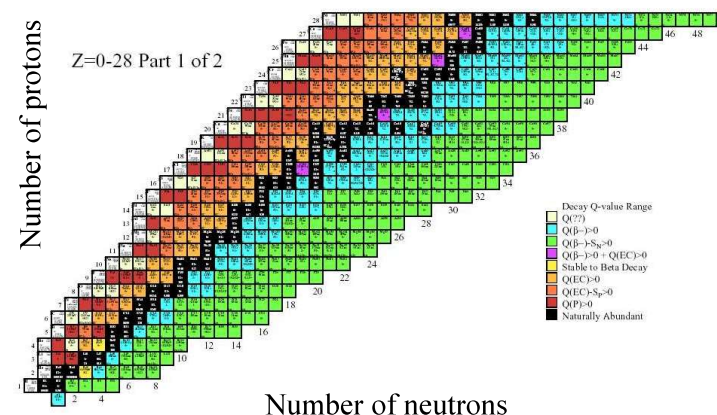
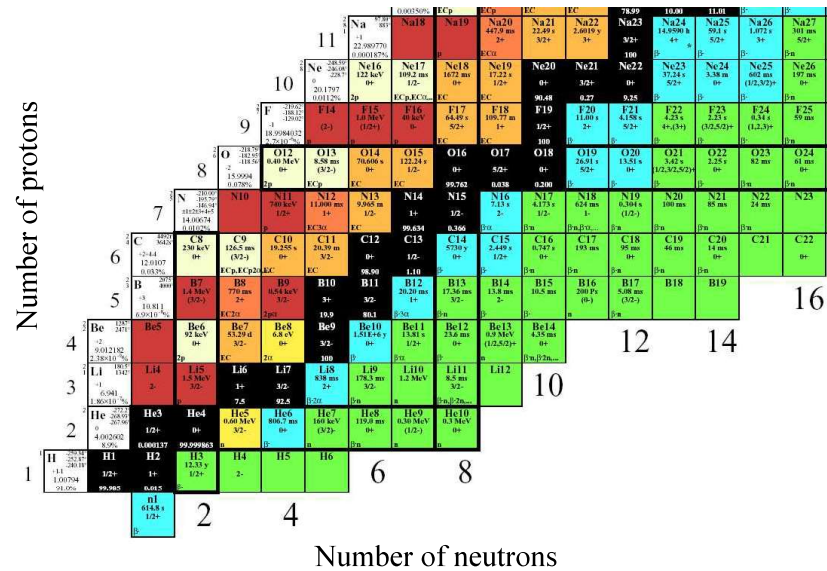


Table of isotopes



Radioactive decays and particles



α - decay

α - particle = ${}^4_2\text{He}$ nucleus

β - decay : β^-
 β^+

β^- particle = electron

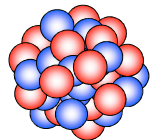
β^+ particle = positron

Isomeric transition

γ -ray

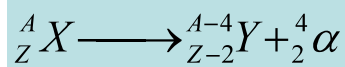
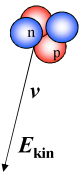
K-electron capture

characteristic x-ray photon

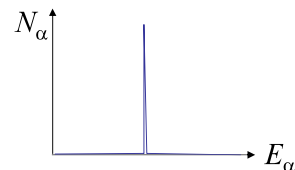


α - decay

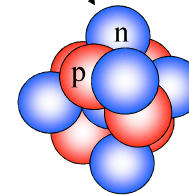
an α particle (${}^4\text{He}$ nucleus) will be emitted
typical for the heavy atoms



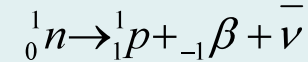
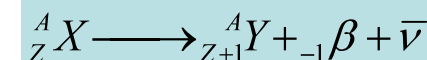
Line spectrum
 $E_\alpha \sim \text{MeV}$



neutron surplus

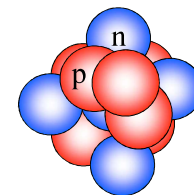
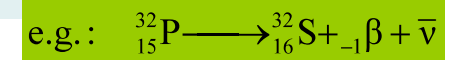
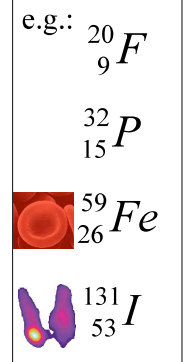


β^- - decay



remains in
the nucleus

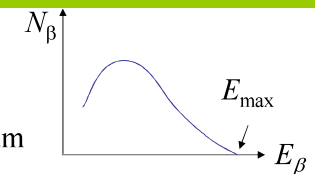
leave the
nucleus



β^- -ray

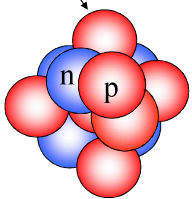
continuous
energy spectrum

$\beta^- = {}_{-1}^0\beta = e^-$



β^+ - decay

proton surplus

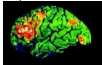


$${}^A_Z X \longrightarrow {}^A_{Z-1} Y + {}^0_{+1} \beta + \nu$$

$${}^1_1 p \longrightarrow {}^1_0 n + {}^0_{+1} \beta + \nu$$

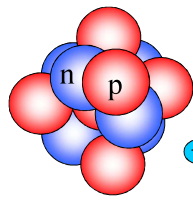
remains in the nucleus leave the nucleus

e.g.: ${}^{11}_6 C$, ${}^{15}_8 O$, ${}^{18}_9 F$, ${}^{52}_{26} Fe$



e.g.: ${}^{30}_{15} P \longrightarrow {}^{30}_{14} Si + {}^0_{+1} \beta + \nu$


β^+ -ray



continuous energy spectrum

These isotopes must be produced artificially (e.g. in cyclotron)

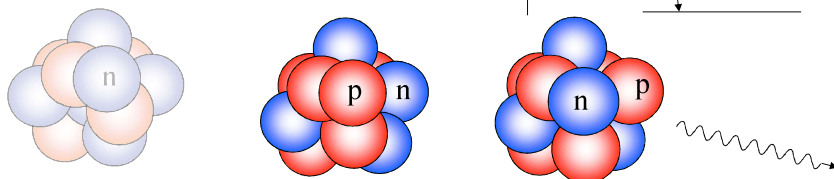
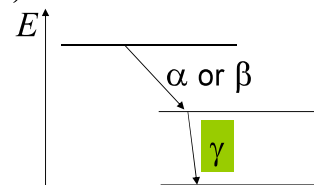
Medical application: PET




Prompt γ -radiation

The daughter nucleus might have an **energetically unfavoured** arrangement of nucleons. (excited state)

The surplus energy will normally be emitted immediately ($< ps$) in form of the γ radiation



Atomic number, mass number are unchanged.

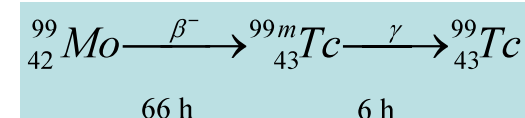
Isomeric transition

In some rare cases the excited state of the daughter nucleus is metastable, the γ -radiation will be emitted later.

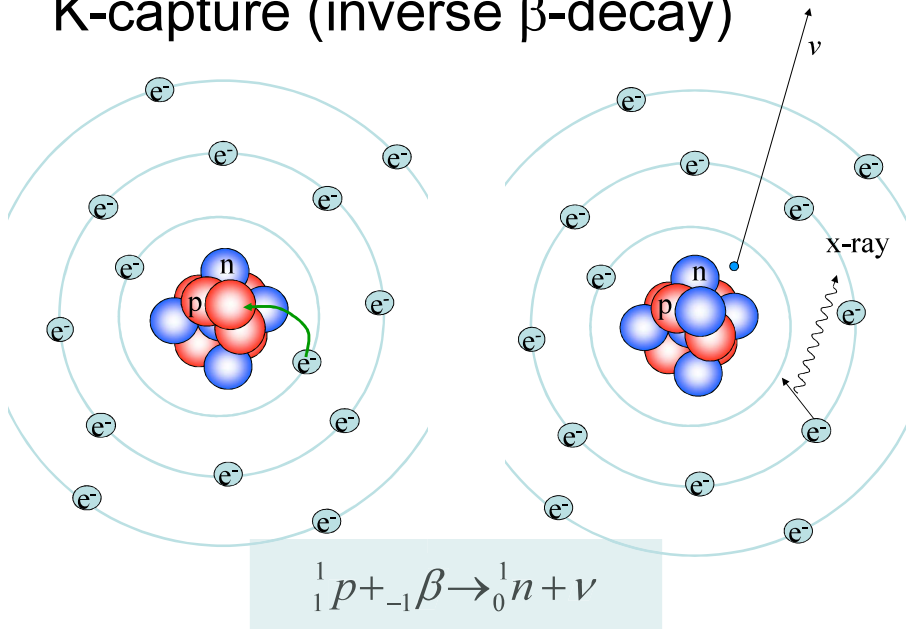
The parent and daughter atoms can be separated: the daughter atom emits **only γ -radiation!**

=> **Isotope diagnostics (nuclear imaging)**

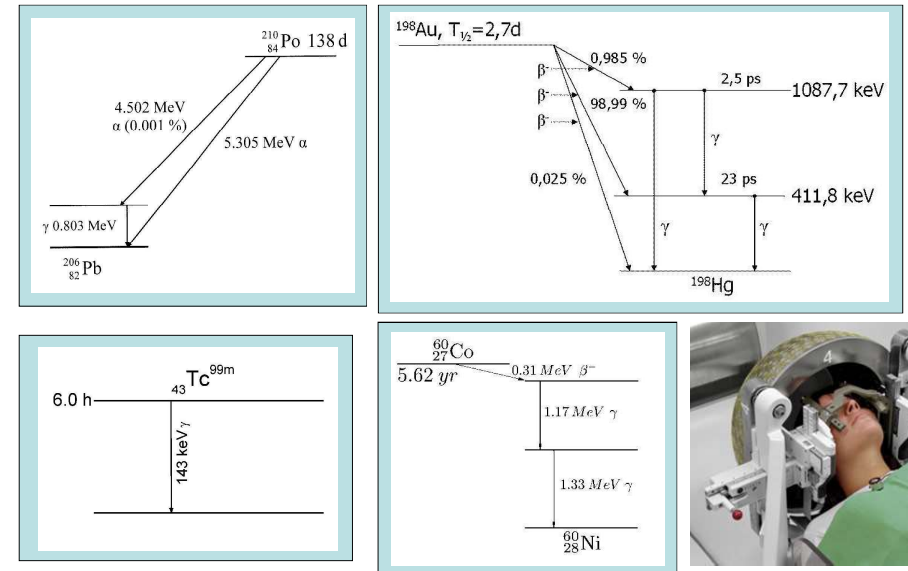
E.g.: ${}^{99m}Tc$



K-capture (inverse β -decay)



Some examples of the decay paths



Characteristics of radioactive decays in general

activity	characterizes the source
half life time	characterizes the speed of the decay*
particle energy	characterizes the radiation*

*depends on the type of the isotope

Activity (Λ)

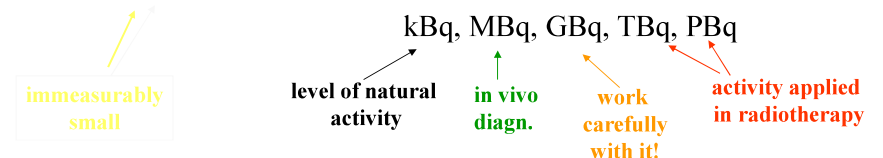
$$\Lambda = \left| \frac{\Delta N}{\Delta t} \right|$$

N = Number of undecayed atoms
 t = time
 $-\Delta N$ = Number of decays during Δt time

Activity = number of decays in a unit time

unit: becquerel Bq
 1 Bq = 1 decay/sec

old unit: Ci = 37 GBq



Law of radioactive decay

$$\Delta N = -\lambda N \Delta t$$

N : Number of undecayed nuclei
($-\Delta N$ = Number of decays during Δt time)

$$\frac{\Delta N}{\Delta t} = -\lambda N$$

λ : decay constant (probability of the decay [1/s])
 $1/\lambda = \tau$ average lifetime

Differential equation

solution:

$$N(t) = N_0 e^{-\lambda t}$$

Exponential decrease

number of undecayed atoms at $t=0$

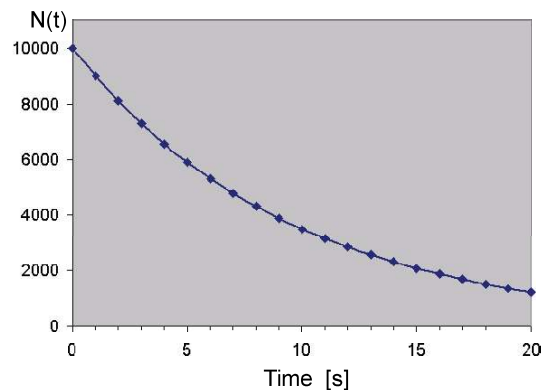
An example

- At start: $N_0 = 10000$ (number of atoms)
 $\lambda = 0.1 \text{ 1/s}$ (decay probability)
 $\Delta N = -\lambda N \Delta t$
- after 1 sec: 9000 (10000x0.1=1000 decayed)
- after 2 sec: 8100 (9000x0.1=900 decayed)
- after 3 sec: 7290 (8100x0.1=810 decayed)
- after 4 sec: 6561 (7290x0.1=729 decayed)
-

An example

- At start: $N_0 = 10000$ (number of atoms)
 $\lambda = 0.1 \text{ 1/s}$ (decay probability)

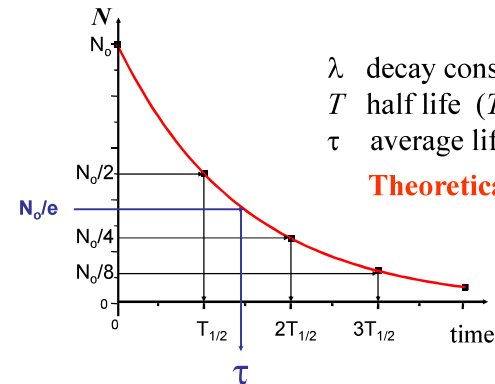
- after 1 s: 9000
- after 2 s: 8100
- after 3 s: 7290
- after 4 s: 6561
-



Law of radioactive decay

$$N(t) = N_0 e^{-\lambda t} = N_0 2^{-\frac{t}{T}}$$

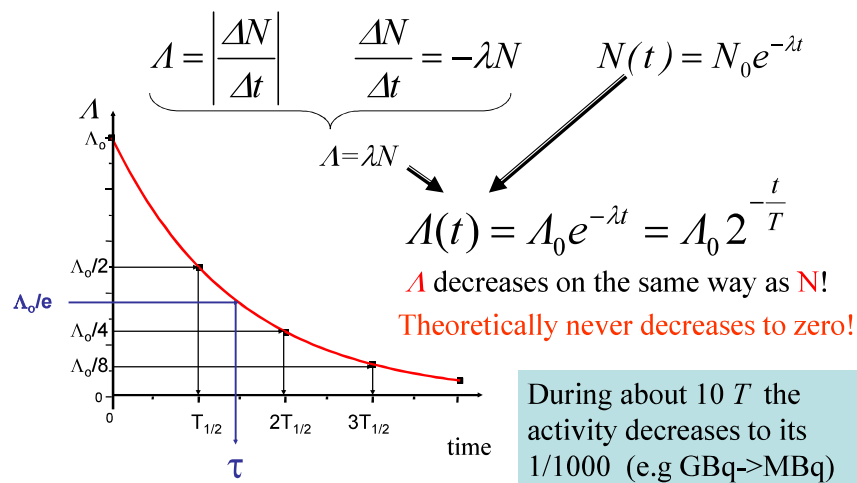
$$T = \frac{\ln 2}{\lambda} = \frac{0,693}{\lambda}$$



λ decay constant (probability of the decay)
 T half life ($T_{1/2}$)
 τ average lifetime

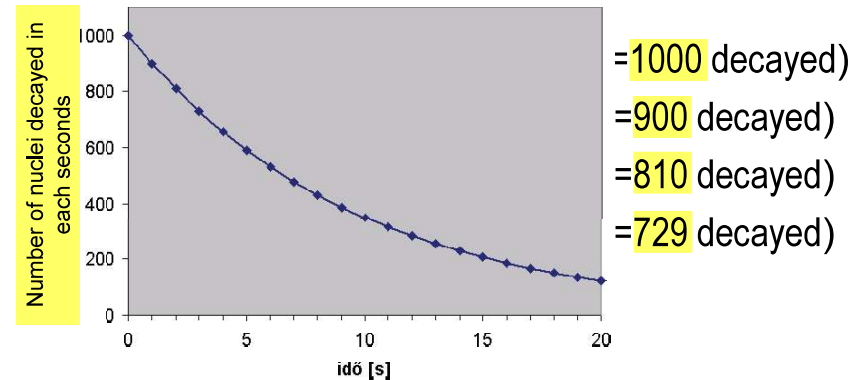
Theoretically never decreases to zero !

Decrease of the activity as a function of time



An example

- At start: $N_0 = 10000$ (number of atoms)
 $\lambda = 0.1 \text{ 1/s}$ (decay probability)



Few examples for half life

^{232}Th	$1.4 \cdot 10^{10} \text{ y}$	^{60}Co	5.3 y
^{238}U	$4.5 \cdot 10^9 \text{ y}$	^{59}Fe	1.5 m
^{40}K	$1.3 \cdot 10^9 \text{ y}$	^{56}Cr	1 m (28 d)
^{14}C	5736 y	^{131}I	8 d
^{137}Cs	30 y	$^{99\text{m}}\text{Tc}$	6 h
^3H	12.3 y	^{18}F	110 min
		^{11}C	20 min
		^{15}O	2 min
		^{222}Th	2.8 ms

Don't learn these numbers!

Typical energy levels in the microworld

Excitation of the outer electrons

eV (aJ)

light



Electron transition between inner electrons

keV (fJ)

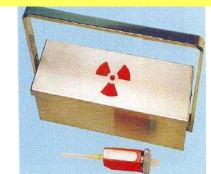
X-ray



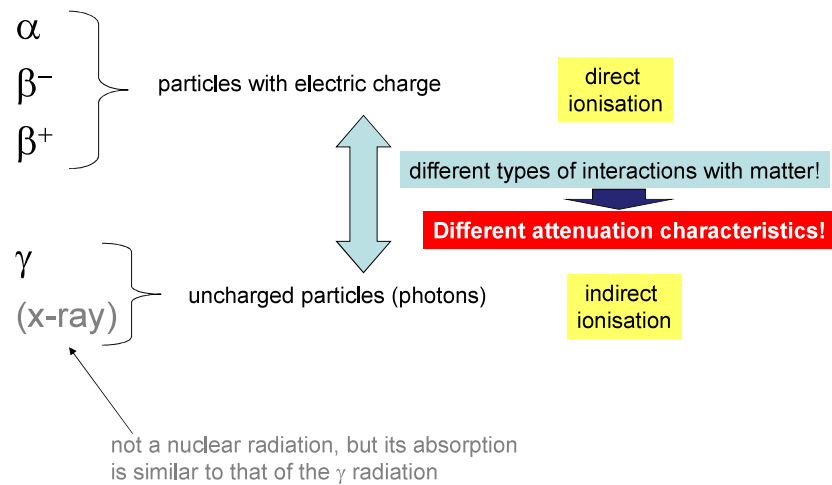
Transformation of the nucleus (decay)

MeV (pJ)

Nuclear radiation
 α, β, γ



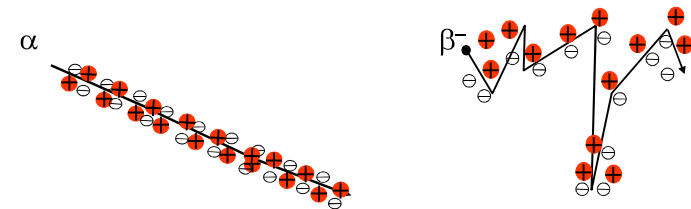
Absorption of the nuclear radiation



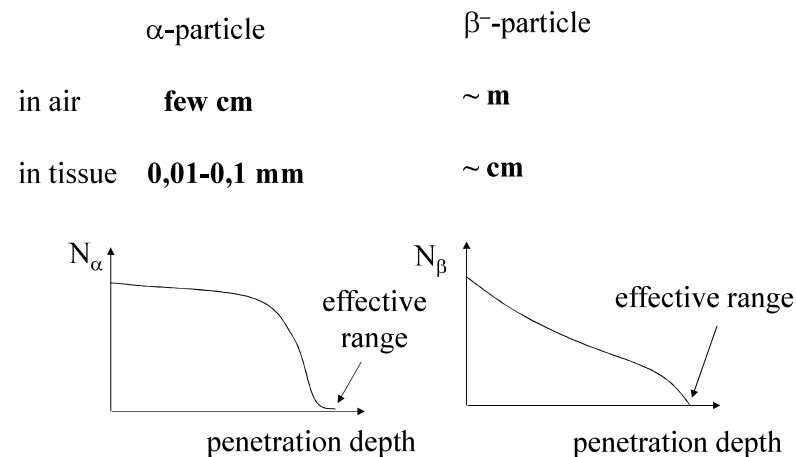
Absorption of the charged particles

Ionizing during the path => continuous decrease of the particle energy
 The energy after a given path length decreases to the thermal value

effective range

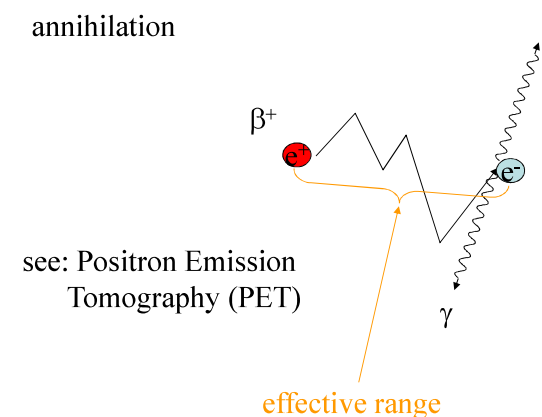


Effective range



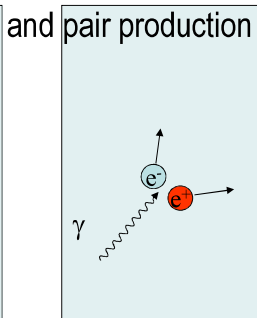
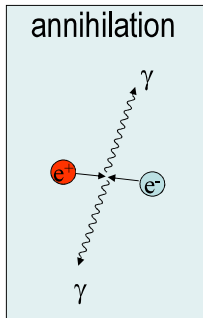
β^+ -radiation

annihilation



Electron and positron

- particle - antiparticle
- same mass,
- charge: same value, but different sign

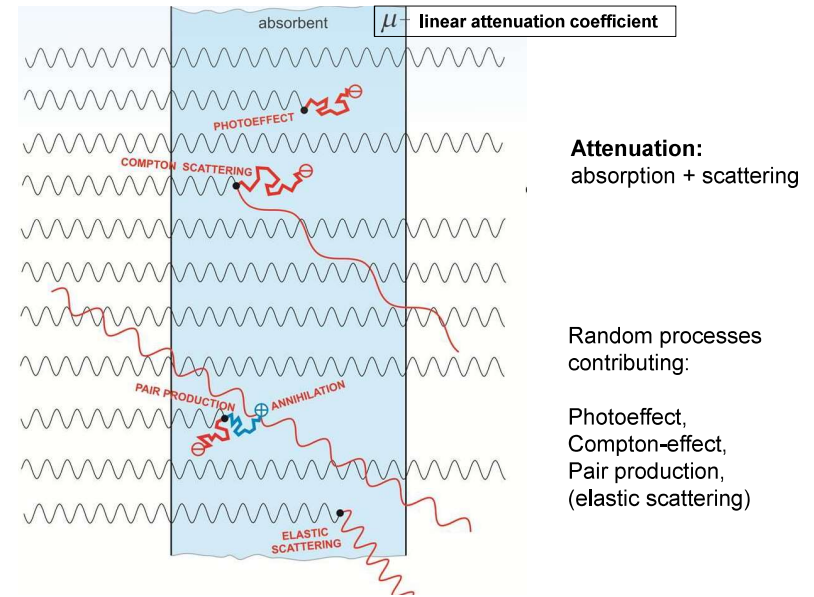


Einstein:
mass-energy
ekvivalence

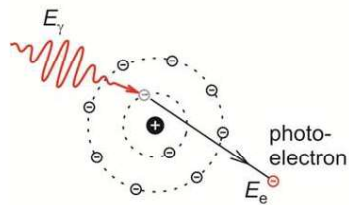
$$E=mc^2$$

$$m_e c^2 = 511 \text{ keV} \approx 0.5 \text{ MeV}$$

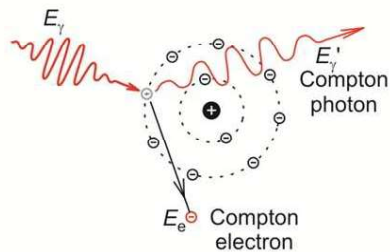
Absorption of the γ -radiation (and x-ray)



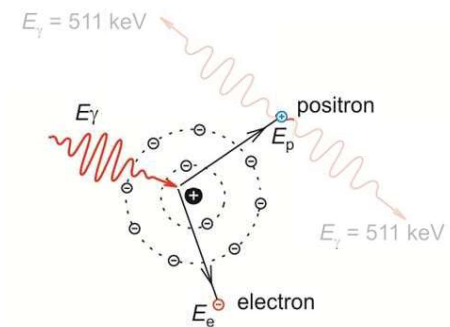
Photoeffect



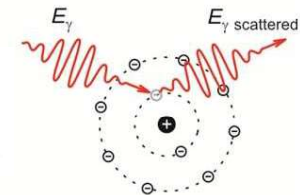
Compton effect
Compton-scattering



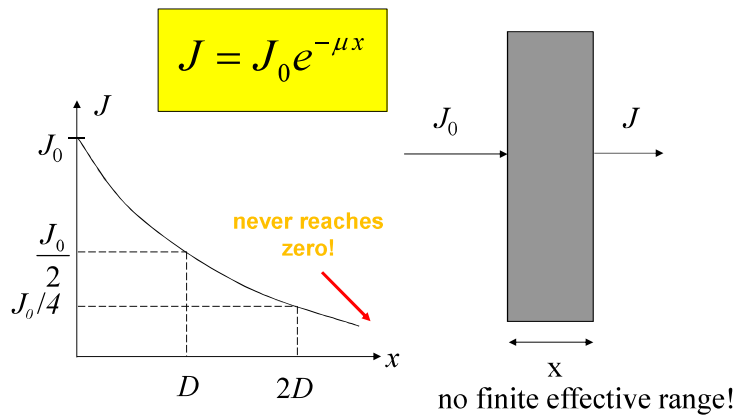
Pair production



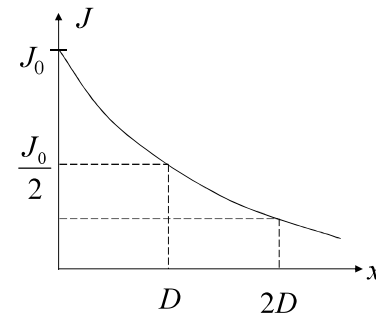
elastic scattering



Attenuation of the γ -radiation and x-ray



few rules of thumb: $x_{1/10} = 3,32 D$ $x_{1/1000} = 10 D$



$$J = J_0 e^{-\mu x}$$

μ : (linear) attenuation coefficient
its units are: 1/m, 1/cm

$\delta = \frac{1}{\mu}$ „penetration depth”
Intensity decreases to the e-th part (c.a. 37%)

μ (material, number of absorbing centers, energy of the radiation)
 $= \mu(\text{material}, \rho, E_{\text{photon}}) \sim \rho$

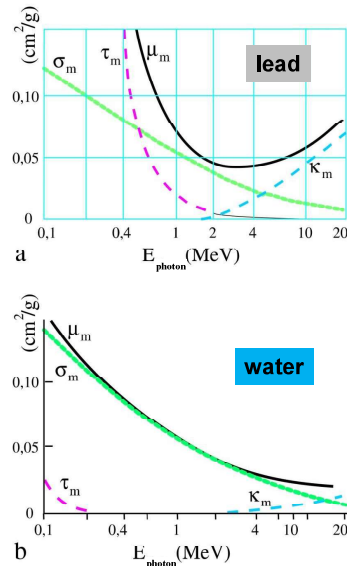
$$\mu_m = \frac{\mu}{\rho} \quad \text{mass attenuation coefficient}$$

mass attenuation coeff.

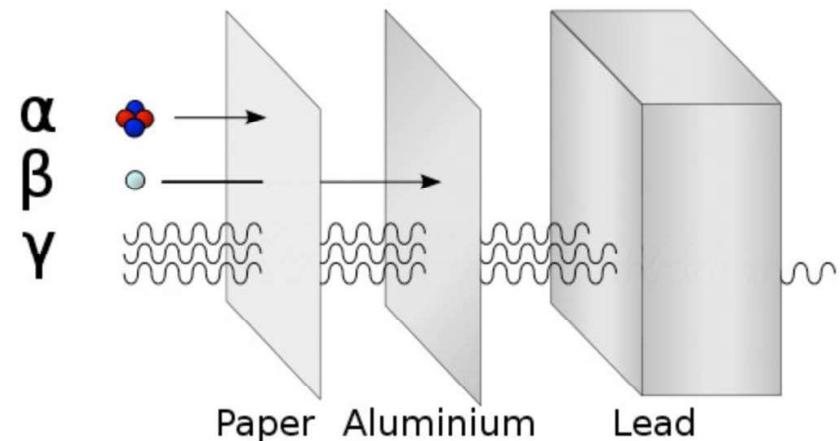
$$\mu_m = \frac{\mu}{\rho}$$

$$\mu_m = \tau_m + \sigma_m + \kappa_m$$

$$\tau_m = c \lambda^3 Z^3$$



Summary of the absorption of α , β and γ radiation



Applications (attenuation)



lead glass

protective apron



Applications: isotopes and nuclear radiation

