

## Nuclear radiation, radioactive isotopes

## Constituents of atoms

Particle	Symbol	Rest Energy (MeV)	Relative Charge*	Mass (kg)	Relative Mass (AMU)**
electron	e	0.51100	1-	$9.11 \times 10^{-31}$	$5.4858 \times 10^{-4}$
proton	p	938.272	0	$1.6726 \times 10^{-27}$	1.0072765
neutron	n	939.566	1+	$1.6749 \times 10^{-27}$	1.0086649

\* electrons have an electric charge of  $-1.602 \times 10^{-19}$  C

\*\*The atomic mass unit is defined as 1/12 of the carbon ( $^{12}\text{C}$ ) atom

## Nuclear notation

Mass number  
 $A = Z + N$

24

12

**N**

Chemical symbol for  
the element

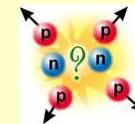
Atomic number =  
Number of protons

$N$  = number of neutrons

## Nuclear stability

- There are very large *repulsive electrostatic forces* between protons

These forces should cause the nucleus to fly apart



- $\Delta M = [Zm_p + (A-Z)m_n] - M(A,Z)$  (?)

It must be that a different type  
of force exists within the  
nucleus



1911 Rutherford, Geiger and Marsden

## Nuclear stability

$$\Delta M = [Zm_p + (A-Z)m_n] - M(A,Z)$$

Instead of two separate conservation laws, a single conservation law states that the sum of mass and energy is conserved. Mass does not magically appear and disappear at random. A decrease in mass will be accompanied by a corresponding increase in energy and vice versa.

$$\Delta E = \Delta Mc^2$$

Another, short-range force is present, called the *nuclear force* (Rutherford, 1911)

## Nuclear stability

- Another, short-range force is present, called the *nuclear force* (Rutherford, 1911)

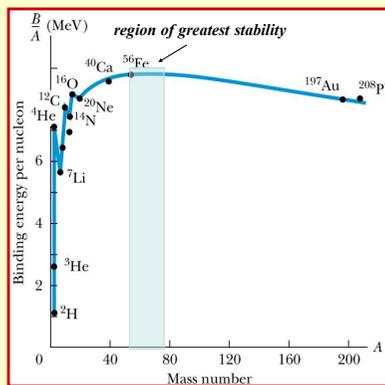
This is an *attractive force* that acts between all nuclear particles  
The nuclear attractive force is stronger than the Coulomb repulsive force at the short ranges within the nucleus

$$\Delta E = \Delta Mc^2$$

The nuclear force is  
- independent of charge  
- the range of action is extremely short (~fm)

## Binding energy per nucleon

- The curve increases rapidly
- Sharp peaks for the even-even nuclides  ${}^4\text{He}$ ,  ${}^{12}\text{C}$ , and  ${}^{16}\text{O}$
- Maximum is around  $A=56$



## Nuclear stability

*The balance of proton and neutron number is extremely important for the stability of the nucleus*

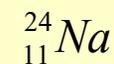
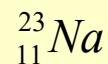
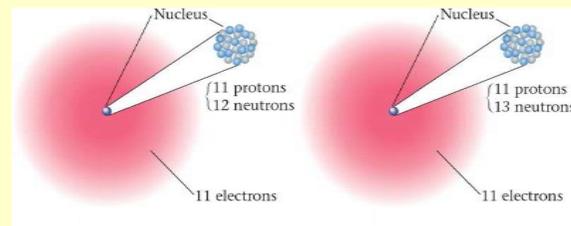
## Isotopes

Greek *isos topos* = equal place

Isotopes of an element have nuclei with

- the same number of protons
- different numbers of neutrons
- different mass number

## Example of isotopes

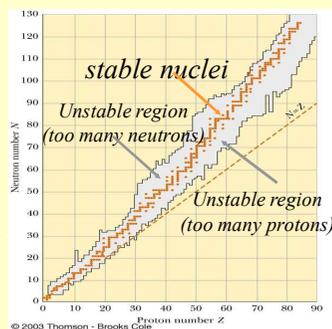


*What is/are the stable combination(s)? 1:1 ?*

## Nuclear stability chart

- Light nuclei are most stable if  $N=Z$
- Heavy nuclei are most stable when  $N > Z$

As the number of protons increase, the Coulomb force increases and so more nucleons are needed to keep the nucleus stable



- No nucleus is stable when  $Z > 83$



**Antoine Becquerel**  
1903 Nobel Prize in Physics  
for discovering radioactivity



Image of Becquerel's photographic plate which has been fogged by exposure to radiation from a uranium salt. The shadow of a metal Maltese Cross placed between the plate and the uranium salt is clearly visible. (1896)

## Radioactive decay

- *Radioactivity* is the spontaneous release of energy in the form of radioactive particles or waves
- Experiments suggested that radioactivity was the result of the decay, or disintegration, of unstable nuclei
- Three types of radiation can be emitted
  - Alpha ( $\alpha$ ) particles
  - Beta ( $\beta$ ) particles
  - Gamma ( $\gamma$ ) rays (Rutherford 1896, see details later)

- The nuclides, as with most things in nature, want to be at their *lowest energy state* which is a stable nucleus.
- Radioactive decay occurs in nuclides where the *nucleus is unstable*.

• *The nuclide reaches its stable state by undergoing radioactive decay.*

## Characteristics of radioactive decay

- it is *statistical process* □ individual disintegrations occur *randomly*
- it results in a decrease over time of the original amount of the radioactive nuclei



decrease of the original amount of the radioactive nuclei over time

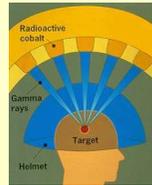
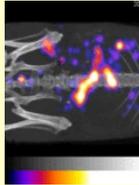
$$\text{Activity : } \Lambda = \left| \frac{dN}{dt} \right| \begin{array}{l} N: \text{ number of nuclei} \\ \text{to be decayed} \\ t: \text{ time} \end{array}$$

*number of nuclei decayed in a unit time*

*measure: bequerel (Bq)*  
*1Bq = 1 decay/sec*

## Typical activities in the practice

<b><i>kBq,</i></b> natural background	<b><i>MBq,</i></b> <i>in vivo</i> diagnostics	<b><i>GBq,</i></b> laboratory practice	<b><i>TBq</i></b> <i>therapy</i>
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## Radioactive decay law

$$\text{Differential form } \left| \frac{dN}{dt} \right| = -\lambda N$$

$\lambda$  : decay constant (measure: 1/s)  
*constant for a certain isotope*

*Activity depends both on the*

- size of the population of radioactive atoms
- type of the isotope

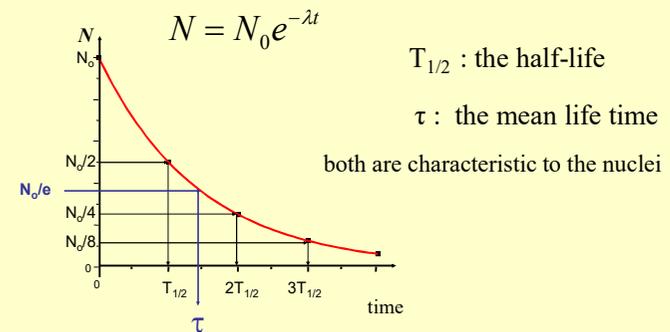
$$\left| \frac{dN}{dt} \right| = -\lambda N$$

Solution of this equation yields

$$N = N_0 e^{-\lambda t} \quad \text{Integral form}$$

$N_0$  : number of radioactive nuclei at  $t = 0$ ,  
 $N$  : the number radioactive of nuclei remaining  
after a period  $t$

## Graphical representation

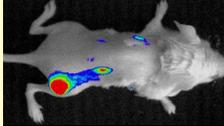
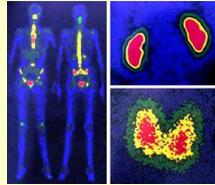


## Half-lives in Medical Uses



**Iodine - 131 ( $^{131}\text{I}$ ) -  $T_{1/2} = 8$  days**  
**Thyroid treatment**

**Technetium-99m ( $^{99\text{m}}\text{Tc}$ ) -  $T_{1/2} = 6$  hours**  
**Isotope diagnostics**



**Gold-198 ( $^{198}\text{Au}$ ) -  $T_{1/2} = 2.7$  days**  
**Tumor therapy**

## Further considerations

$$N = N_0 e^{-\lambda t}$$

If  $t = T_{1/2} \longrightarrow N_0 / 2 = N_0 e^{-\lambda T_{1/2}}$

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{T_{1/2}}$$

If  $t = \tau \longrightarrow N_0 / e = N_0 e^{-\lambda \tau}$

$$\lambda = \frac{1}{\tau}$$

Definition of decay constant

## Alteration of activity in time

$$N = N_0 e^{-\lambda t}$$



$$\Lambda = \lambda N$$

$$\Lambda = \Lambda_0 e^{-\lambda t}$$

**Specific activity** : activity in a unit mass ( $\Lambda/m$ ),  
 unit: Bq / kg

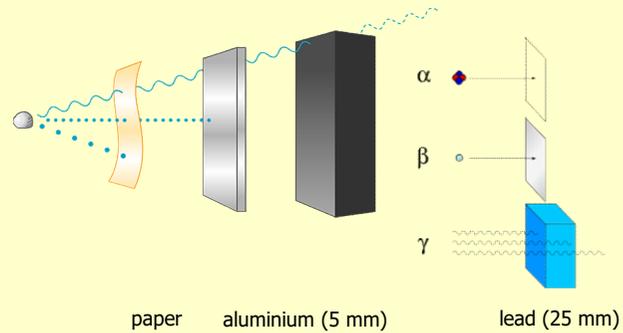
## Characteristics of radioactive isotopes

**Activity** : depends both on the nucleus and the size of its population

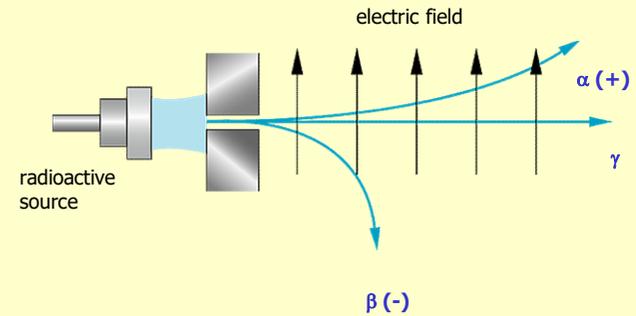
**Half-life** : physical parameter of each radioactive isotope

**Type of radiation**: physical characteristic of the nucleus

## Types and nature of nuclear radiation

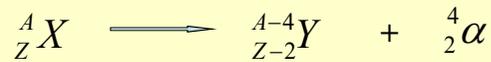
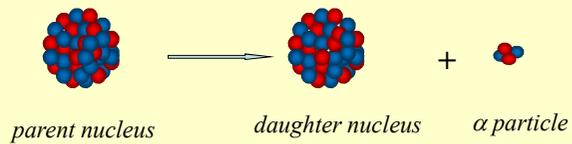


## Deflection of radiation in electric field



Radiation from a radium source is split by an electric field.

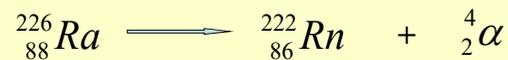
## $\alpha$ decay



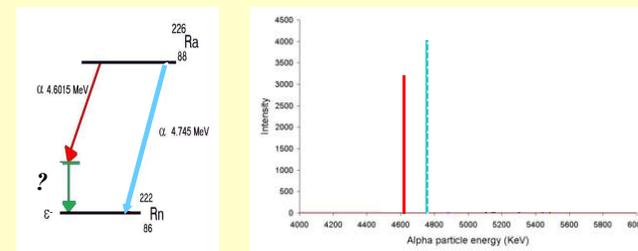
**$\alpha$  particle** is a nucleus of helium containing two neutrons and two protons

Heavy nuclei with mass numbers higher than 150 can disintegrate by emission of an  $\alpha$  particle

example



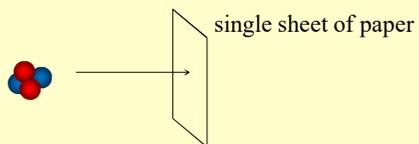
## Energy spectrum of $\alpha$ radiation



line spectrum

Energy is characteristic for the nucleus

## Penetration depth of $\alpha$ particles

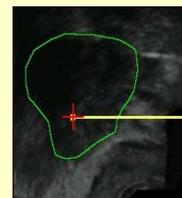


absorber	density	alpha range
air (STP)	1.2 mg/cm <sup>3</sup>	3.7 cm
paper (20lb)	0.89 g/cm <sup>3</sup>	53 $\mu$ m
water (soft tissue)	1.0 g/cm <sup>3</sup>	45 $\mu$ m

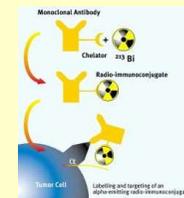
## Medical application of $\alpha$ radiation

Diagnostics: none

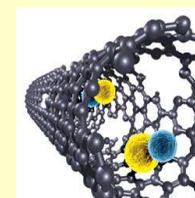
Targeted alpha **therapy** of cancer



Seed implantation  
by needle



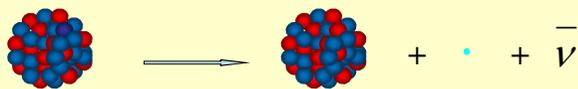
Monoclonal  
antibody



Carbon nano-tube

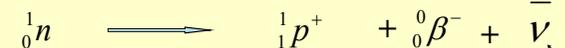
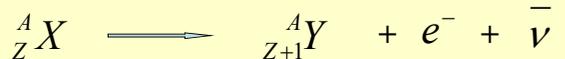
## $\beta$ decay

### 1. Neutron excess: $\beta^-$ decay

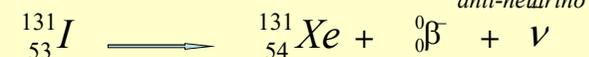


parent nucleus

daughter nucleus

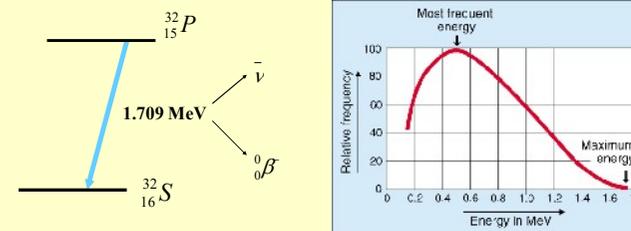


example



anti-neutrino

## Energy spectrum of $\beta$ radiation



Energy distribution of the  $\beta^-$  particles emitted during the  $\beta^-$  decay of  ${}^{32}P$ .

**continuous spectrum**  
with maximum kinetic energy for the  $\beta$  particle

## Penetration depth of $\beta^-$ particles

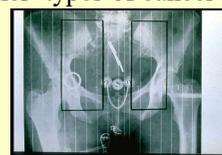


absorber	density	maximum beta range (2.3 MeV) (1.1 MeV)	
air	1.2 mg/cm <sup>3</sup>	8.8 m	3.8 m
water (soft tissue)	1.0 g/cm <sup>3</sup>	11 mm	4.6 mm
aluminum	2.7 g/cm <sup>3</sup>	4.2 mm	2.0 mm
lead	11.3 g/cm <sup>3</sup>	1.0 mm	0.4 mm

## Medical application of $\beta^-$ radiation

Diagnostics: none

Targeted therapy: hyperthyroidism, thyroid and several other types of cancer



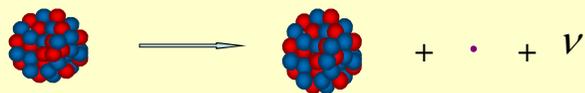
Brachytherapy:  
implants into the  
tumours



Endovascular  
irradiation

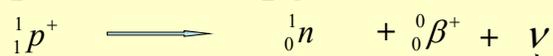
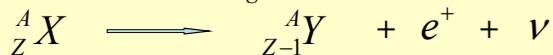
## $\beta$ decay

### 2. Proton excess: $\beta^+$ decay

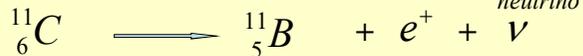


parent nucleus

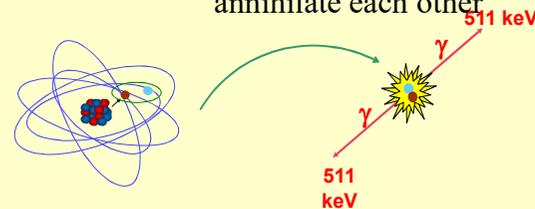
daughter nucleus



example



## Annihilation - particle-antiparticle pairs can annihilate each other



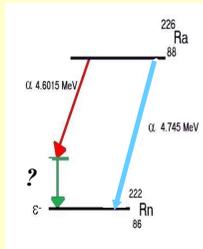
1. Conservation of momentum : two photons with opposite direction are produced

2. Energy balance:

$$m_e c^2 + m_p c^2 = 2 hf$$

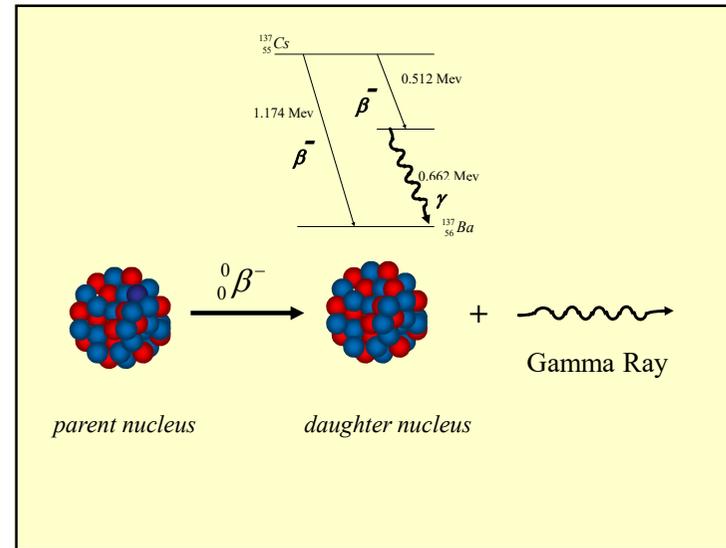
*mass - energy equivalence*

## $\gamma$ decay

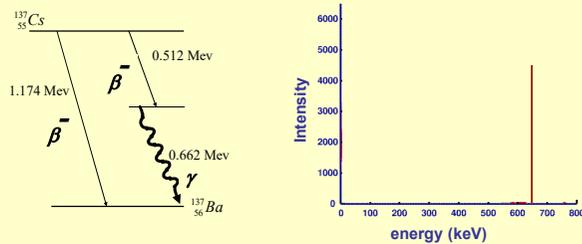


Sometimes the newly formed isotopes (after  $\alpha$  or  $\beta$  decay) appear in the excited state.

Excited nuclides have tendency to release the excess of energy by **electromagnetic radiation** - emission of gamma rays.



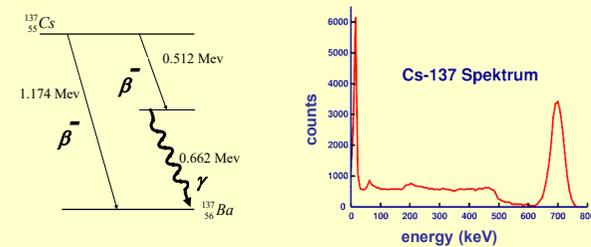
## Energy spectrum of $\gamma$ radiation



*line spectrum*

*Energy is characteristic for the nucleus*

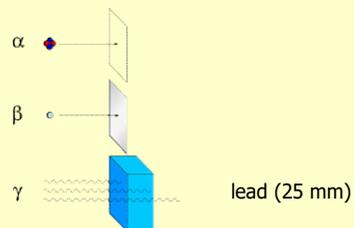
## Energy spectrum of $\gamma$ radiation



*line spectrum*

*Energy is characteristic for the nucleus*

## Penetration depth of $\gamma$ radiation



Penetration distance is higher than that of  $\alpha$  or  $\beta$  particles, but it is highly energy dependent.

Gamma rays can travel from **1 to 100s of meters in the air** and can easily go right **through people (~dm)**.

## Timing of $\gamma$ emission

Lifetime of the excited nucleus:

1. *Prompt  $\gamma$  decay*:  $\sim 10^{-13} - 10^{-18} \text{ s}$

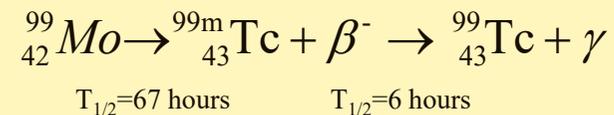
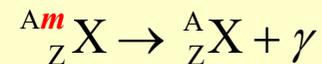
2. *Isomeric transition*:  $\geq 10^{-10} \text{ s}$

## Checklist

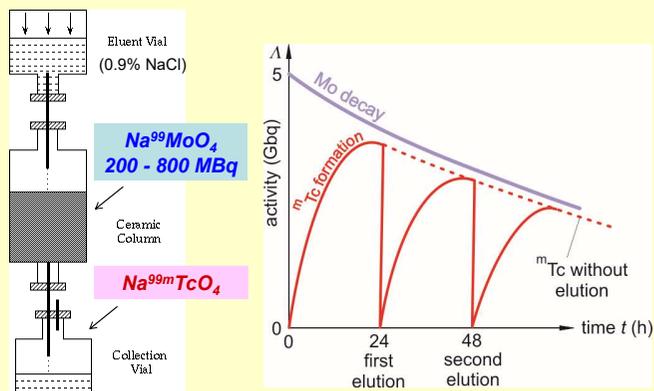
- Composition and stability of the nucleus
- Origin of nuclear force
- Radioactive decay law – differential and integral form
- Decay constant
- Types and characteristics of nuclear radiation

## Isomeric transition

Some excited states may have a half-lives ranging from hours up to more than 600 years

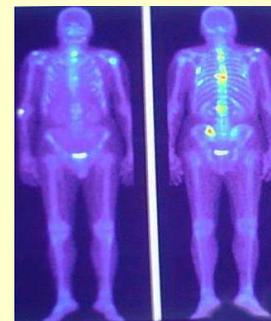


## Technetium-99m generator



## Medical application

Diagnostics: ideal for isotope diagnostics



Bone scan using  $^{99m}\text{Tc}$  labeled phosphate compound

<b>Radionuclide</b>	<b>Compound</b>	<b>Organ</b>	<b>Function</b>
$^{99m}\text{Tc}$	sodium pertechnetate	brain	blood flow
$^{99m}\text{Tc}$	coagulated albumin	lung	blood flow
$^{99m}\text{Tc}$	colloidal suspension	liver	liver function
$^{99m}\text{Tc}$	complex phosphate	bone	bone metabolisms
$^{99m}\text{Tc}$	red blood cells	heart	blood circulation
$^{123}\text{I}$	iodide	thyroid	metabolisms
$^{123}\text{I}$	hippuran	kidneys	renal function
$^{133}\text{X}$	gas	lungs	ventilation

Damjanovich, Fidy, Szöllösi: Medical Biophysics

I. 1.5	1.5.1
	1.5.2
	1.5.4
II.3.2	3.2.1
	3.2.2
	3.2.3
	3.2.4