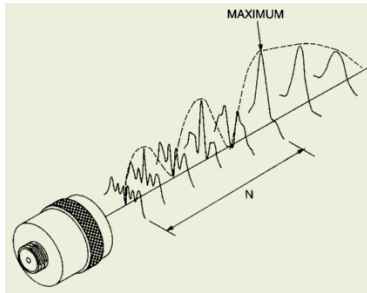
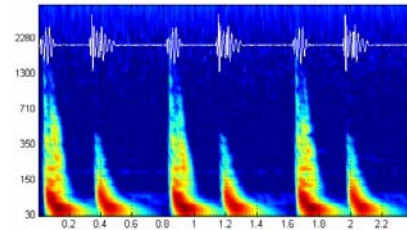


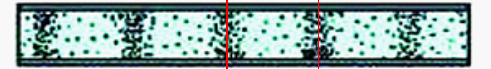
Physics of ultrasonography



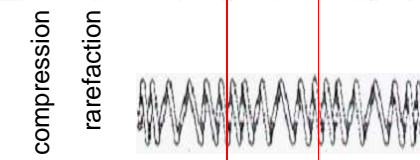
KAD 2020.02.19

Sound: mechanical wave (model)

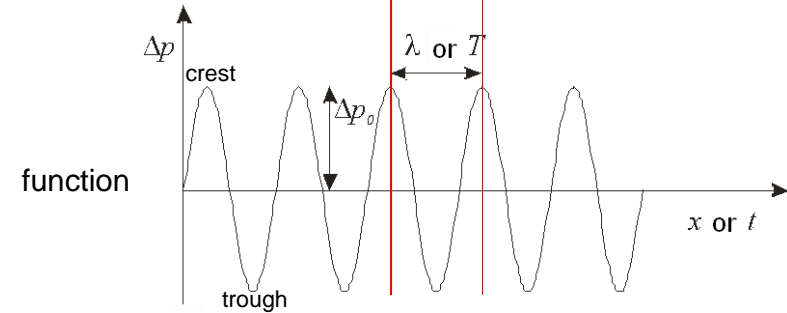
whistle



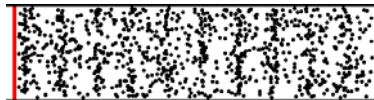
spring



spatial and
temporal
periodicity



2



longitudinal wave
(in the interior of liquids and
gases only this type)



transverse wave

hydrostatic pressure pressure change,
pressure sound pressure

$$p_{\text{total}} = p_{\text{hydrostat}} + \Delta p$$

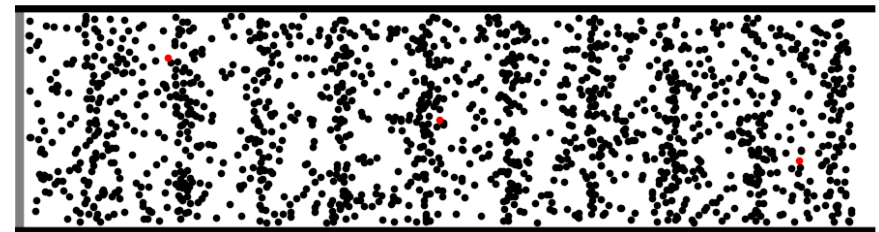
pressure DC + AC amplitude phase

$$\Delta p(t, x) = \Delta p_{\text{max}} \sin \left[2\pi \left(\frac{t}{T} - \frac{x}{\lambda} \right) \right]$$



$$c \cdot T = \lambda, \quad c = f \cdot \lambda$$

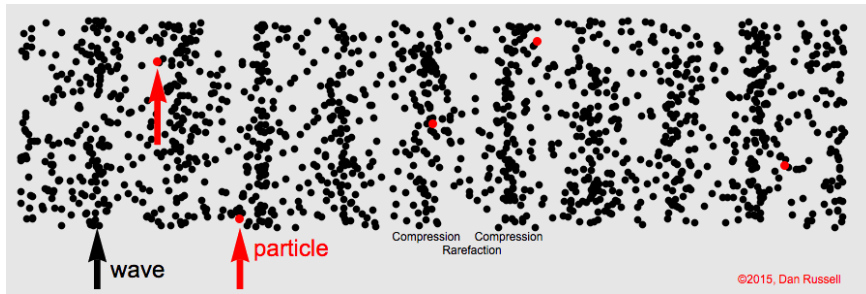
longitudinal wave



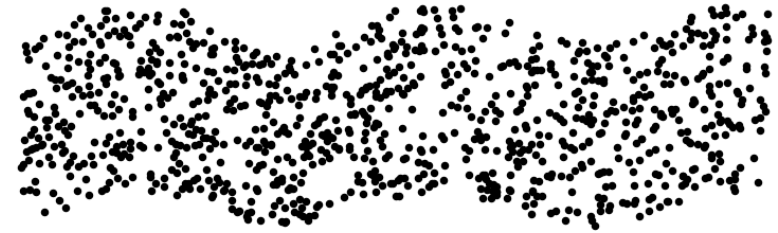
©2011. Dan Russell

moving surface (source of wave)

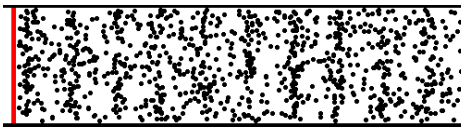
longitudinal wave



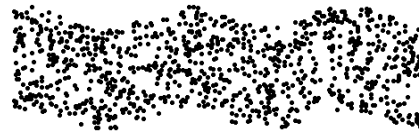
transverse wave



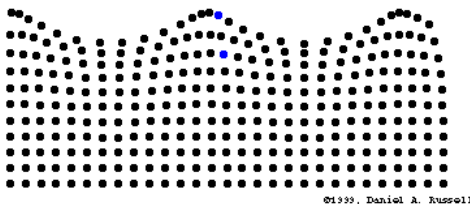
longitudinal wave



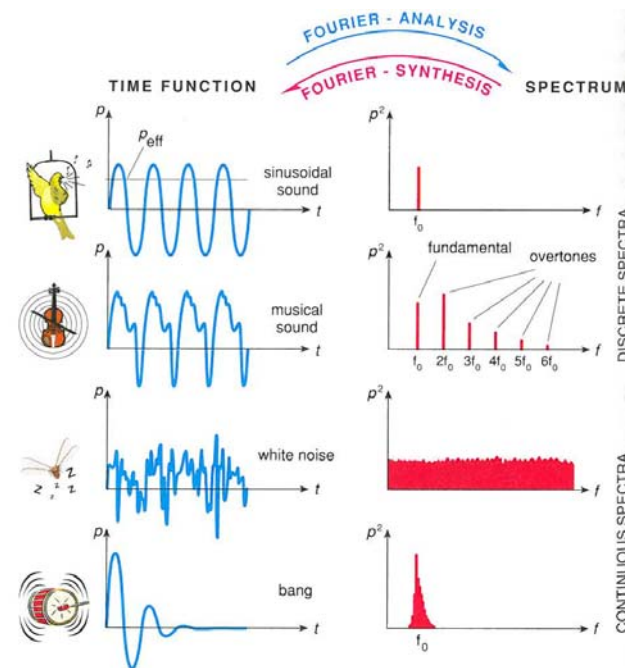
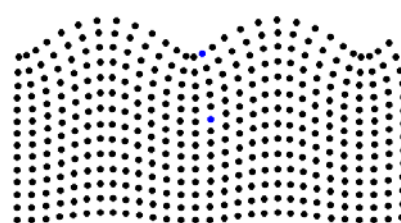
transverse wave



surface wave



Rayleigh wave

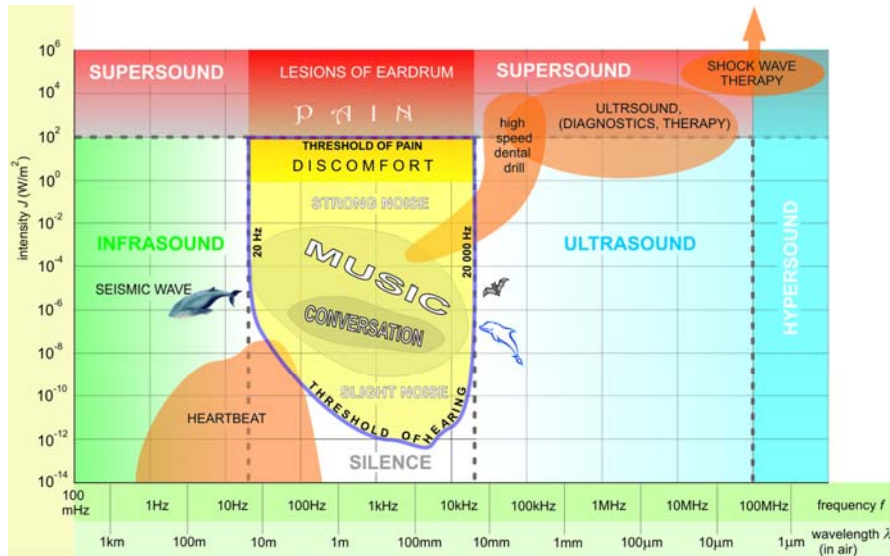


Textbook, Fig. IV.23.

pitch:
frequency of the
fundamental

timbre (tone colour):
relative strengths of
overtones/harmonics
(spectrum)

Frequency and intensity regions of sounds



Lab. manual, Audiometry.

9

The role of elastic medium

$$\kappa = -\frac{\frac{\Delta V}{V}}{\Delta p}$$

compressibility
relative volume decrease
over pressure

$$c = \frac{1}{\sqrt{\rho \kappa}}$$

speed of sound

$$Z = \frac{p}{v} = \frac{p_{\max}}{v_{\max}}$$

acoustic impedance
(definition)

$$Z_{\text{el}} = \frac{U}{I}$$

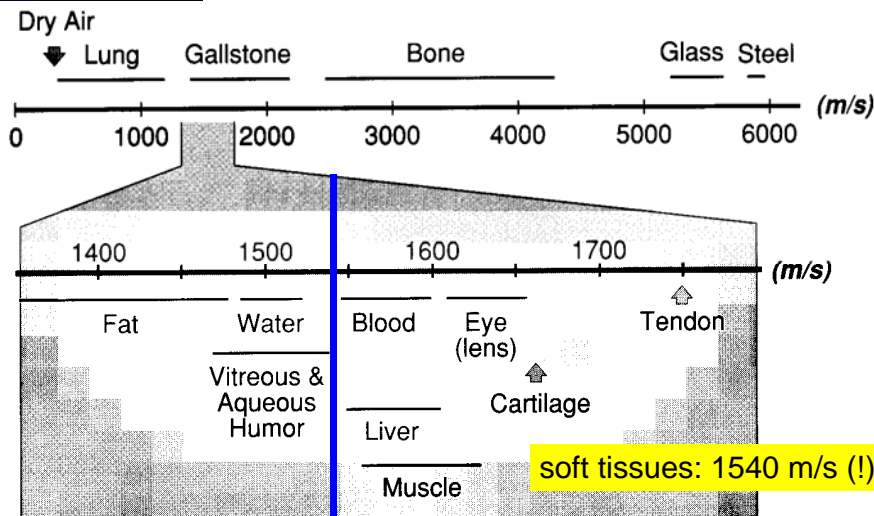
$$Z = c\rho = \sqrt{\frac{\rho}{\kappa}}$$

acoustic impedance
(useful form)



10

Speed of sound/US in different media



soft tissues: 1540 m/s (!)

1

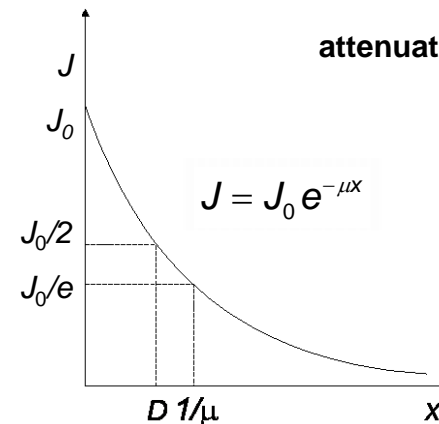
Intensity of US

$$J = \frac{1}{Z} \Delta p_{\text{eff}}^2$$

$$P_{\text{el}} = \frac{1}{Z_{\text{el}}} U_{\text{eff}}^2$$

intensity =
energy-current density electric analogy

Loss of energy during propagation (absorption)



$$\text{attenuation: } \alpha = 10 \cdot \lg \frac{J_0}{J} \text{ dB}$$

$$\alpha = 10 \cdot \mu \cdot x \cdot \lg e \text{ dB}$$

μ is proportional to
frequency in the
diagnostic range

**specific
attenuation:**

$$\frac{\alpha}{f \cdot x}$$

12

μ is proportional to frequency in the diagnostic range

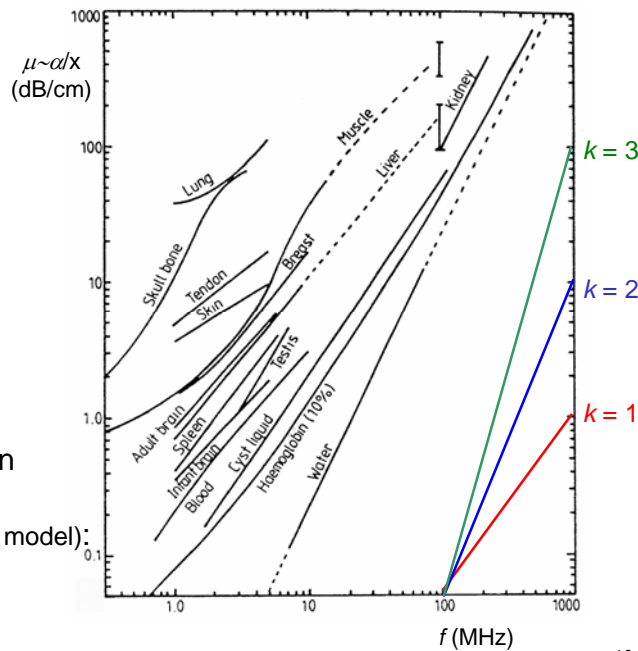
$$\mu \sim f^k, \quad k \sim 1(?)$$

$$\log \mu \sim k \log f$$

if the graph is a linear, the power function approximation is valid

specific attenuation for soft tissues (homogeneous tissue model):

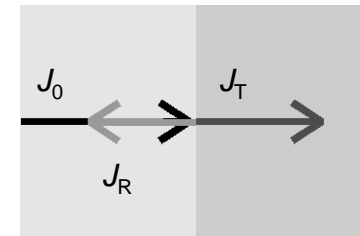
$$\frac{\alpha}{f x} \sim 1 \frac{\text{dB}}{\text{cm MHz}}$$



13

Phenomena at the boundary of different media

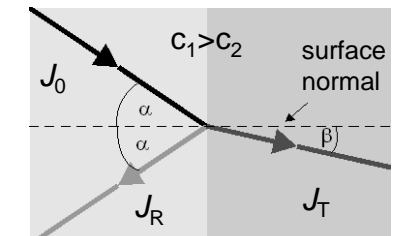
normal/perpendicular incidence



$$J_0 = J_R + J_T$$

reflection and transmission (penetration)

skew incidence



$$\frac{\sin \alpha}{\sin \beta} = \frac{c_1}{c_2}$$

Snellius-Descartes

Textbook, Fig. II.47.

14

Reflection (normal incidence)

reflectivity:

$$R = \frac{J_{\text{reflected}}}{J_{\text{incident}}} = \left(\frac{Z_1 - Z_2}{Z_1 + Z_2} \right)^2$$

"full" reflection:

$$Z_1 \ll Z_2, \quad R \approx 1$$

optimal coupling:

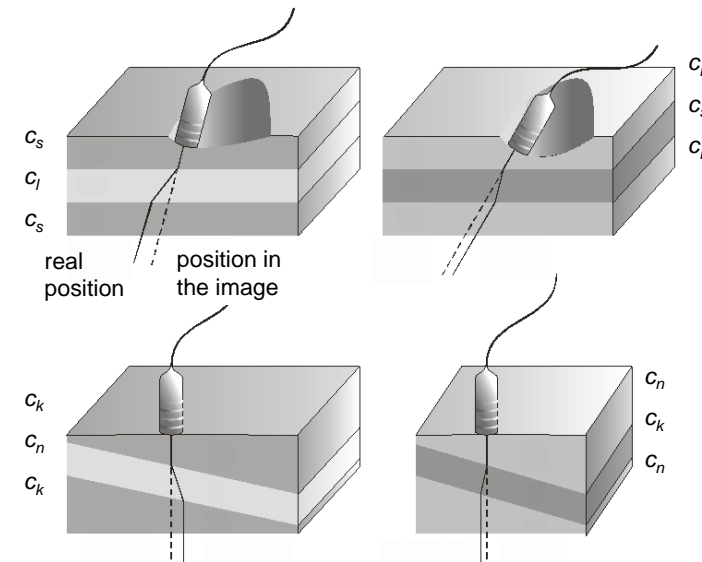
$$Z_{\text{connecting}} \approx \sqrt{Z_{\text{source}} Z_{\text{skin}}}$$



boundary surface	R
muscle/blood	0.001
fat/liver	0.006
fat/muscle	0.01
bone/muscle	0.41
bone/fat	0.48
soft tissue/air	0.99

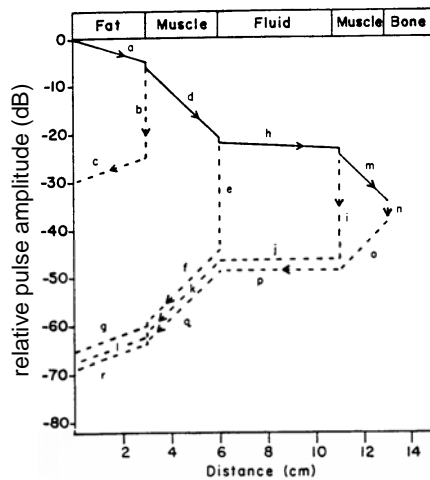
15

Phenomenon of skew incidence or normal incidence and skew boundaries



Textbook, Fig. on pg. 153

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Absorption and reflection

the later comes back the reflection, the deeper lays the reflecting surface and the weaker is the intensity

run time dependent amplification

TGC: time gain compensation

DGC: depth gain control

boundary surface	R	$10\lg R$ (dB)	T	$10\lg T$ (dB)
fat/muscle	0.01	-20.0	0.990	-0.044
muscle/blood	0.001	-30.0	0.999	-0.004
muscle/bone	0.41	-3.9	0.590	-2.291

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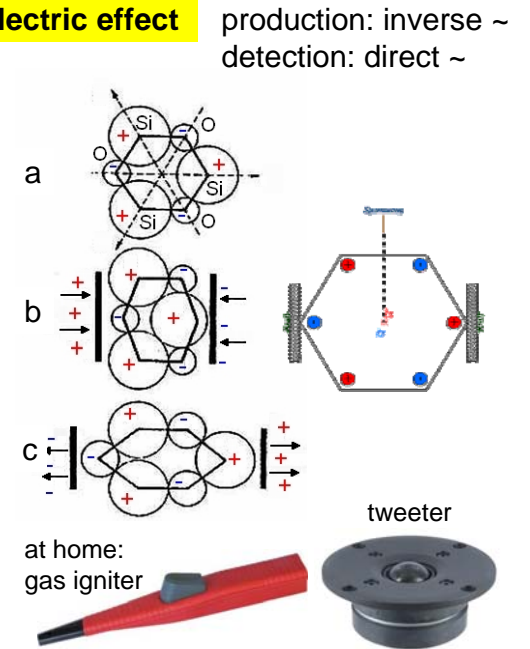
Generation of US. Piezoelectric effect

source of electric signal
(sine wave oscillator)+
transducer (piezo-crystal)

(a) Center of charge of positive and negative charges coincides.

(b) and (c) As a result of pressure, the charge centers are separated, i.e. a potential difference arises (direct ~).

The crystal is deformed when voltage is applied (inverse ~).

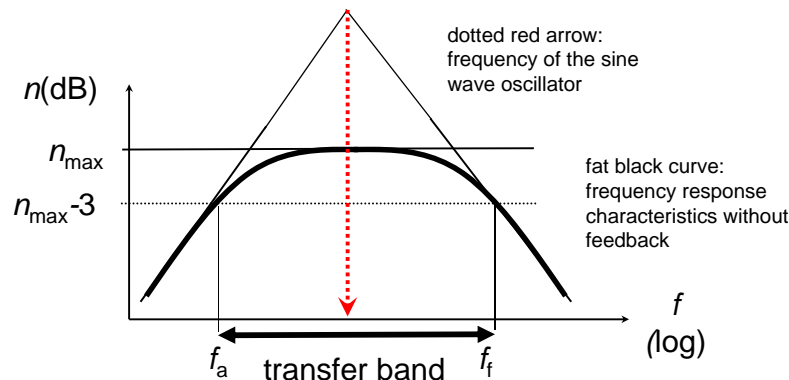


Source of electric signal : sine wave oscillator

amplifier with positive feedback

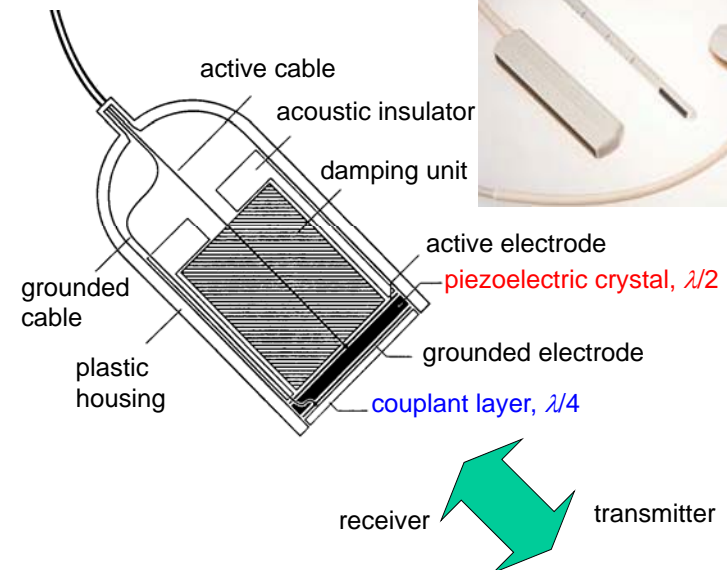
$$A_{U, \text{feedback}} = \frac{A_U}{1 - \beta A_U}$$

$\beta A_U = 1$, amplification = „infinity“ → sine wave oscillator
no input signal, output signal: sine voltage



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Ultrasound transducer

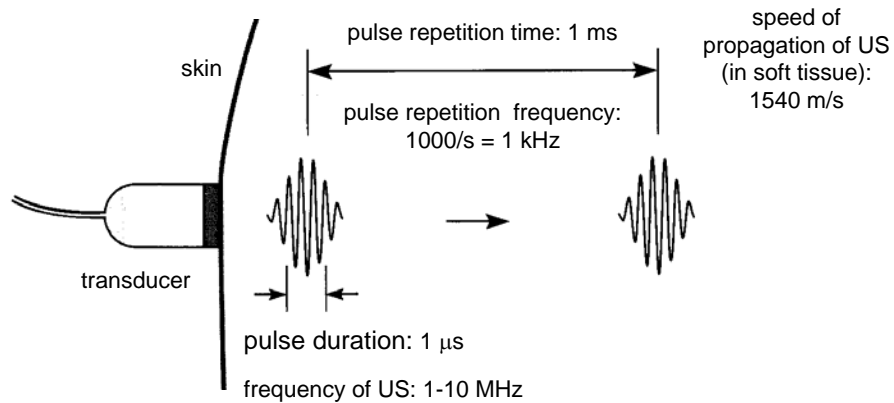


20

Characteristic of US pulses

transducer: transmitter and receiver is the same unit

time sharing mode: pulses instead of continuous wave US



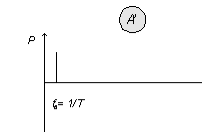
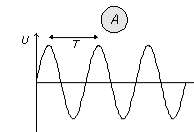
Textbook, Fig. VIII.32.

21

Time function

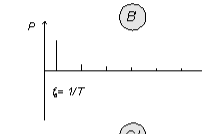
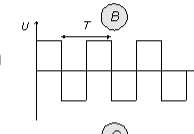
Spectrum

sine function



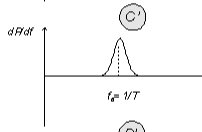
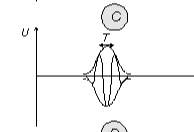
line spectrum (1 line)

square function



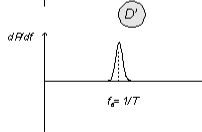
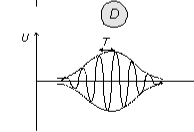
line spectrum

sine wave pocket (some „periods”)



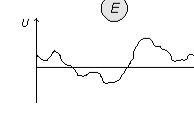
band spectrum

sine wave pocket (several „periods”)



band spectrum

aperiodic function

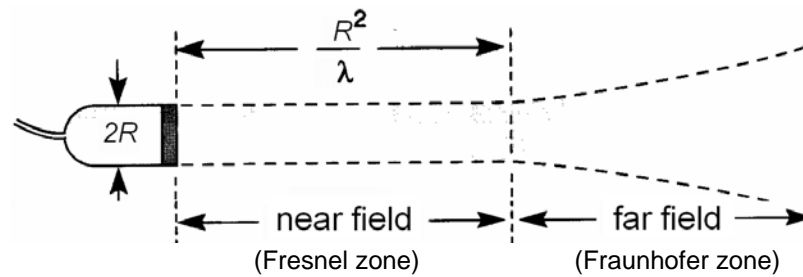


continuous spectrum

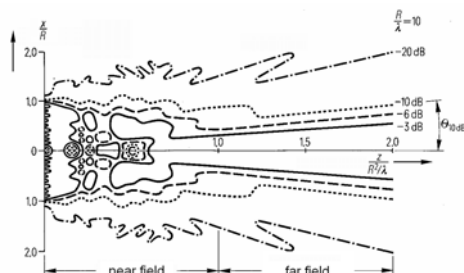
US pulse

22

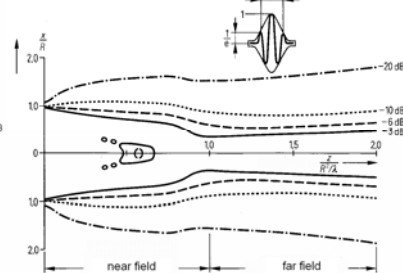
US beam shape (simplified version)



Beam shape, continuous wave US

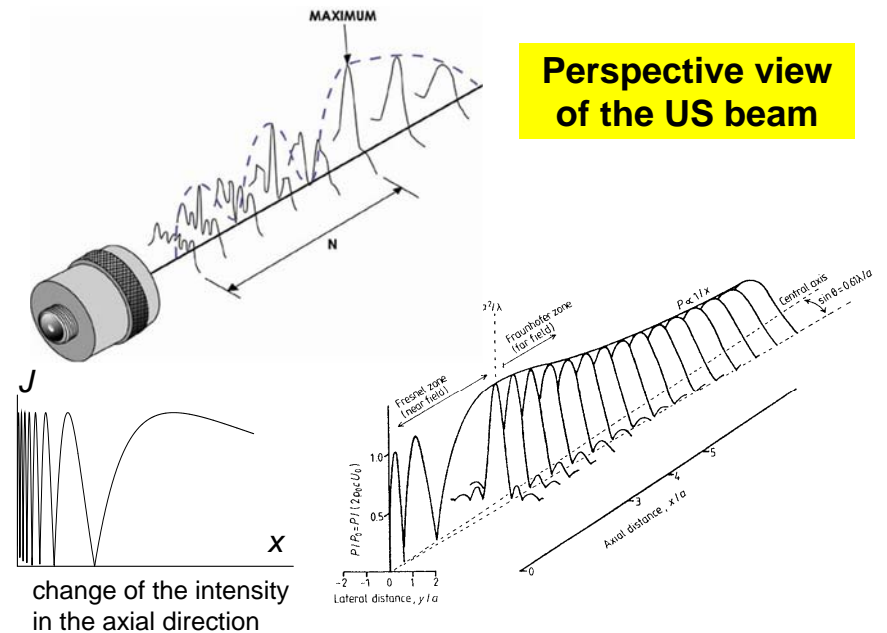


Beam shape, pulsed wave US



13

Perspective view of the US beam



cf. Textbook. Fig. on p.505

24

Resolving limit, resolution

Resolving limit is the distance between two object details which can be just resolved as distinct objects (the smaller the better).

Resolution (resolving power): the reciprocal of the resolving limit (the greater the better)

Axial resolving limit depends on the pulse length. Pulse length is inversely proportional to the frequency.

Lateral resolving limit is the minimum separation of two interfaces aligned along a direction perpendicular to the ultrasound beam. It depends on the beam width

Typical values	frequency (MHz):	2	15
	wavelength (in muscle) (mm):	0.78	0.1
	penetration depth (cm):	12	1.6
	lateral resolving limit (mm):	3.0	0.4
	axial resolving limit (mm):	0.8	0.15

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Axial resolving limit

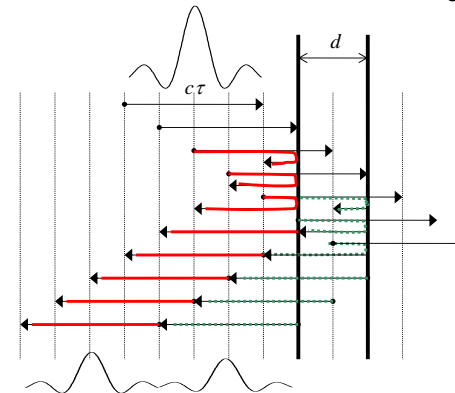
τ : pulse duration

$c_1\tau \cong c_2\tau = c\tau$ pulse length

$\delta_{ax} = d = \frac{c\tau}{2}$ resolving limit

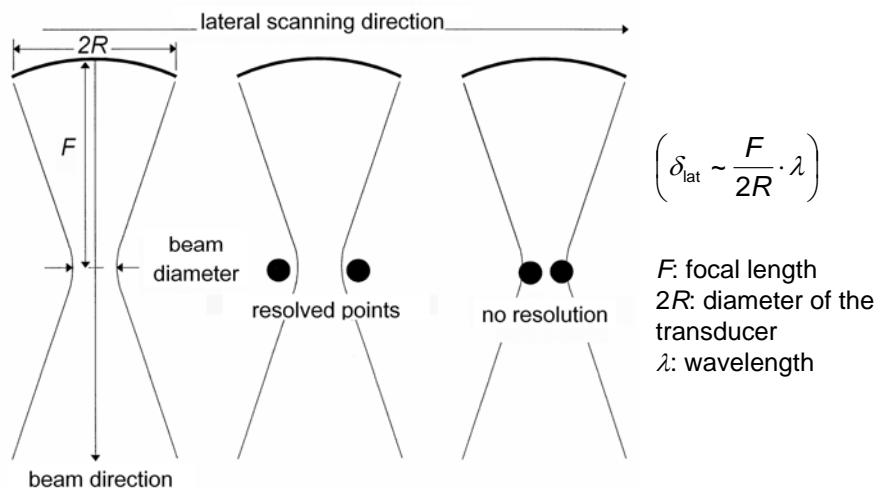
The axial resolving limit is the half of the pulse length. The echos from the adjacent surfaces in this case just hit another.

$$\tau \sim T = \frac{1}{f}$$



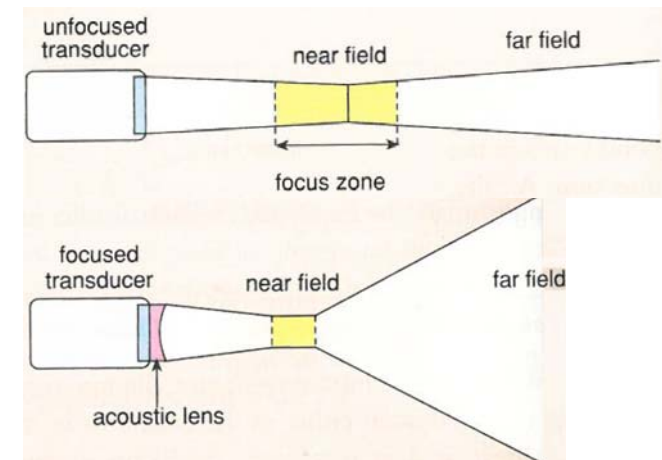
26

Lateral resolving limit



27

Focusing of the beam

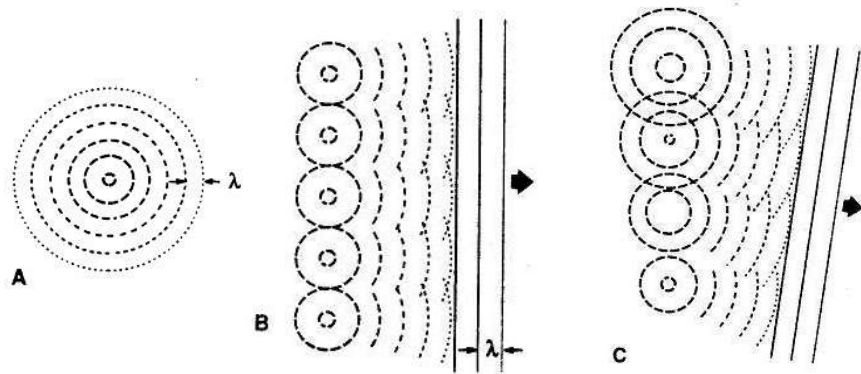


Focusing increases the divergence of the beam in the far field regime and reduces the depth sharpness.

cf. Textbook Fig. on p.506

28

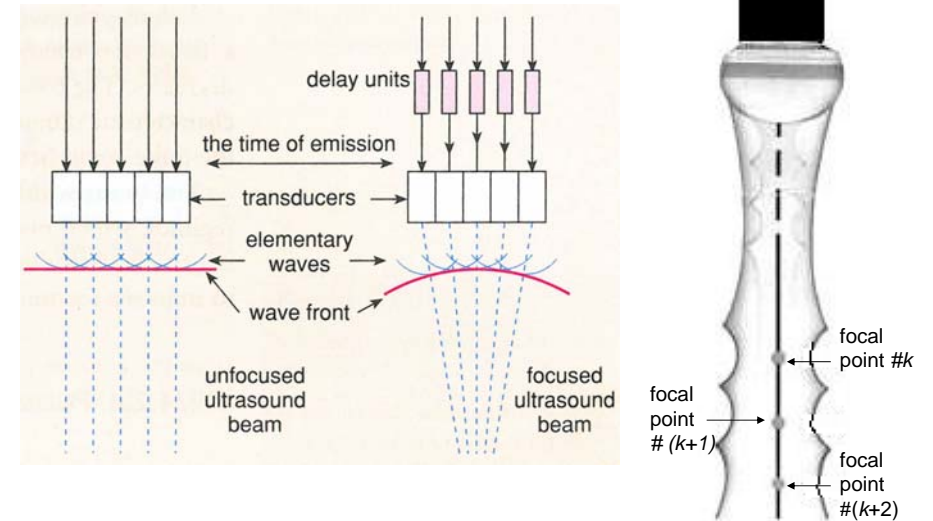
Huygens' principle



Any wave propagates so, that each point on a primary wavefront serves as the source of spherical secondary wavelets that advance with a speed and frequency equal to those of the primary wave. The primary wavefront at some later time is the envelope of these wavelets.

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Electronic focusing

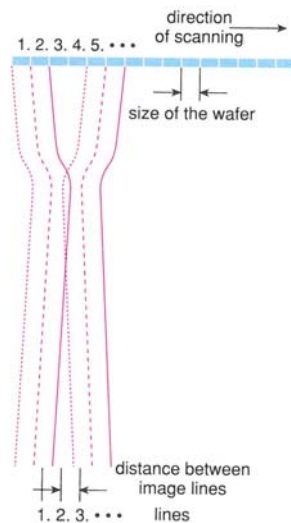


cf. Textbook Fig. on p.507

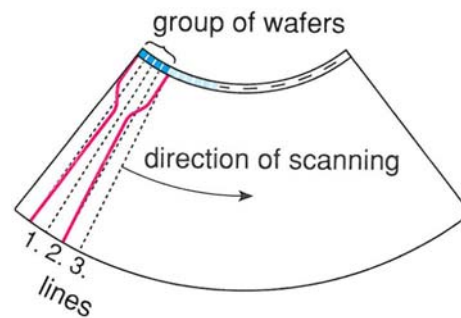
30

Scanning

multi unit linear array



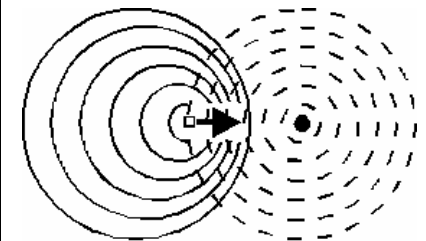
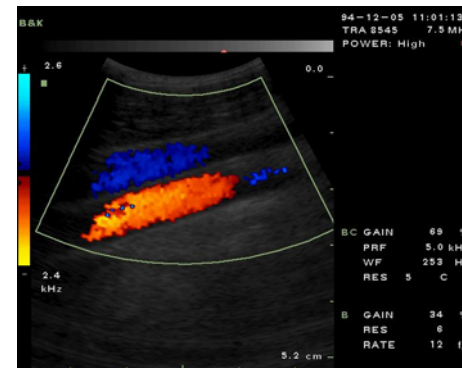
multi unit curved array



cf. Textbook Fig. VII. 36-37

31

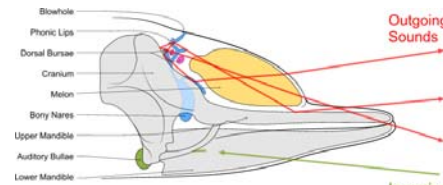
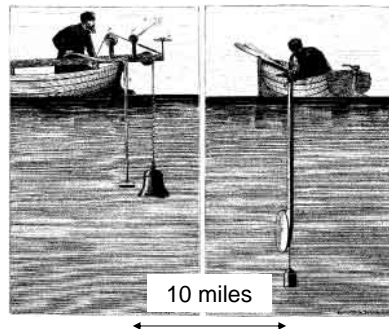
US imaging. Modes of sonography. Doppler-echo.



Echo principle

1794 Spallanzani:
bat's navigation

1822 Colladen
measured the speed of
sound in water

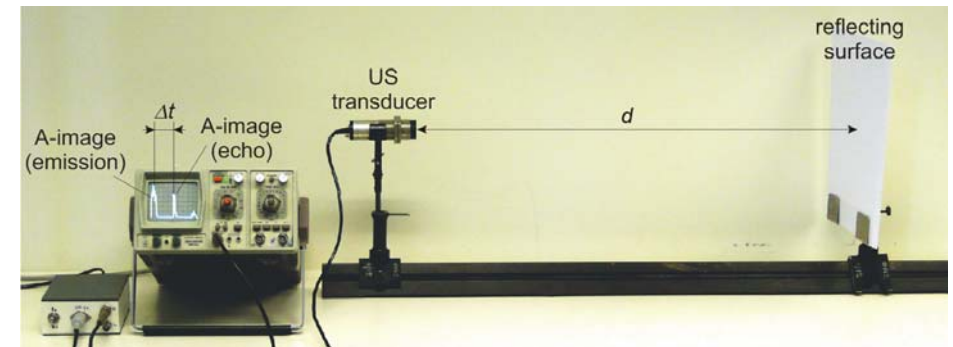


bottlenose dolphin

33

Echo principle

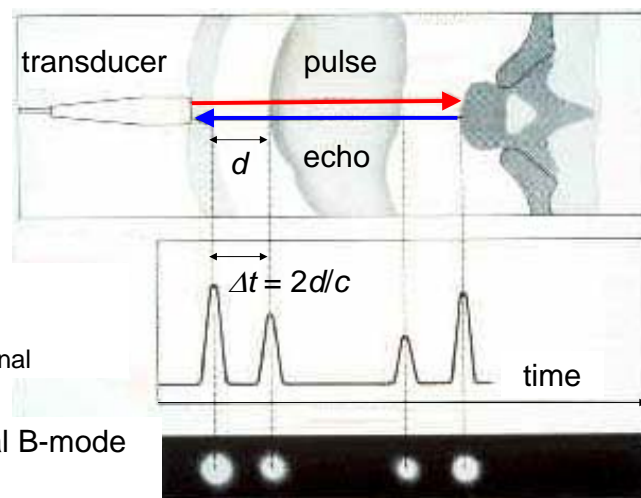
using a special US-head, short pulses are emitted in the air towards a
reflecting surface, and the same US-head detects the echo signal



$$c\Delta t = d + d = 2d$$

34

Receiving the echos

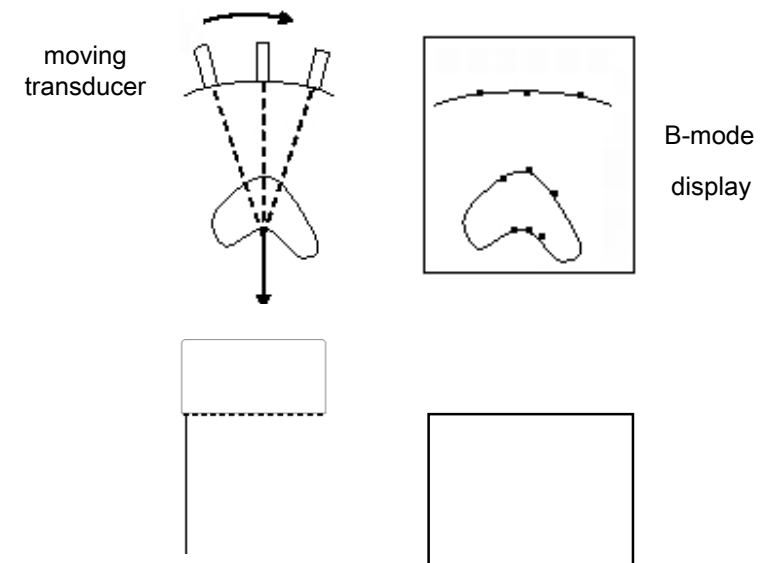


A-mode
(**A**mplitude)
only 1-dimensional

1-dimensional B-mode
(**B**rightness)

35

2-dimensional B-mode

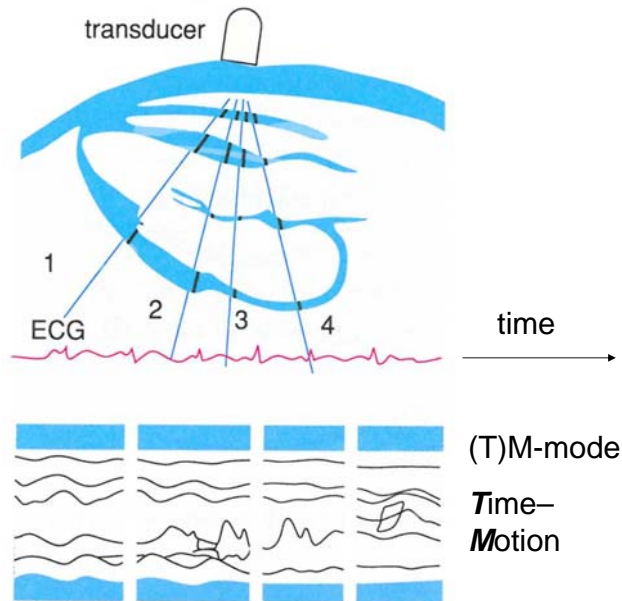


36

TM-mode

ECG signal
for reference

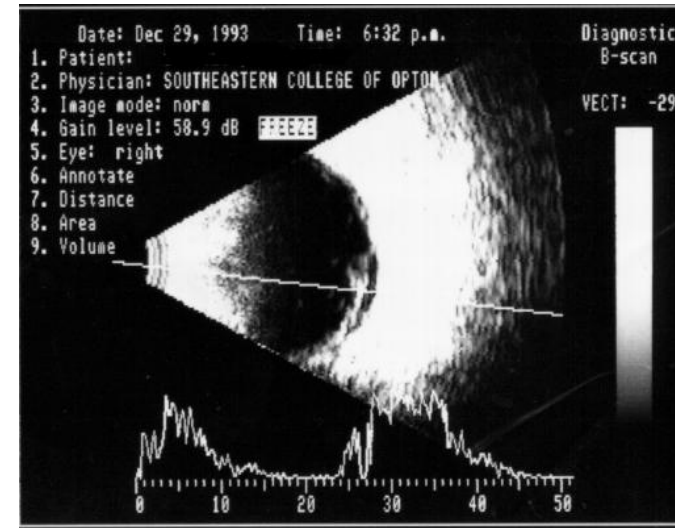
(vertical)
time-dependent
1-dimensional
B-mode



Textbook Fig. VIII.34

37

2-dimensional B-mode and A-mode (used in ophthalmology)



real speed of
propagation for
the accurate
determination of
distances:

cornea: 1641 m/s

aqueous humour:
1532 m/s

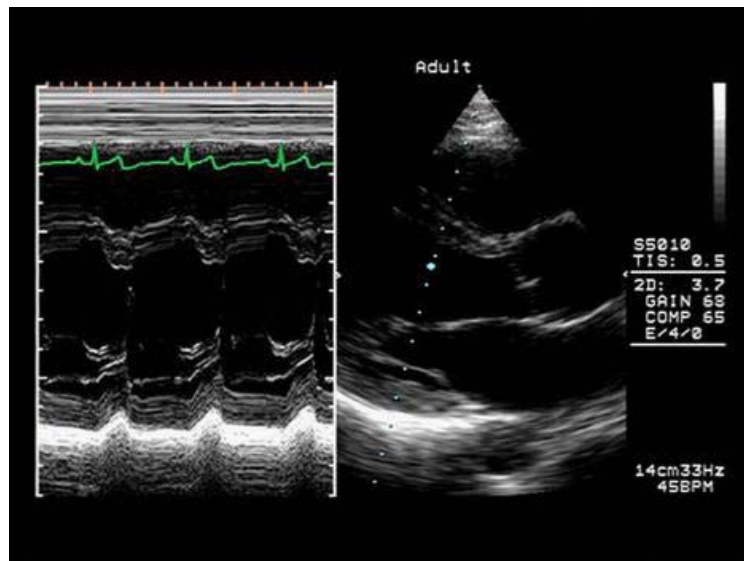
crystalline lens:
1641 m/s

vitreous body:
1532 m/s

38

TM-mode

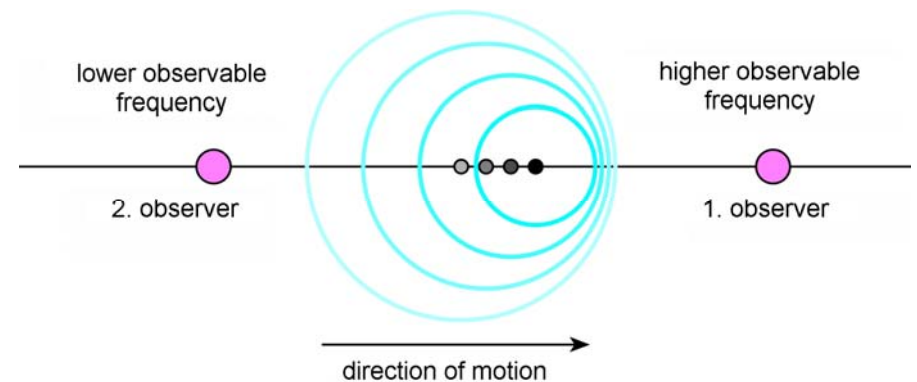
B-mode



39

Doppler phenomenon

„The pitch of a train whistle seems to get higher as it approaches, then seems to lower as the train whistle moves away.” (C. Doppler, 1842)



Textbook Fig. VIII.39

40

f' : **observed frequency**, f : original frequency

- (a) standing source and moving observer (v_o)
 +: observer approaches the source
 -: observer moves away from the source

$$f' = f \left(1 \pm \frac{v_o}{c} \right)$$

- (b) moving source and standing observer
 (if $v_s \ll c$, then „same” as (a))

$$f' = \frac{f}{1 \mp \frac{v_s}{c}}$$

- (c) moving source and moving observer

$$f' = f \frac{1 \pm \frac{v_o}{c}}{1 \mp \frac{v_s}{c}}$$

- (d) moving reflecting object (surface),
 (if $v_R \ll c$)

$$f' = f \left(1 \pm \frac{2v_R}{c} \right)$$

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Doppler frequency = frequency change = frequency shift

if $v_i, v_R \ll c$ (i= S or O)

rearranging equation (a)

moving source or observer:

$$\Delta f = f_D = \pm \frac{v_i}{c} f$$

rearranging equation (d)

**moving reflecting object
or surface:**

$$\Delta f = f_D = \pm 2 \frac{v_R}{c} f$$

if v and c are not parallel, then $v \cos \theta$ should be used
 instead of v (remark: if $\theta = 90^\circ$, $f_D = 0$)

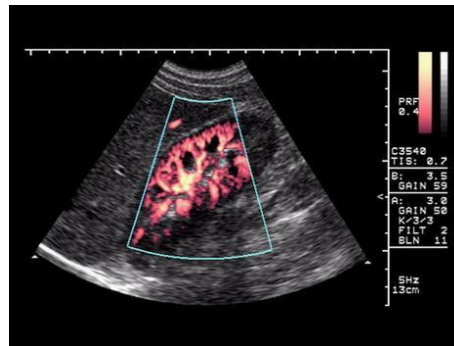
42

Colour coding

towards the transducer: warm colours
 away from the transducer: cold colours



BART: Blue Away Red Towards



power Doppler

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1-dimensional CW Doppler apparatus for measuring average flow velocity. Red blood cells as sound scatterers

CW: continuous wave

source and detector are separated

$$|f_D| = 2 \frac{v_R \cos \theta}{c} f$$

e.g. $f = 8000$ kHz

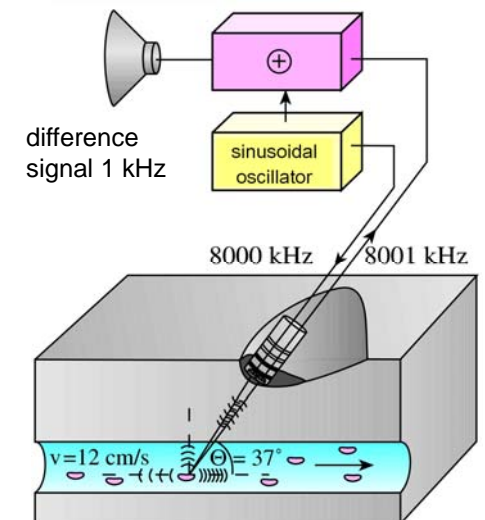
$v = 12$ cm/s

$c = 1600$ m/s

$\theta = 37^\circ$

$\Rightarrow f_D = 1$ kHz

(beating phenomenon)

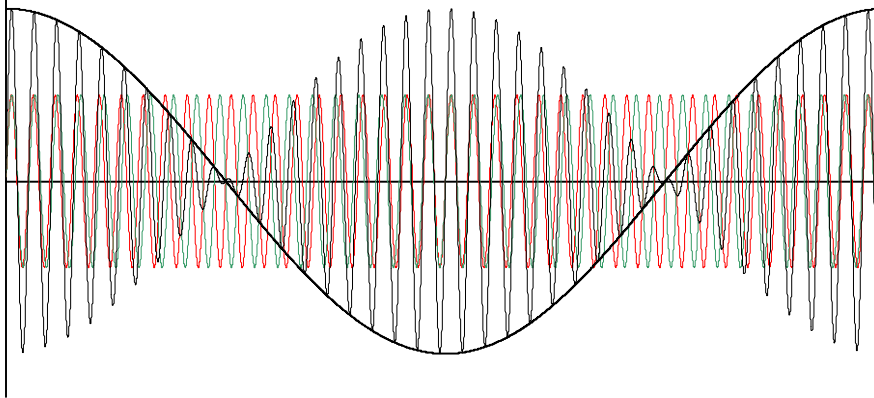


Textbook Fig. VIII. 41

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Beating phenomenon

$f_{\text{red}} \geq f_{\text{green}}$ the beating frequency equals to the difference of the two interfering frequency

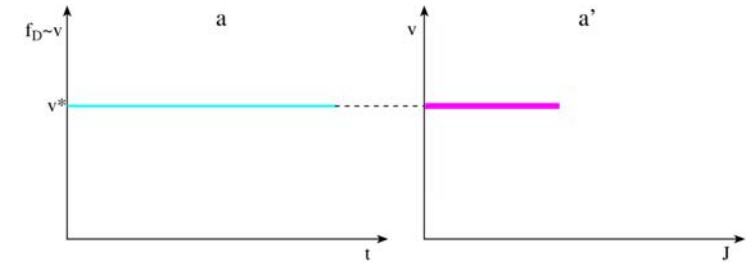


reminder: $\sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$

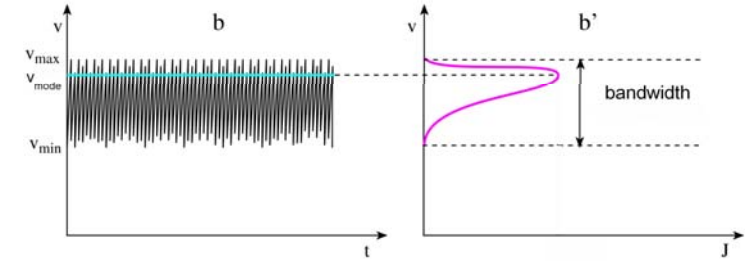
45

Doppler curves

one constant velocity (v^*)



frequency distribution (with v_{mode})



velocity distribution in TM-mode

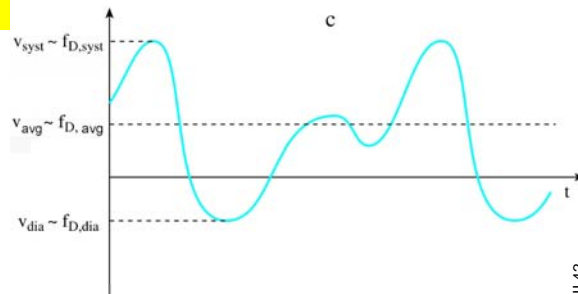
velocity distribution at a certain time

Textbook Fig. VIII.42

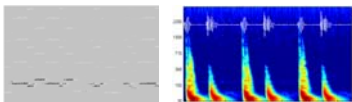
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Doppler curves

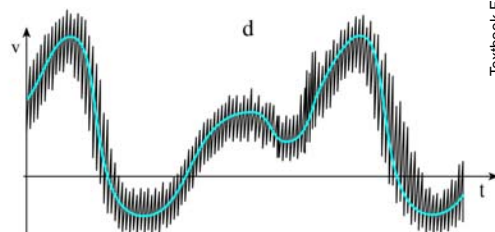
flow can be represented by one velocity in each moment



flow can be represented by a velocity distribution in each moment



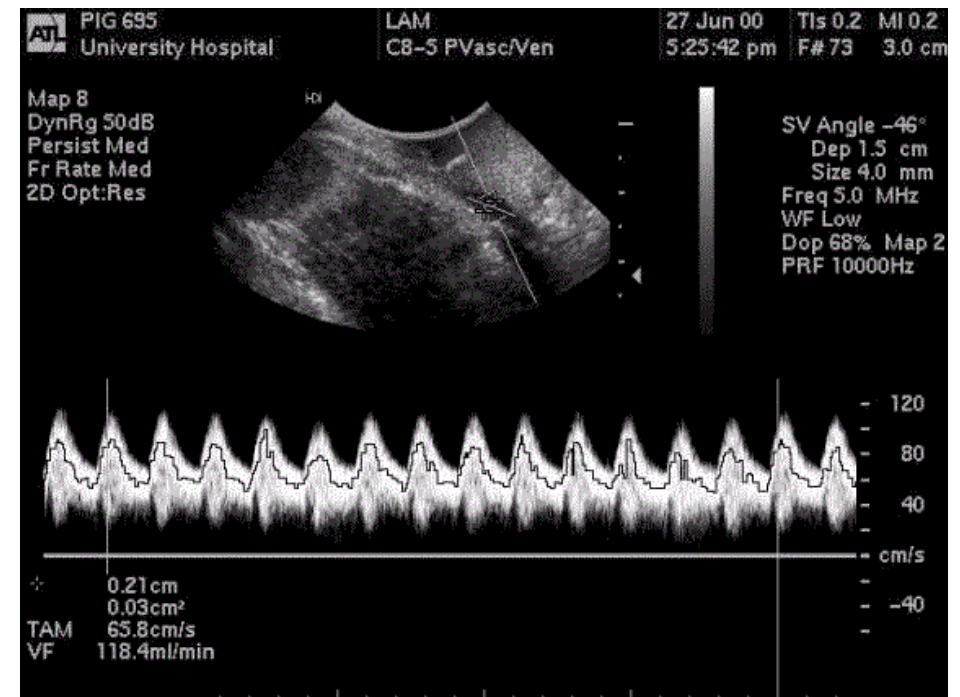
cf. voiceprint, music/heart beats in time-frequency representation

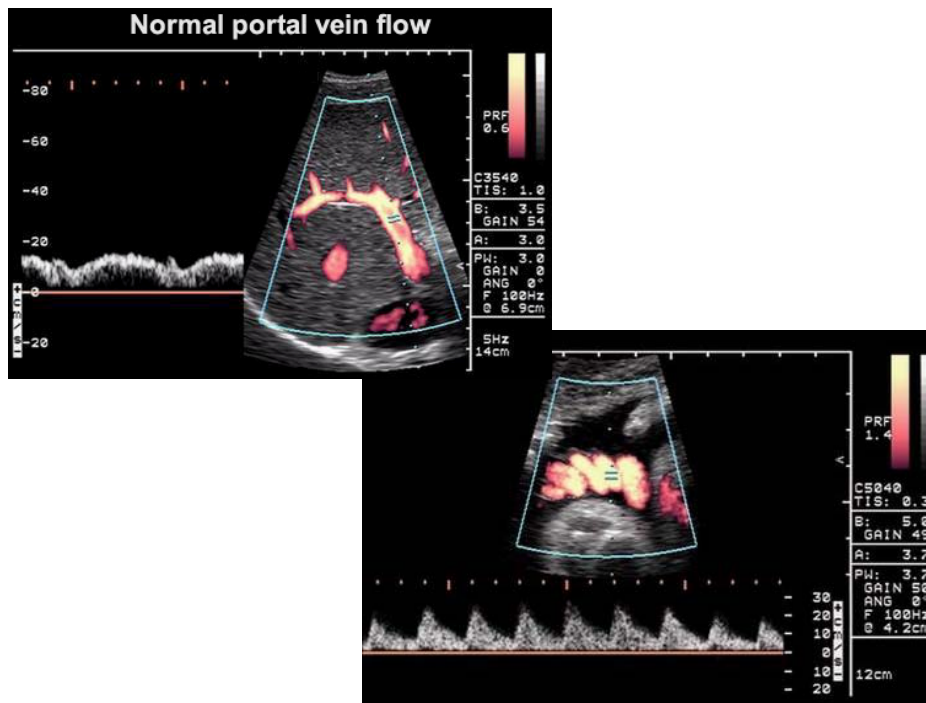


velocity distribution in TM-mode

Textbook Fig. VIII.42

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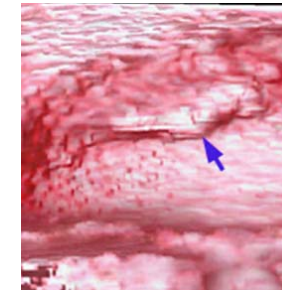


3D reconstruction

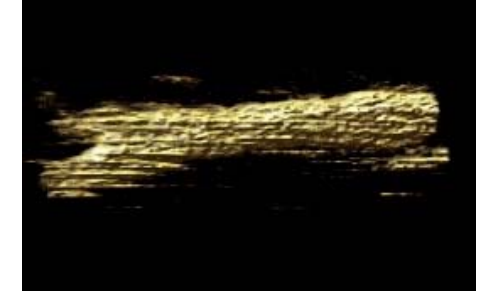
face of a fetus



bladder



carotis



Safety

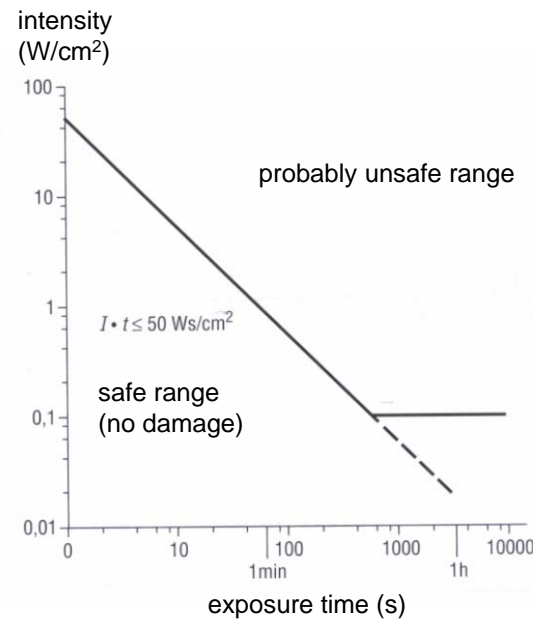
in the diagnostics:

$10 \text{ mW/cm}^2 = 100 \text{ W/m}^2$

cf. pain threshold: 10 W/m^2

in the therapy: 1 W/cm^2

spatial average temporal average (SATA) intensity;
 spatial peak temporal peak (SPTP) intensity;
 spatial peak temporal average (SPTA) intensity;
 spatial peak pulse average (SPPA) intensity;
 spatial average pulse average (SAPA) intensity



more:

in Ultrasound practice

