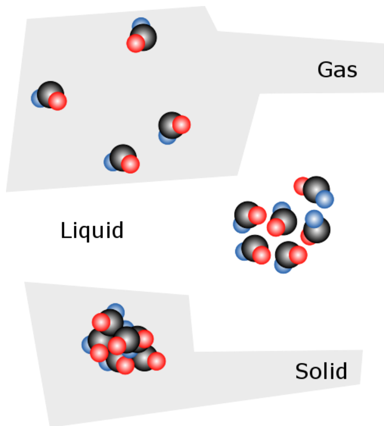
TRANSPORT PHENOMENA, FLOW OF FLUIDS

A transport phenomenon is any of various mechanisms by which extensive thermodynamic quantities (particle number, mass, electric charge, heat) move from one place to another. Transport phenomena include flow of liquids or gases, diffusion, convection, electric current, heat conduction, etc. In these transport phenomena particles can move collectively, in a synchronized way (e.g. flow of fluids), or individually (e.g. diffusion). In spite of the obvious differences in the details, there are many similarities in the description of different transport processes, and in the general principles that underlie these processes. The foundational axioms of all transport phenomena are the conservation laws, specifically, conservation of mass, energy, linear momentum, charge, particle number, etc.

Fundamental Concepts of Fluid Dynamics



Fluid is a common name for gases and liquids. The gas and the liquid states are two of the principle states of matter.

Gas is a collection of particles (molecules, atoms, ions) moving in a largely random fashion, without significant inter-particle interactions. The distance between gas particles is large compared to the particle size, and gases lack definite shape or volume. In liquids, inter-particle interactions play significant role. Particles in a liquid are more densely packed, but free to move compared to each other. Liquids are relatively incompressible, so while a liquid has a definite volume, it does not have a definite shape.

When a body comes in contact with a fluid, the fluid exerts a force perpendicular to the surface of the body. We call pressure (p) the force exerted per unit area:

$p = \frac{F}{A}$; the SI unit of pressure is pascal (Pa), 1 Pa = 1 N/m^2.

Fluid dynamics is the sub-discipline of fluid mechanics dealing with fluid flow – i.e. fluids (liquids and gases) in motion.

The foundational axioms of fluid dynamics are the conservation laws, specifically, conservation of mass, conservation of energy, and conservation of linear momentum.

In addition to the above, fluids are assumed to obey the continuum assumption. Fluids are composed of small particles that collide with one another and solid objects. Since these particles are very small, however, we can treat the macroscopic behavior of the fluid assuming that it is a continuum. Consequently, properties such as density, pressure, temperature, and velocity are taken to be well-defined at every point in the fluid. The fact that the fluid is made up of discrete molecules is ignored.

The movement pattern of a flowing fluid can be described by streamlines. Streamlines are a family of curves that are instantaneously tangent to the velocity vector of the flow.

Laminar flow occurs when a fluid flows in parallel layers, with no disruption between the layers. In such a flow, streamlines do not cross or mix. Fluids flow laminarly at low velocities. Above a critical velocity, the flow becomes turbulent: the flow swirls irregularly, and the streamlines mix chaotically.

A steady state flow is one in which all physical parameters (e.g. velocity, pressure, etc.) are unchanged in time.

The volumetric flow rate is the volume of fluid which passes through a given surface per unit time:

$I_V = \frac{\Delta V}{\Delta t}$,

where I_V is the volumetric flow rate, ΔV is the volume that passed through the studied surface in Δt time ($[I_V]_{SI} = m^3 \cdot s^{-1}$).

The mass flow rate is the mass of fluid which passes through a given surface per unit time:

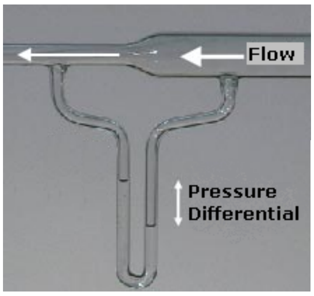
$I_m = \frac{\Delta m}{\Delta t}$,
where I_m is the mass flow rate, Δm is the mass that passed through the studied surface in Δt time ($[I_m]_{SI} = kg \cdot s^{-1}$).
Medical application:

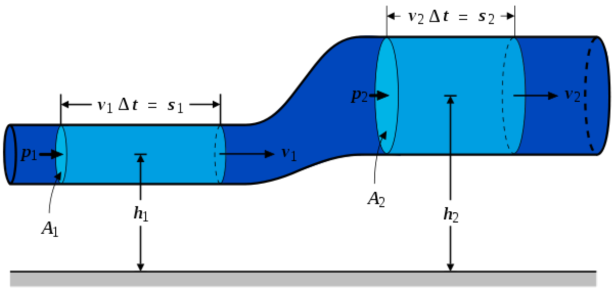
- Dilution method for measuring blood flow rate.

Continuity Equation

The continuity equation states that in any steady state process, the rate at which a fluid enters a system is equal to the rate at which the fluid leaves the system (conservation of mass). For a tube with changing diameter:
$I_V = A_1 \cdot v_1 = A_2 \cdot v_2 = \text{constant}$,
$I_m = \rho \cdot A_1 \cdot v_1 = \rho \cdot A_2 \cdot v_2 = \text{constant}$,
where I_V is the volumetric flow rate in the tube, I_m the mass flow rate, ρ is the density of the fluid, v_1 is the velocity of the flow through the cross section A_1, and v_2 is the velocity of the flow through the cross section A_2.

Bernoulli's Principle





Bernoulli's principle is a special case of the work-energy theorem. Let us consider a fluid flowing in a tube that changes elevation as well as cross section area. We can apply the work-energy theorem to the fluid that is initially contained between the cross section A_1 and A_2 in the tube. After some time Δt, the fluid will have moved along the tube. The only difference between the two situations is in the fluid contained in the regions indicated in light blue. This part of the fluid with a volume ΔV, and mass Δm was lifted from elevation h_1 to h_2 and its velocity changed from v_1 to v_2.
The change in the potential energy is:
$\Delta E_{pot} = \Delta m \cdot g \cdot h_2 - \Delta m \cdot g \cdot h_1 = \rho \cdot \Delta V \cdot g \cdot h_2 - \rho \cdot \Delta V \cdot g \cdot h_1$.
The change in the kinetic energy is:
$\Delta E_{kin} = \frac{1}{2} \cdot \Delta m \cdot v_2^2 - \frac{1}{2} \cdot \Delta m \cdot v_1^2 = \frac{1}{2} \cdot \rho \cdot \Delta V \cdot v_2^2 - \frac{1}{2} \cdot \rho \cdot \Delta V \cdot v_1^2$.
The work done on the volume can be calculated from the pressures (p_1, p_2), cross section areas, and displacements (s_1, s_2):
$W = F_1 \cdot s_1 - F_2 \cdot s_2 = p_1 \cdot A_1 \cdot s_1 - p_2 \cdot A_2 \cdot s_2 = p_1 \cdot \Delta V - p_2 \cdot \Delta V$.
The work-energy theorem gives:
$W = \Delta E_{pot} + \Delta E_{kin}$;
$p_1 \cdot \Delta V - p_2 \cdot \Delta V = \rho \cdot \Delta V \cdot g \cdot h_2 - \rho \cdot \Delta V \cdot g \cdot h_1 + \frac{1}{2} \cdot \rho \cdot \Delta V \cdot v_2^2 - \frac{1}{2} \cdot \rho \cdot \Delta V \cdot v_1^2$.
After simplifying with ΔV and rearranging, we get Bernoulli's equation:
$p_1 + \rho \cdot g \cdot h_1 + \frac{1}{2} \cdot \rho \cdot v_1^2 = p_2 + \rho \cdot g \cdot h_2 + \frac{1}{2} \cdot \rho \cdot v_2^2 = \text{constant}$.
Physiological implications:

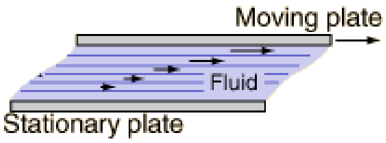
- Formation of aneurysms.
- Blood plasma skimming.

Viscosity

The resistance to flow of a fluid and the resistance to the movement of an object through a fluid are due to the internal friction or viscosity of the fluid. In the case of a moving plate in a liquid, it is found that there is a layer or lamina which moves with the plate, and a layer which is essentially stationary if it is next to a stationary plate. A constant force is needed to overcome the drag force and keep the upper plate moving at constant speed. The force F is found to be directly proportional to the speed (v) and the surface area (A) of the plate, and inversely proportional to the separation (Δh) between the two plates. The proportionality constant is called the coefficient of viscosity (η).

$F = \eta \cdot \frac{v \cdot A}{\Delta h}$.

The SI unit of viscosity is $N \cdot s/m^2 = Pa \cdot s$

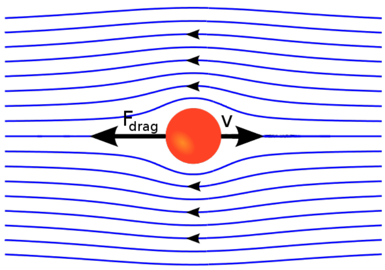


Stokes Law

The frictional force (drag force) exerted on spherical objects with very small Reynolds numbers (e.g. small particles) in a continuous viscous fluid can be calculated the following way:

$F_{drag} = 6 \cdot \pi \cdot \eta \cdot r \cdot v$,

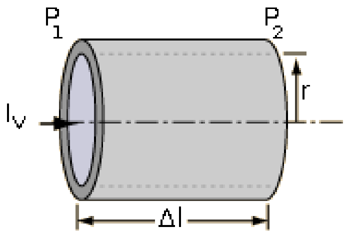
where η is the coefficient of viscosity, r is the radius of the sphere, v is the speed of flow around the sphere.



Hagen-Poiseuille Law

In the case of laminar flow in a tube, the volumetric flow rate is given by the pressure difference (Δp) divided by the viscous resistance (R). This resistance is directly proportional to the viscosity of the fluid (η) and the length of the tube (Δl), but it is inversely proportional to the fourth power of the radius of the tube (r).

$I_V = \frac{\Delta p}{R} = -\frac{\pi}{8 \cdot \eta} \cdot r^4 \cdot \frac{\Delta p}{\Delta l}$



Physiological consequences:

- A small amount of arterial occlusion can have a surprisingly big effect.
- A small change in the diameter of the blood vessels can regulate blood flow efficiently.

Turbulent Flow

The Hagen-Poiseuille law is valid only for laminar flow. Above some critical velocity, the flow will become turbulent with the formation of eddies and chaotic motion which do not contribute to the volume flow rate. This turbulence increases the resistance of the fluid dramatically.

The Reynolds number Re is a dimensionless number that gives a measure of the ratio of inertial forces to viscous forces:

$Re = \frac{\bar{v} \cdot \rho \cdot r}{\eta}$,

where $\bar{v}$ is the average flow rate of the fluid, ρ denotes density, r is a characteristic length of the system (e.g. radius of the tube), and η is viscosity. If the Reynolds number becomes larger than a critical value, the flow becomes turbulent. The critical Reynolds number depends on the actual experimental arrangement. For flow in a tube with circular cross section, the critical Reynolds number is $Re_{crit} = 1160$.

Physiological consequence:

- Blood flow in the human body is remarkably free of turbulence. If it becomes turbulent, the heart has to make extra effort to overcome the increased resistance of the flow.