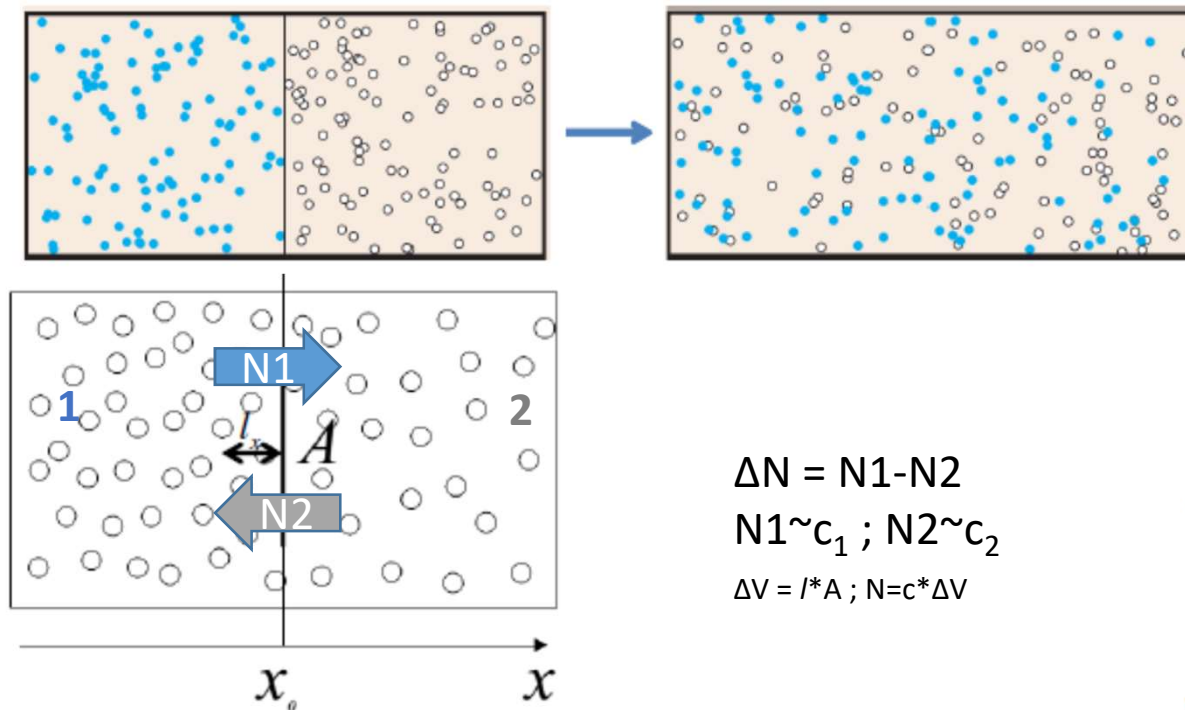


# Diffusion

- Random Walk
- Diffusion of potassium permanganate
- Image evaluation

**Brownian motion:** Random uncorrelated motion of particles due to the thermal motion and random collisions.

**Diffusion:** A net observable material transport, which lasts until there is an equilibration of the concentration over space (in thermal equilibrium, and free diffusion)



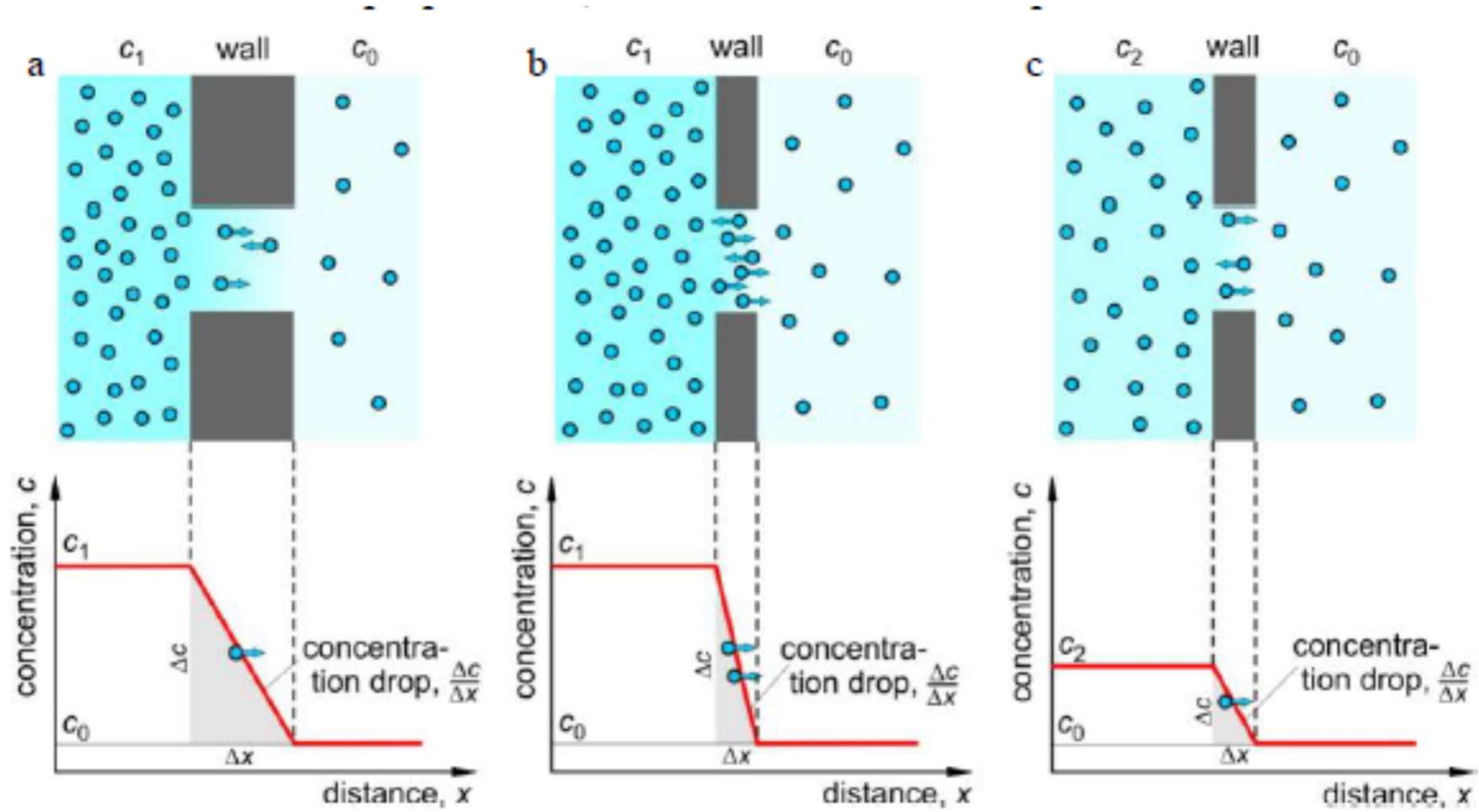
$$\Delta N = N1 - N2$$

$$N1 \sim c_1 ; N2 \sim c_2$$

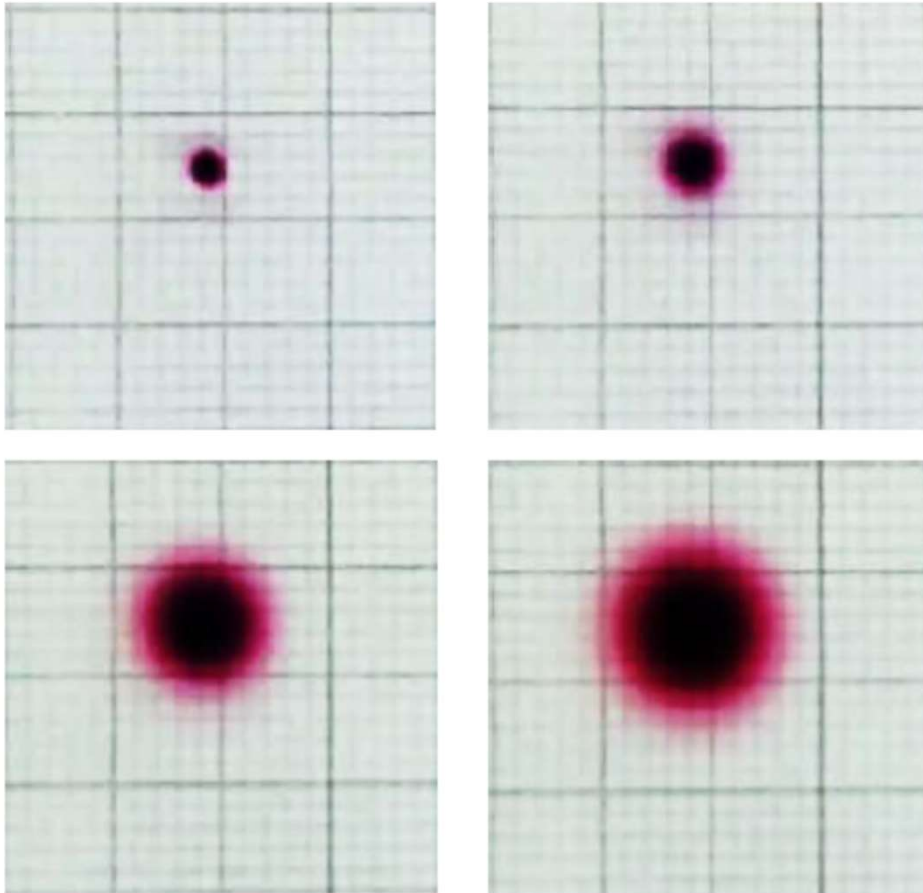
$$\Delta V = l \cdot A ; N = c \cdot \Delta V$$

$$J_v = -D \cdot \frac{\Delta c}{\Delta x}$$

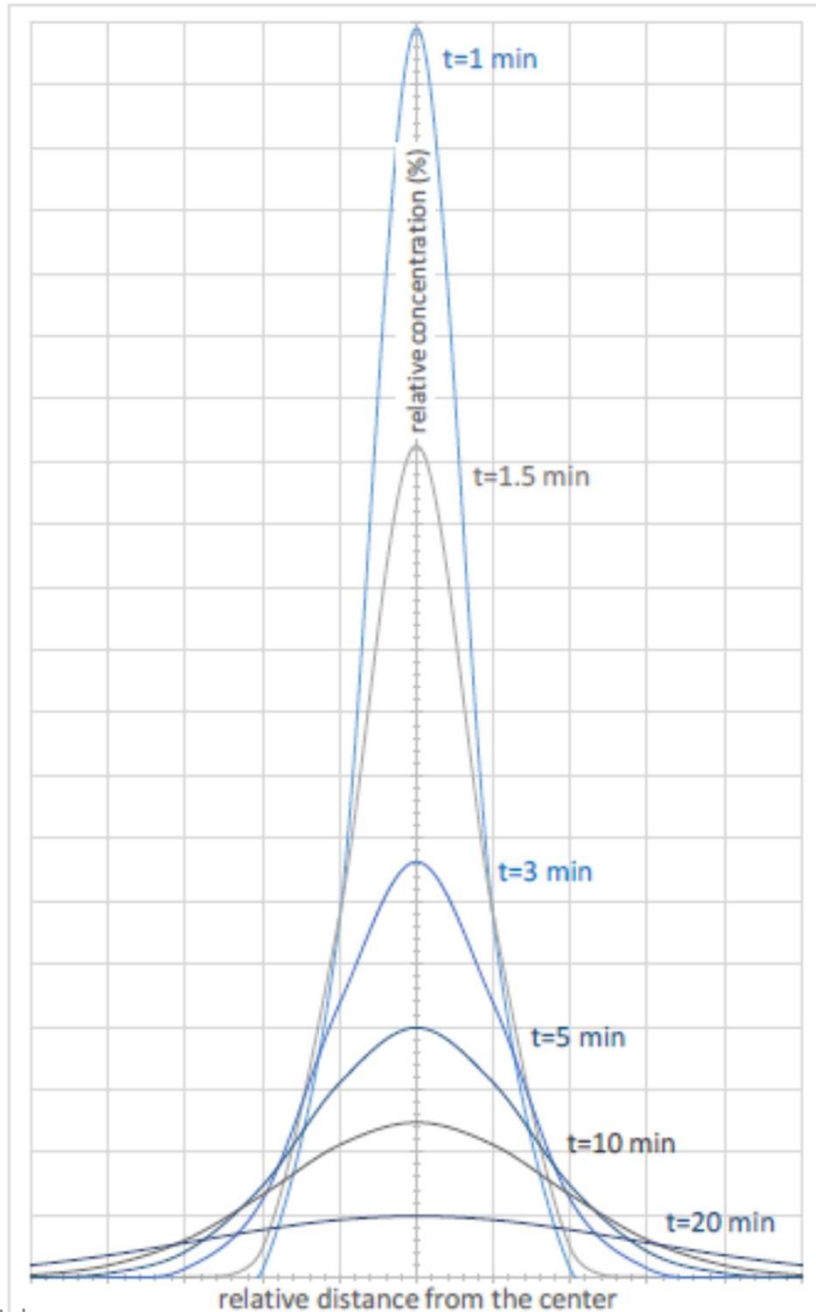
$$D \cdot \frac{\Delta \left( \frac{\Delta c}{\Delta x} \right)}{\Delta x} = \frac{\Delta c}{\Delta t}$$

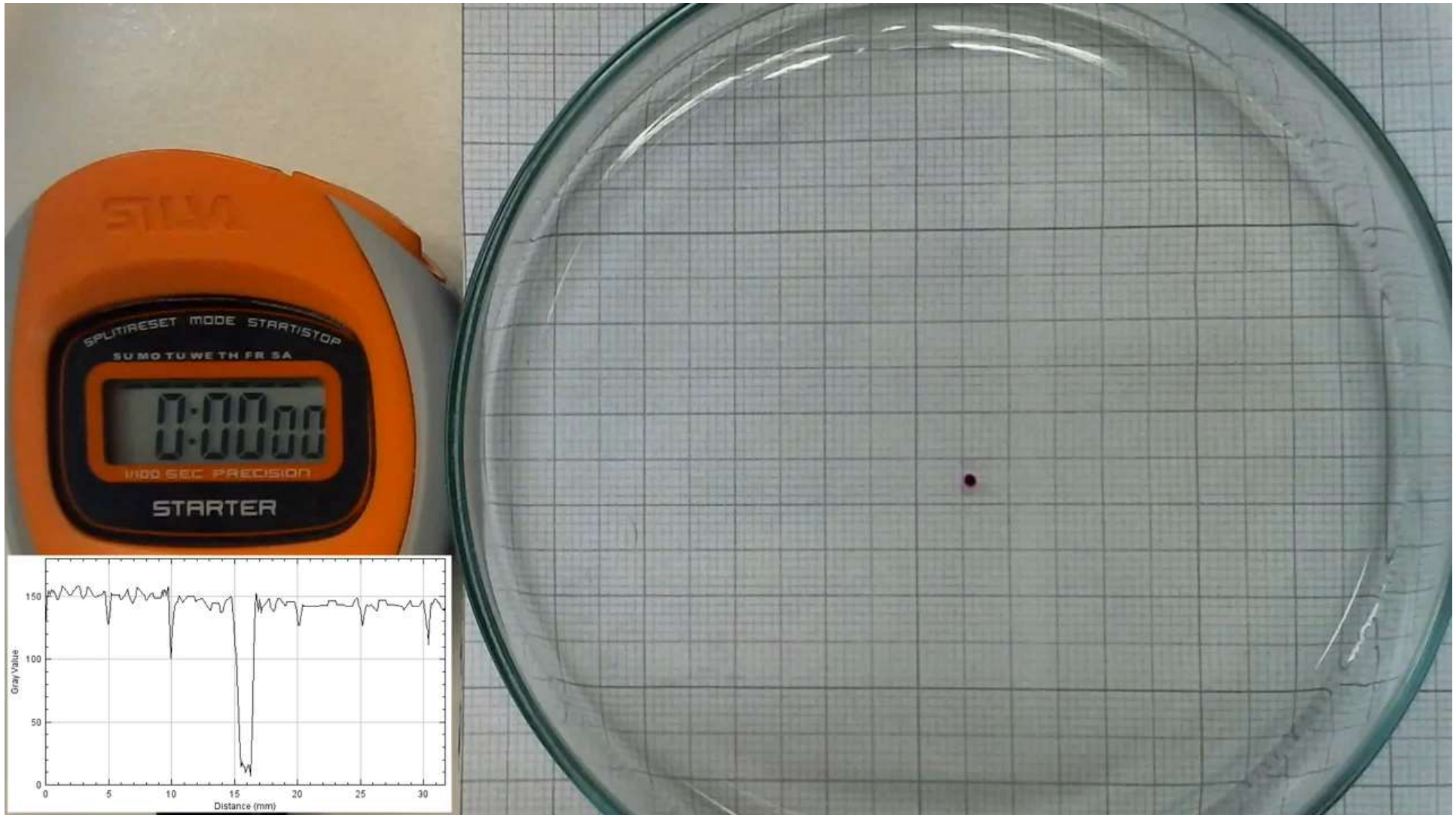


# Potassium permanganate diffusion from a starting spot over time



$$D \cdot \frac{\Delta \left( \frac{\Delta c}{\Delta x} \right)}{\Delta x} = \frac{\Delta c}{\Delta t}$$





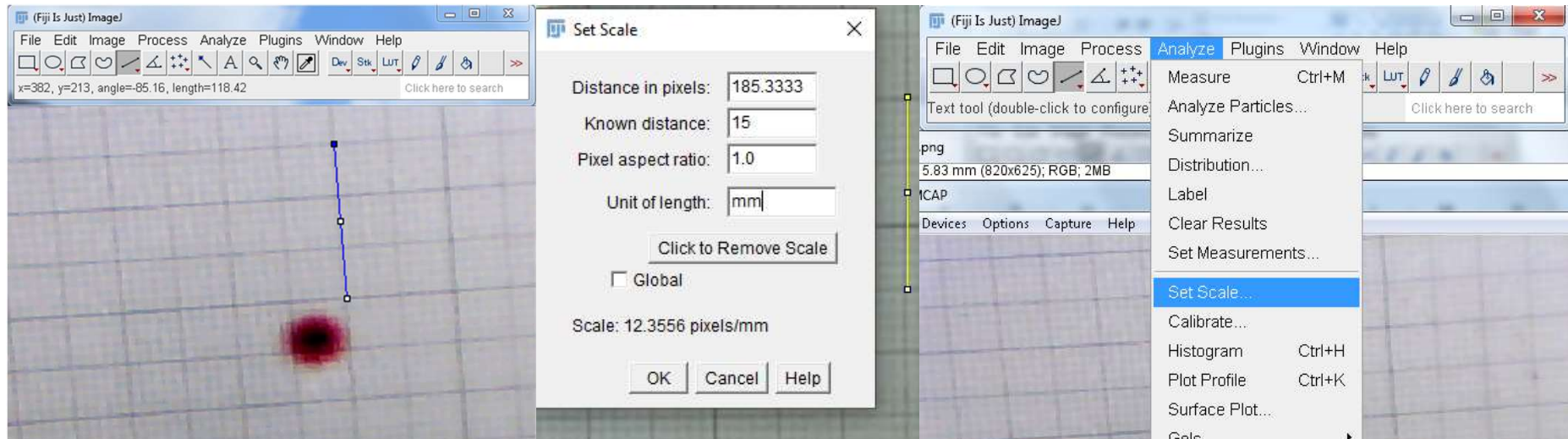
2020. 04. 06.

diffusion material

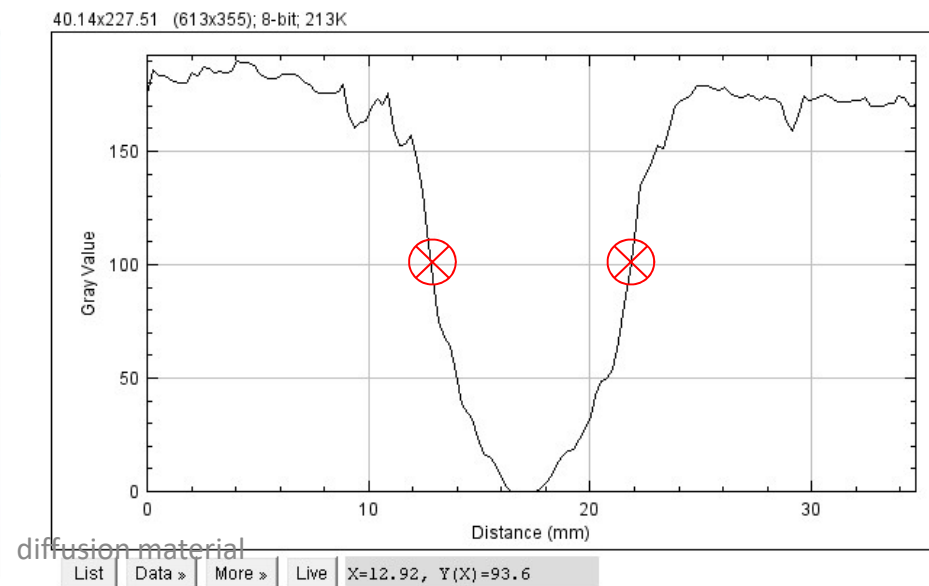
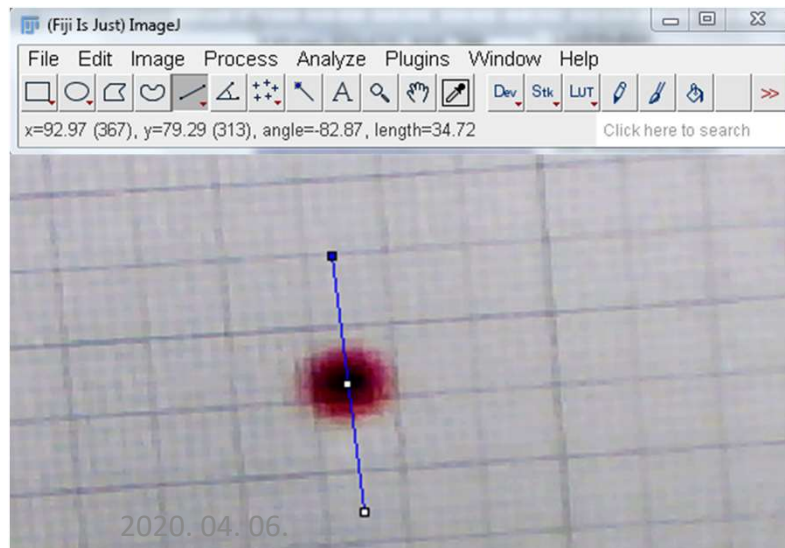


## Image analysis:

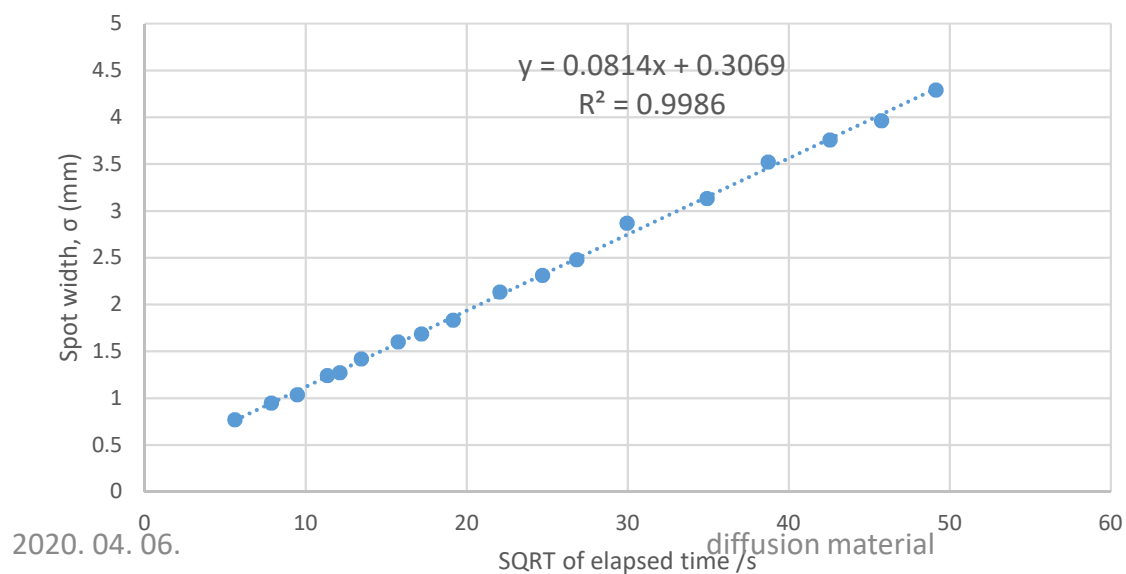
1. Calibrate the image



2. Plot the intensity profile

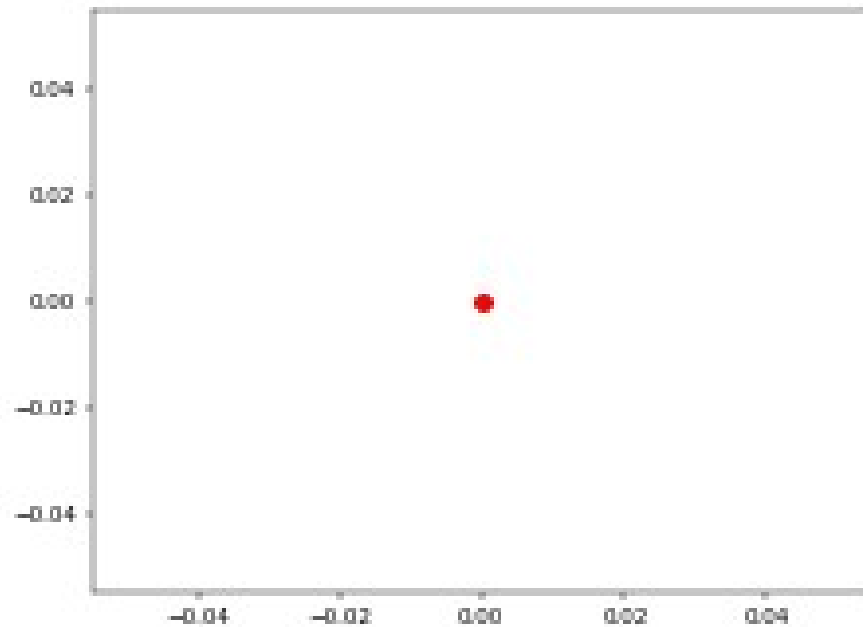


|  | Time (min) | Real<br>timepoint<br>of the<br>image (s) | left side of<br>the spot<br>(mm) | right side of<br>the spot<br>(mm) |  |  | square-root<br>of the time<br>(/s) | diameter<br>(FWHM) of<br>the spot<br>(mm) | width parameter<br>of the bell-<br>shaped curve<br>$\sigma$ (mm) |  |
|--|------------|--|----------------------------------|-----------------------------------|--|--|------------------------------------|---|--|--|
|  | 0.5        | 31.5                                     | 5.74                             | 7.87                              |  |  | 5.61248608                         | 2.13                                      | 0.76884856   |  |
|  | 1          | 62                                       | 4.84                             | 7.46                              |  |  | 7.87400787                         | 2.62                                      | 0.945719825  |  |
|  | 1.5        | 90                                       | 4.26                             | 7.13                              |  |  | 9.48683298                         | 2.87                                      | 1.035960266  |  |
|  | 2          | 129                                      | 4.84                             | 8.28                              |  |  | 11.3578167                         | 3.44                                      | 1.241708472  |  |
|  | 2.5        | 147                                      | 4.92                             | 8.44                              |  |  | 12.1243557                         | 3.52                                      | 1.270585413  |  |
|  | 3          | 181                                      | 8.77                             | 12.7                              |  |  | 13.453624                          | 3.93                                      | 1.418579737  |  |
|  | 4          | 248                                      | 7.7                              | 12.13                             |  |  | 15.7480157                         | 4.43                                      | 1.599060619  |  |
|  | 5          | 296                                      | 8.36                             | 13.03                             |  |  | 17.2046505                         | 4.67                                      | 1.685691443  |  |
|  | 6          | 367                                      | 7.21                             | 12.29                             |  |  | 19.1572441                         | 5.08                                      | 1.833685767  |  |
|  | 8          | 487                                      | 7.29                             | 13.2                              |  |  | 22.0680765                         | 5.91                                      | 2.133284032  |  |
|  | 10         | 611                                      | 6.8                              | 13.2                              |  |  | 24.7184142                         | 6.4                                       | 2.310155297  |  |
|  | 12         | 720                                      | 4.59                             | 11.46                             |  |  | 26.8328157                         | 6.87                                      | 2.479807326  |  |
|  | 15         | 898                                      | 4.75                             | 12.7                              |  |  | 29.9666481                         | 7.95                                      | 2.869646033  |  |
|  | 20         | 1220                                     | 4.43                             | 13.11                             |  |  | 34.9284984                         | 8.68                                      | 3.133148121  |  |
|  | 25         | 1500                                     | 4.26                             | 14.02                             |  |  | 38.7298335                         | 9.76                                      | 3.522986828  |  |
|  | 30         | 1811                                     | 8.85                             | 19.26                             |  |  | 42.5558457                         | 10.41                                     | 3.757611975  |  |
|  | 35         | 2095                                     | 6.48                             | 17.46                             |  |  | 45.77117                           | 10.98                                     | 3.963360181  |  |
|  | 40         | 2415                                     | 6.72                             | 18.61                             |  |  | 49.1426495                         | 11.89                                     | 4.291835387  |  |

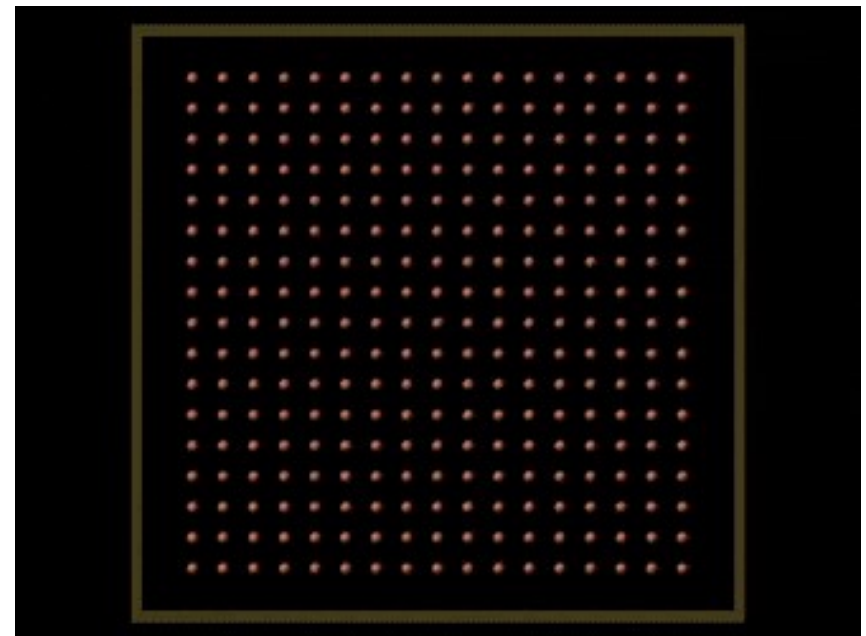


$$D = \text{slope}^2/2$$

The average distance covered by a particle can be calculated from the random walk model

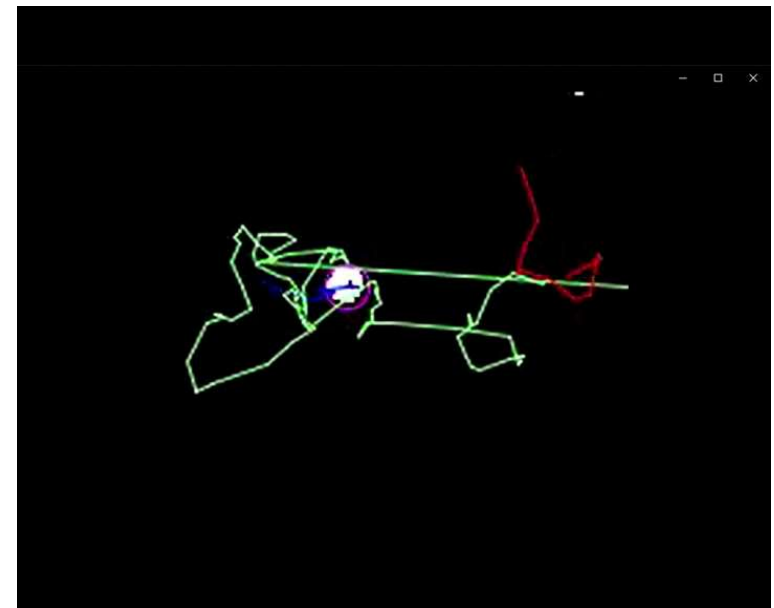


The particles move randomly uncorrelated.  
Each step is in a random direction, generating a zig-zag pattern



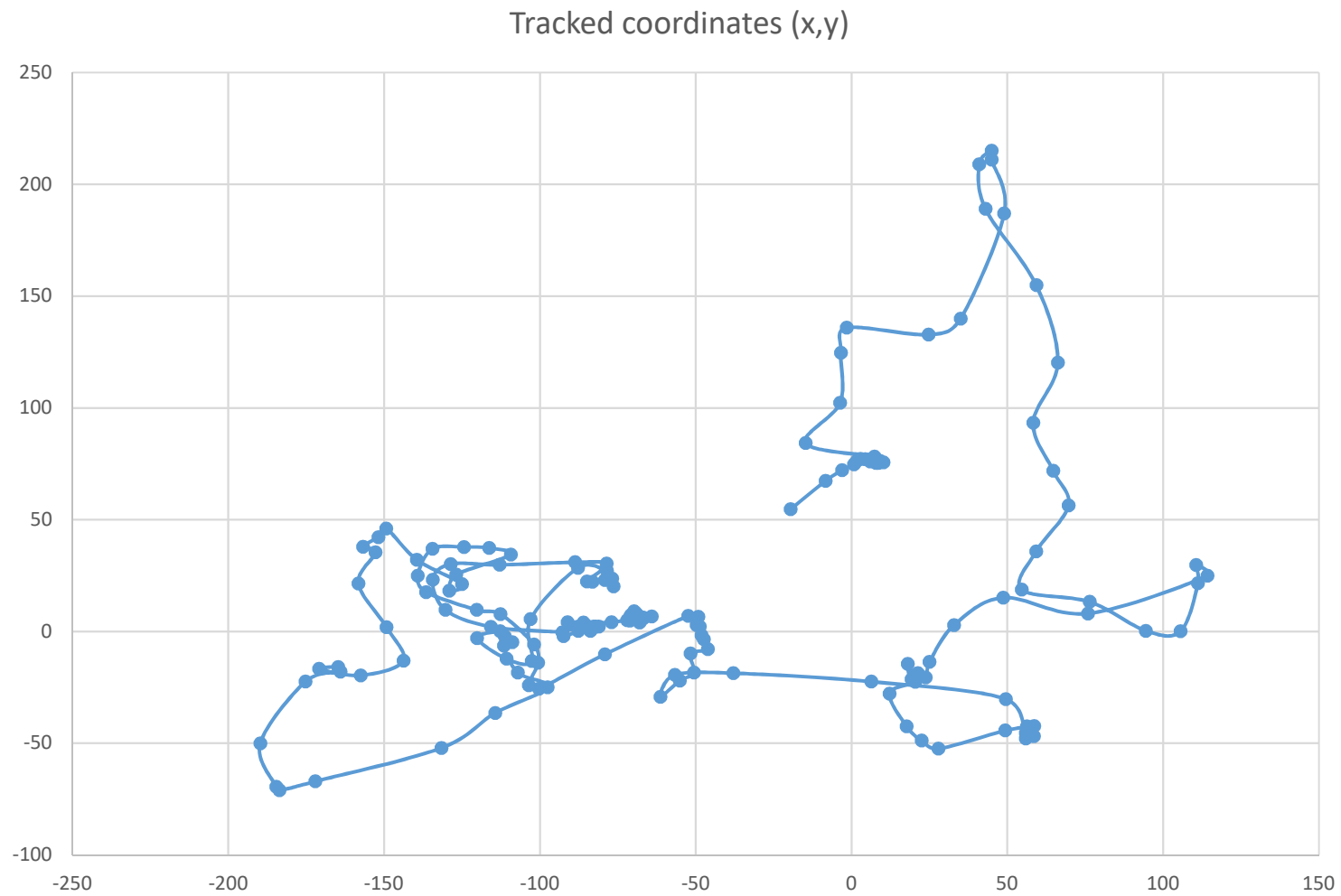


Macroscopic (scaled-up) home made model: the fluidized poppy-seeds act as the solvent. The white foam ball acts as the particle.



ImageJ can track the particle

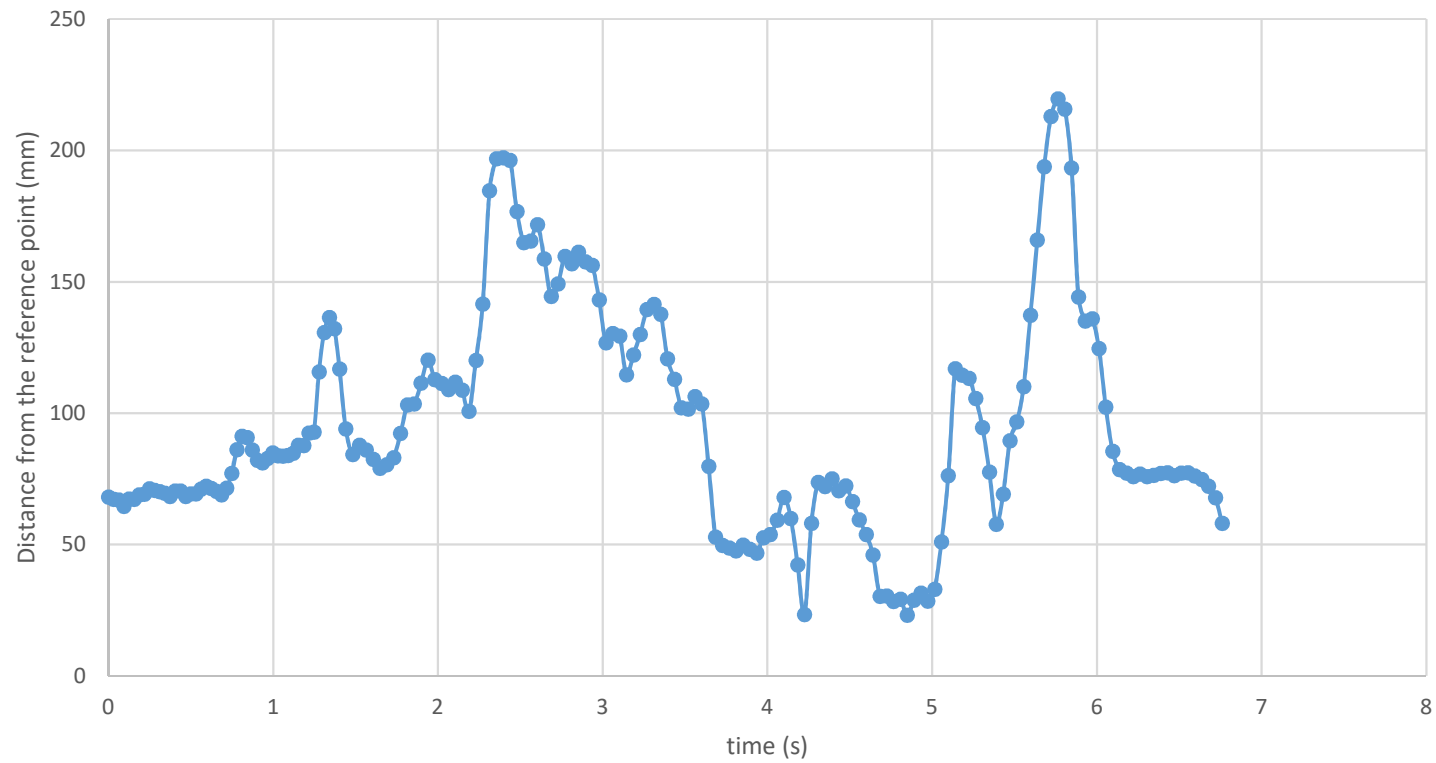
With particle tracking it is possible to convert the video into a series of coordinates.



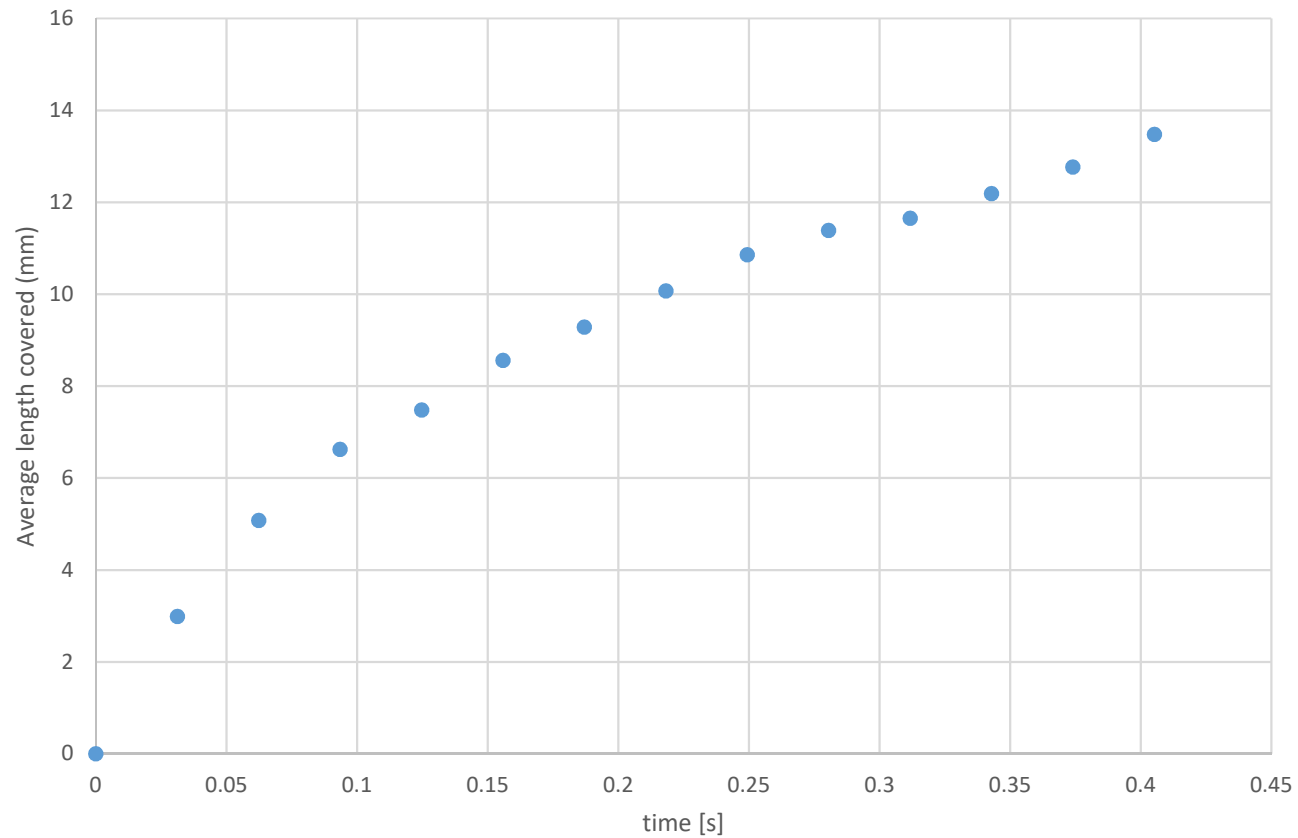
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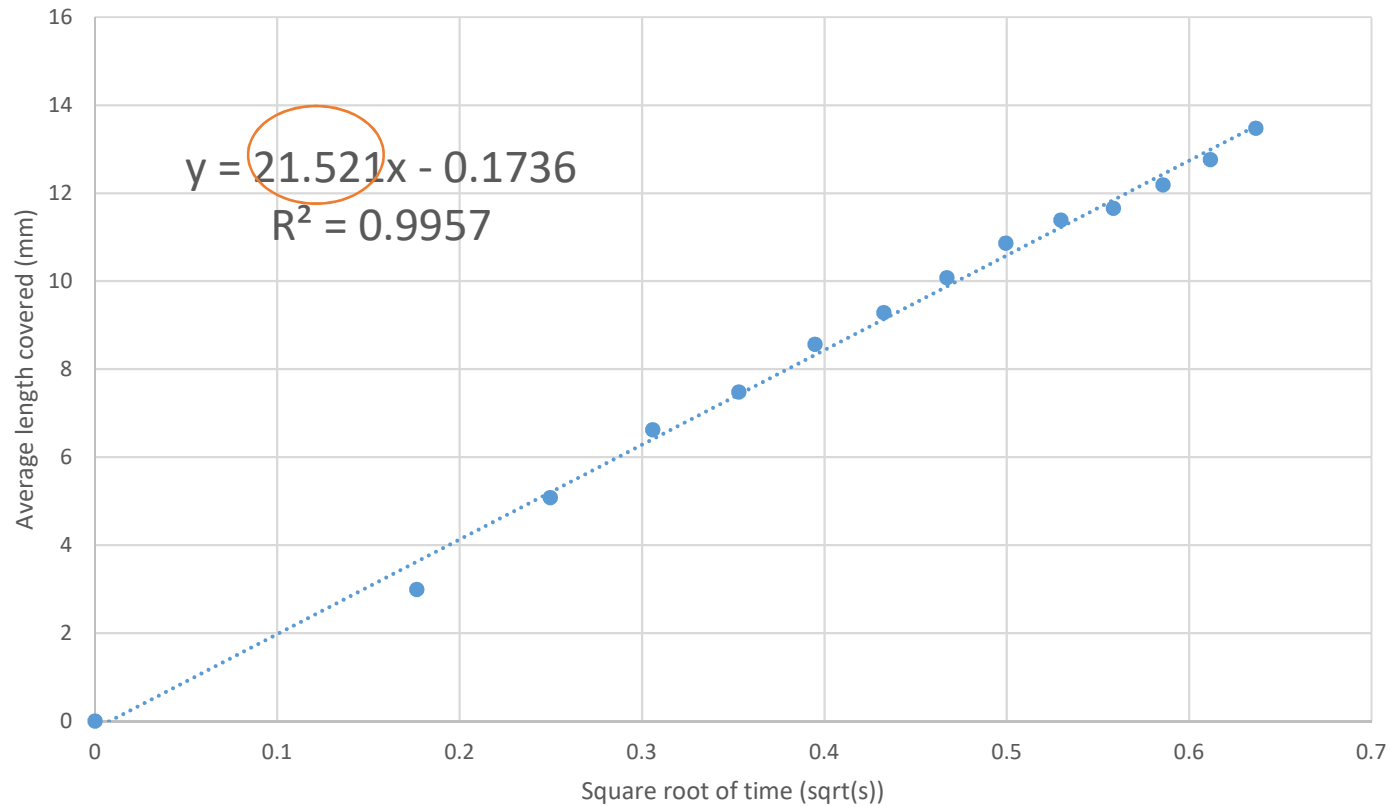
The distance from the origin shows a typical random fluctuation



From multiple particles the average track length over time can be calculated.  
(it is also possible to estimate it from one recording)



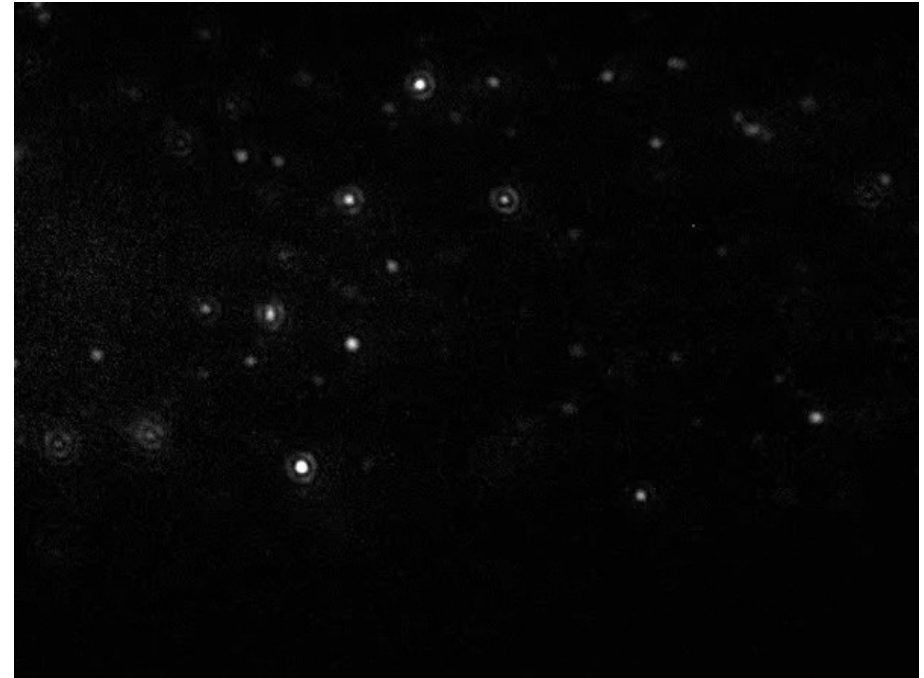
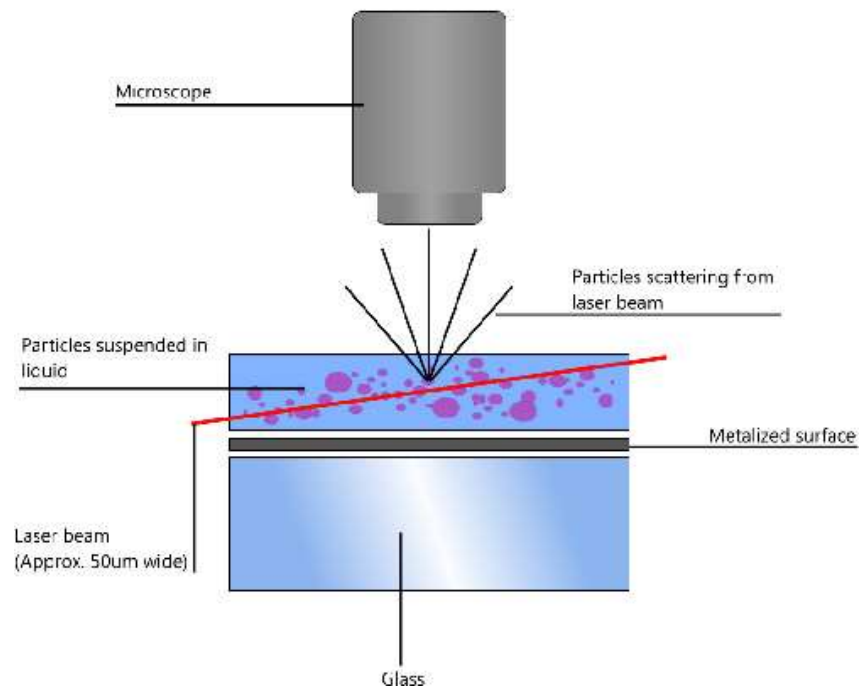
$$R_{average} = \sqrt{2 * D * t}$$



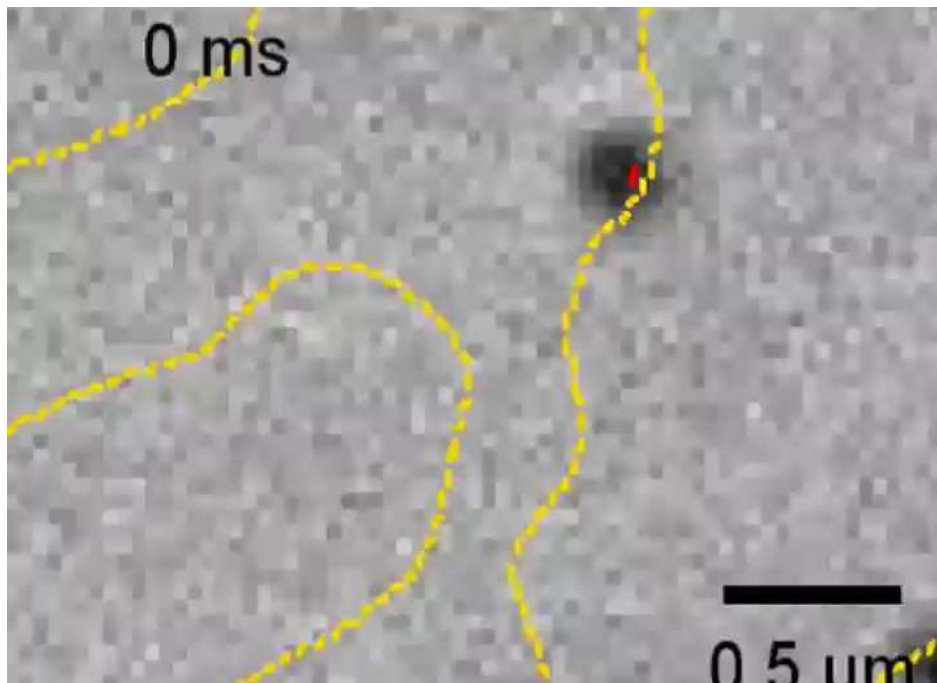
$$\sqrt{2 * D} = 21.521 \frac{mm}{\sqrt{s}}$$

$$D_{apparent} = 231 \text{ mm}^2/s$$

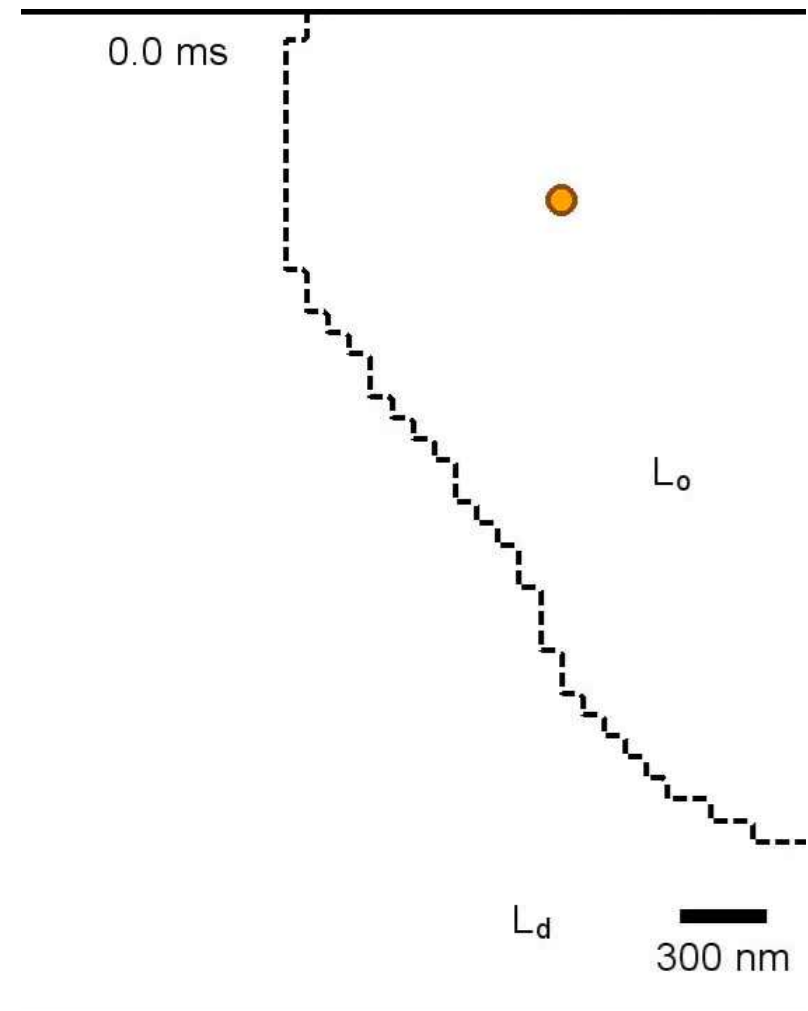
# Nanoparticle tracking analysis (NTA)







Tracking gold nano-spheres in the lipid membrane to see the different viscosity (different diffusion coefficient) pars. Faster diffusion reflects greater diffusion coefficient (lower local viscosity)



2020. 04. 06.

diffusion material

Wu et al, Scientific Reports, Vol 6, 20542 (2016)