Medical Biophysics II.

Biomechanics
Biomolecular and tissue mechanics

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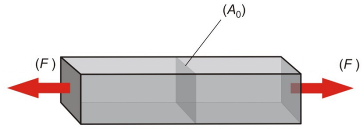
1

Physical bases of biomechnaics

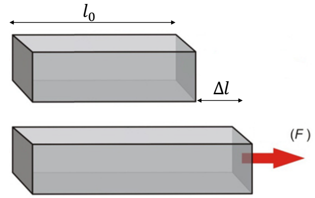
Stress

$$\sigma = \frac{F}{A_0} \qquad \left[\frac{N}{m^2} = Pa\right]$$



Strain (deformation)

$$\varepsilon = \frac{\Delta l}{l_0} \qquad \left[\frac{m}{m}\right] \textit{ no dimension}$$



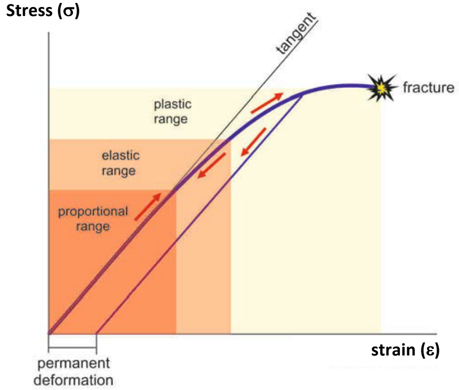
Strain is proportional to stress!

$\sigma \backsim \varepsilon$

2

2

Stress-strain diagram



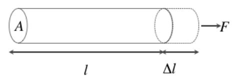
1. Elastic range
Range of reversible deformation. Unloaded length (l_0) recovers when released. Hysteresis may occour.

Proportional range
(part of elastic range)
Deformation is linearly proportional to the load. No hysteresis.

2. Plastic range
After a critical stress value, object undergoes irreversible change of its structure. Unloaded length (l_0) does not recover. Permanent deformation of object.

3

Hooke's law

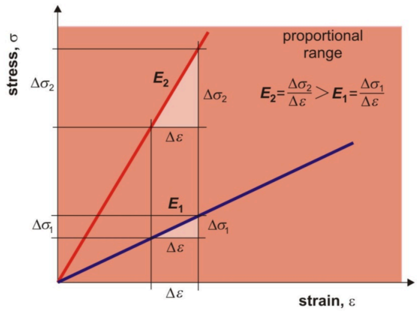


$$\sigma = E \cdot \varepsilon$$

$$\frac{F}{A_0} = E \cdot \frac{\Delta l}{l_0}$$ Hooke's law

$$F = \frac{E \cdot A_0}{l_0} \cdot \Delta l$$

$$F = k \cdot \Delta l$$



$$E_2 = \frac{\Delta\sigma_2}{\Delta\varepsilon} > E_1 = \frac{\Delta\sigma_1}{\Delta\varepsilon}$$

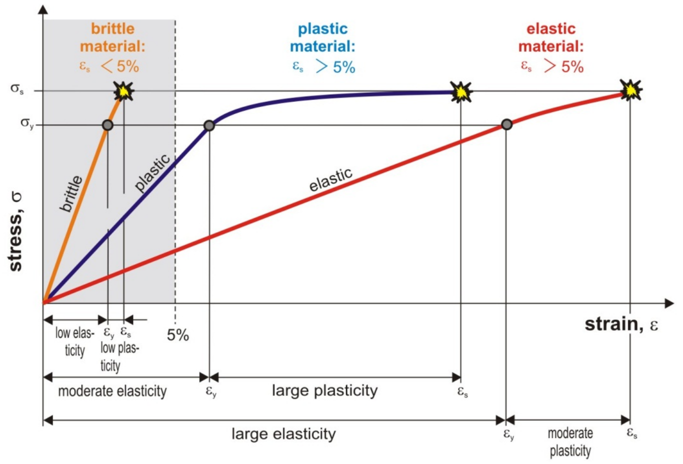
Young's modulus
(material stiffness)

$$E = \frac{\sigma}{\varepsilon} = \frac{F}{A_0} \cdot \frac{l_0}{\Delta l} \qquad E = \left[\frac{N}{m^2} = Pa\right]$$

Spring constant
(body stiffness)

$$k = \frac{F}{\Delta l} \qquad k = \left[\frac{N}{m}\right]$$

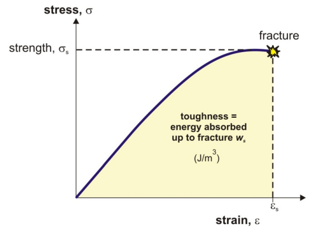
4



5

Toughness

w_s: the work done on unit volume up to fracture.

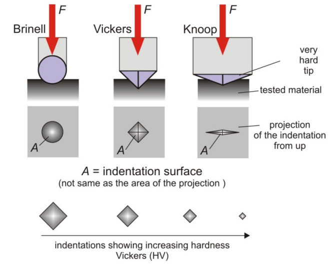


Strength

σ_s: the stress value belonging to fracture. Tensile-, compressive-, etc.

Hardness

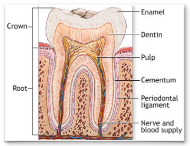
property of the material that characterizes the resistance against plastic deformation during mechanical interaction with a harder material.



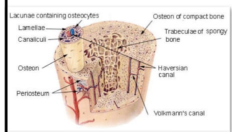
$$H = \frac{F}{A} = [Pa]$$

6

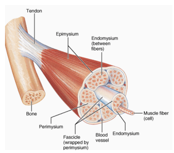
Tooth



Bone

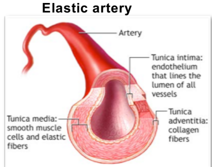


Muscle



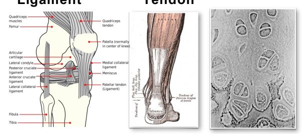
titin, desmin, collagen

Elastic artery



Collagen and elastin

Ligament **Tendon** **Cartilage**



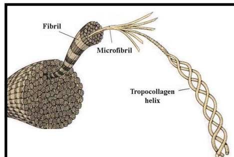
Collagenn, proteoglycans (water)

Composite of collagen (organic) and apatite (inorganic).

Organic component provides: toughness
Inorganic component provides: strength

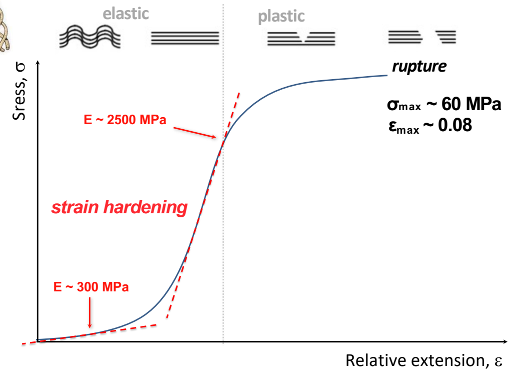
7

Collagen



1400 amino acids/chain
3 chains (triple helix)
Glicine (1/3)
Proline (1/10)
Hidroxiproline

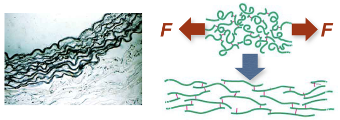
Posttranslational modification is required for mechanical stability: Prolin-hidroxilase and vitamin-C



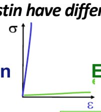
8

Biomechanics of elastic arteries

Elastin – elastic protein network

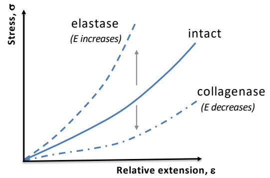


Collagen and elastin have different functions

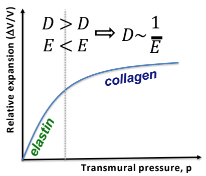


Collagen	Elastin
E = 300 MPa .... 2 500 MPa, $\sigma_{max} \approx 60$ MPa, $\varepsilon_{max} \approx 0{,}08$	E = 0,1 MPa .... 0,4 MPa, $\sigma_{max} \approx 0{,}6$ MPa, $\varepsilon_{max} \approx 3$
Protection against overstretch	Provides distensibility

Effect of proteases on the mechanics of vessel wall



Aorta expansion



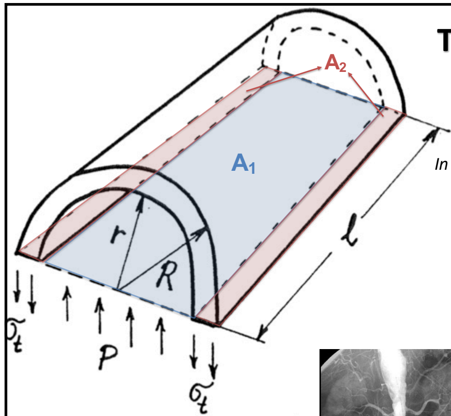
$D > D$, $E < E$ $\Rightarrow D \sim \frac{1}{E}$

Distensibility

The change in vessel volume under pressure

$$D = \frac{\Delta V}{\Delta p \cdot V_0}$$

Tangential stress of blood vessel wall



In case of equilibrium, the forces acting on the two surfaces are equal

$$F_{A_1} = F_{A_2}$$

$$p \cdot A_1 = \sigma_t \cdot A_2$$

$$p \cdot 2r \cdot l = \sigma_t \cdot 2(R - r) \cdot l$$

p: transmural pressure
σ_t: tangential stress
r: inner radius of blood vessel
R: outer radius of blood vessel
R-r: wall thickness

$$\sigma_t = \frac{r}{R - r} \cdot p$$

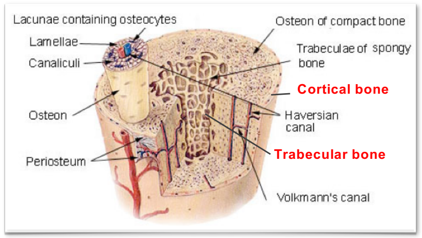
Laplace-Frank equation

Significance: High blood pressure, aneurism

Bone

Due to the different structure of bone tissue along the cross section of long bones, the **Young's modulus distribution is anisotropic**. Denser cortical bone has greater Young's modulus vs. the trabecular bone..
Young's-modulus: 5-20 GPa
Decalcified bone (acid treatment): flexible
Removal of organic compounds (heating): brittle

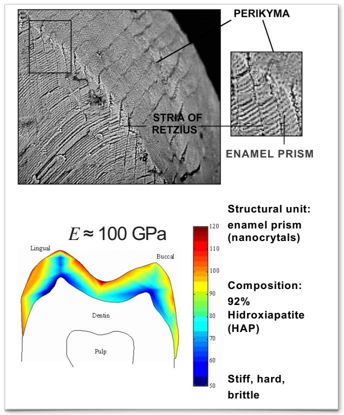
Composite of collagen and hydroxylapatite
collagen: elasticity, toughness
apatite: strength, stiffness, hardness

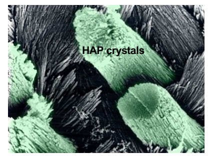


Bone is a composite material composed of an organic polymer (collagen) and an inorganic mineral (hydroxilpatite) which combines the mechanical properties of each.

Stiff, hard and strong but tough and slightly elastic.

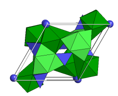
Enamel





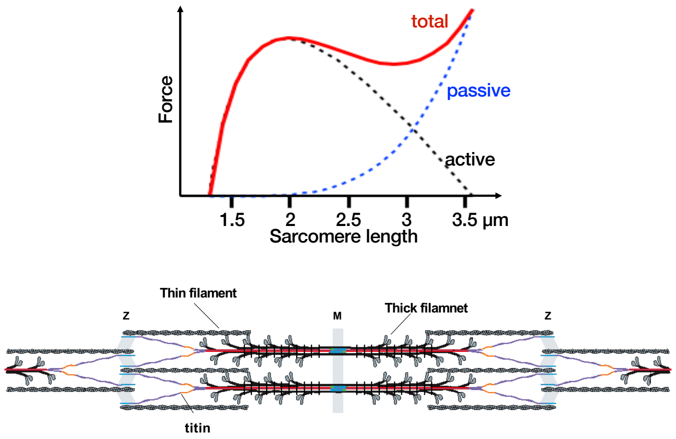
Stiffest material in human body, but brittle!

Ca10(PO4)6(OH)2
Ca10(PO4)6F2



Hexagonal ion crystal
20-60 nm x 6 nm - dentin, bone
500-1000 nm x 30 nm - enamel

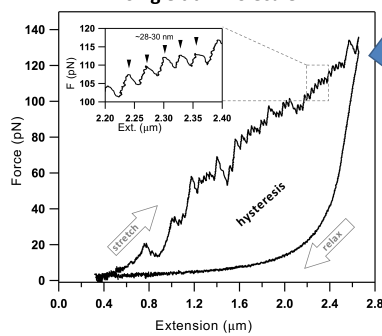
Titin: the elastic filament of the sarcomere



13

Titin is the main determinant of muscle's passive elasticity

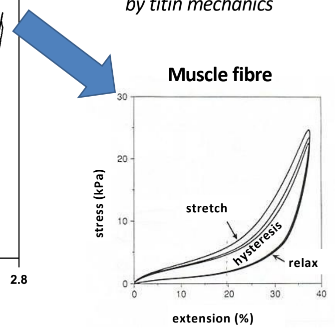
Single titin molecule



From molecule to tissue

Muscle mechanics can be explained by titin mechanics

Muscle fibre

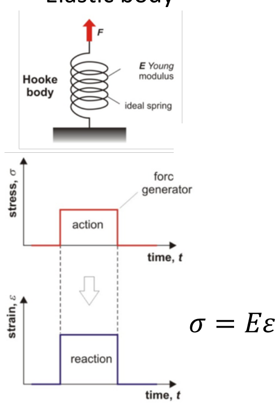


14

Viscoelasticity

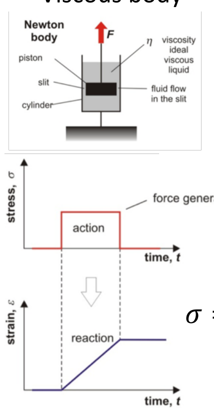
(mechanical model)

Elastic body



$$\sigma = E\varepsilon$$

Viscous body



viscosity

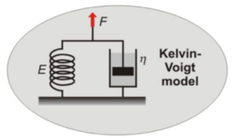
$$\sigma = \eta \frac{\Delta\varepsilon}{\Delta t}$$

Strain rate

15

Viscoelasticity

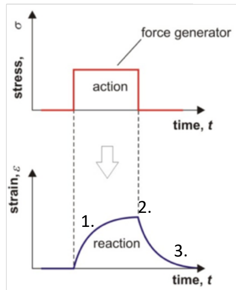
(mechanical model)



Viscoelasticity means the co-appearance of viscous and elastic behavior.

model: parallel connection of spring and dashpot (Kelvin-Voight model)

Spring: ideal elastic (Hooke) body
Dashpot: ideal viscous (Newton) body

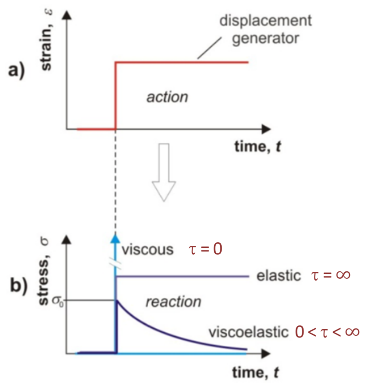


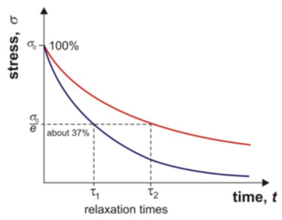
1. Upon stretch, the extension of the spring is slowed down by the dashpot.
2. Extension stops when the elastic spring force equals the external force.
3. When the external force is quenched, the contraction of the spring is slowed down by the dashpot.

16

Stress-relaxation in viscoelastic system

Decrease in stress while strain remains constant





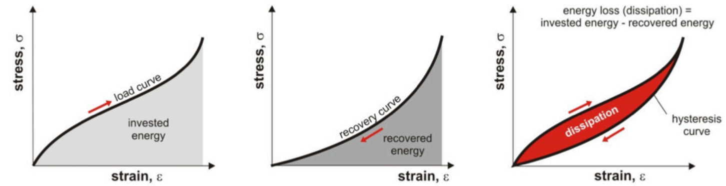
$$\sigma = \sigma_0 \cdot e^{-\frac{t}{\tau}}$$

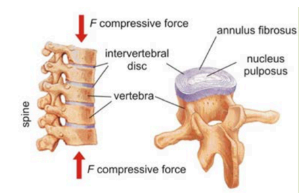
Stretching

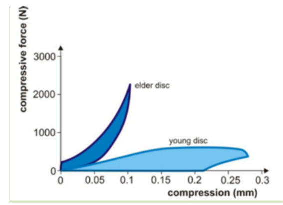
17

Energy dissipation in viscoelastic system

(hysteresis)





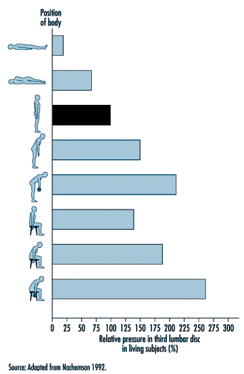


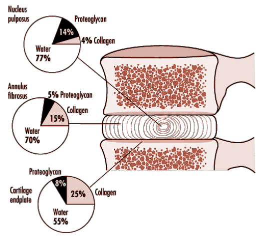
18

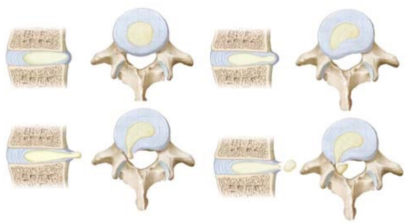
Example I: Consequence of mechanical stress on intervertebral discs

(*discus hernia*)

Stress on the L3 disc at various positions







19

Flashback: What did you learn about US propagation….?

In wich types of tissue does sound propagate faster?

The acoustic properties of each tissue are charahcterized by theeir stiffness

	E (GPa)	κ (GPa^{-1})	c_{sound} (m/s)
Cortical bone	18	0.05	3600
Muscle	7×10^{-5}	0.38	1568

$$c_{sound} = \frac{1}{\sqrt{\rho \cdot \kappa}}$$

$$\kappa = \frac{-\Delta V / V}{\Delta p}$$

Volumetric strain

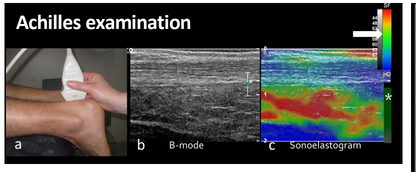
stress

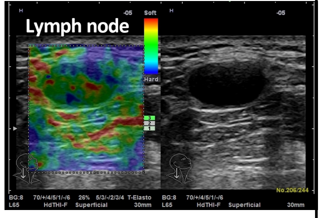
compressibility

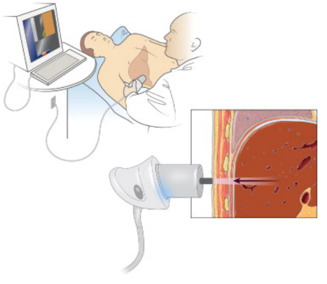
Greater Young-modulus, faster propagation speed

20

Diagnostic application: sonoelastography







Transient elastography (measurement of liver stiffness based on pulse-echo principle)

21