

Sedimentation and electrophoresis methods

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Physical basis of sedimentation methods

Goal: we would like to measure the mass of tiny particles

(this method originates long before the AFM or resonance methods, but is still in use)

Put the particle into a solvent/liquid, and see what happens:

If it's density is higher than that of the liquid, it will sink, or settle down.

This is called **sedimentation**.

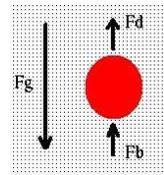
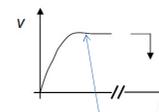


Fig : gravity force, Fd: drag force, Fb: buoyant force.



Drag: a force (Fd) acting on a moving object (usually in a fluid of given viscosity), working against the movement.
 $F_d \sim v, \eta, \text{size}$



The particle will accelerate until the force equilibrium is reached.

(or until the bottom of the holder tube is reached)

Here we have the force equilibrium

Newton-II. Law : $\Sigma F = m \cdot a$

and

$$\frac{\Delta v}{\Delta t} = a$$

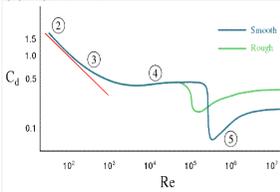
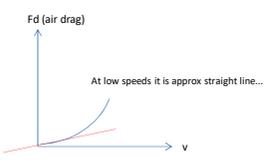
At force equilibrium $a=0$.

The drag force:

General equation: $F_d = \frac{1}{2} \rho v^2 \cdot C_d \cdot A$, where A is the cross-sectional area, and C_d is the **drag coefficient**.

At low speeds $C_d \sim 1/Re$, which means F_d is linearly proportional to the speed.

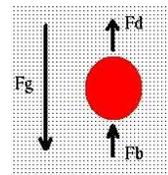
$$Re = \frac{v \cdot L}{\eta/\rho} = \frac{v \cdot L \cdot \rho}{\eta}$$

N.B.: Re is the Reynolds number

F_g : gravity force, F_d : drag force, F_b : buoyant force.

Newton-II. Law: $\Sigma F = m \cdot a$
and
 $\frac{\Delta v}{\Delta t} = a$



$F_g = m \cdot g$
 $F_d = f \cdot v$ ($Re < 0.1$)
where f is the shape constant, embedding C_d .

From Archimedes's law we get: $F_b = g \cdot \rho_{fluid} \cdot V_{particle}$, but $V_{particle} = m/\rho_{particle}$
so $F_b = m \cdot g \cdot \frac{\rho_{fluid}}{\rho_{particle}}$

From the force equilibrium we get:
 $\Sigma F = 0$, which means $F_g - F_b - F_d = 0$,
(or $F_d = F_g - F_b$) thus

$$f \cdot v = m \cdot g \cdot \left(1 - \frac{\rho_{fluid}}{\rho_{particle}}\right)$$

Vav (Volume, velocity)

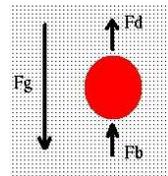
There is one problem with this:
If the particles are really small, then the **Brownian motion** will prohibit settling, and thus the method will not work.

Solution: Centrifuge!



Let's centrifuge the fluid+particles:

F_g : gravity force, F_d : drag force, F_b : buoyant force.



$g = 9.8 \text{ m/s}^2$ would be the gravity force, but in the centrifuge
 $a = \omega^2 r$ is the acceleration felt by the particle. (ω is the angular velocity)

$$f \cdot v = m \cdot g \cdot \left(1 - \frac{\rho_{fluid}}{\rho_{particle}}\right)$$


$$f \cdot v = m \cdot r \cdot \omega^2 \cdot \left(1 - \frac{\rho_{fluid}}{\rho_{particle}}\right)$$

We can rearrange such as:

$$S \equiv \frac{v}{r \cdot \omega^2} = \frac{m}{f} \cdot \left(1 - \frac{\rho_{fluid}}{\rho_{particle}}\right)$$

here S is the sedimentation coefficient. Unit is Svedberg, 1Sv = 10⁻¹³ s

(Theodor Svedberg, Nobel prize 1926)

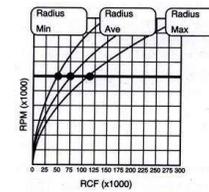


This shows, that mass and density play a crucial role. If the density is identical, then the bigger particle will sediment faster.

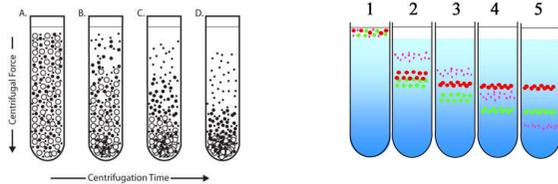
Useful equations

$$\omega = 2\pi \left(\frac{rpm}{60}\right), rpm = \text{revolutions per minute}$$

RCF: relative centrifugal field
 $RCF = a = r\omega^2 = 4\pi^2 rpm^2 / 3600$

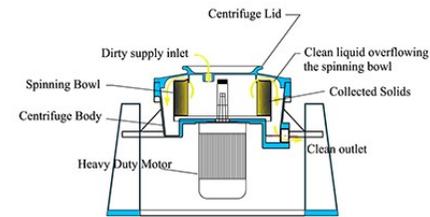


Since the terminal velocities are different, the particles segregate/separate by mass during the process

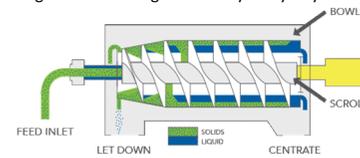


There is always an optimal centrifugation time!
 Too short: no separation
 Too long: every size reaches the bottom, also no separation.

The centrifugal separator



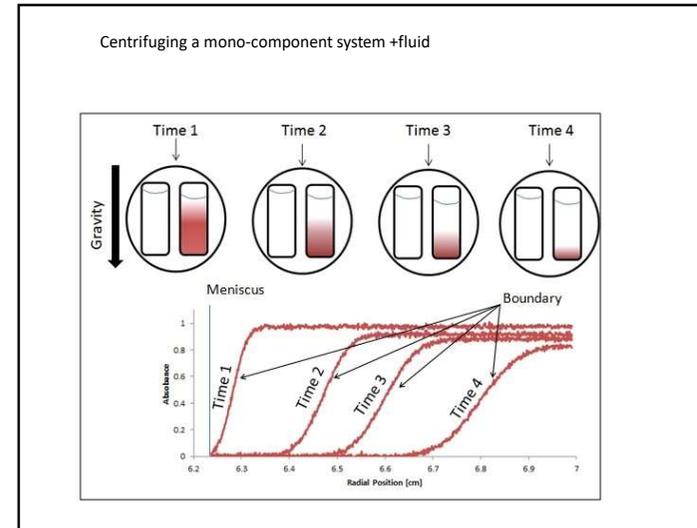
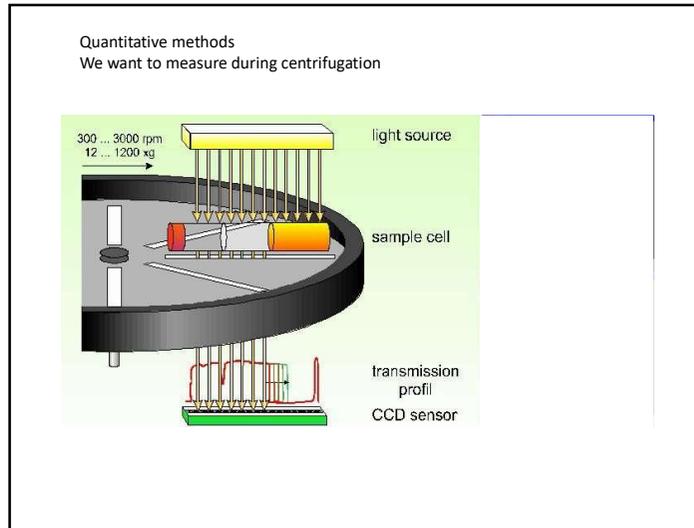
The centrifugal force can be generated by many ways.



Rotating the mixture, or rotating the container, both, etc..

Everyday life:
Fruit centrifuge.





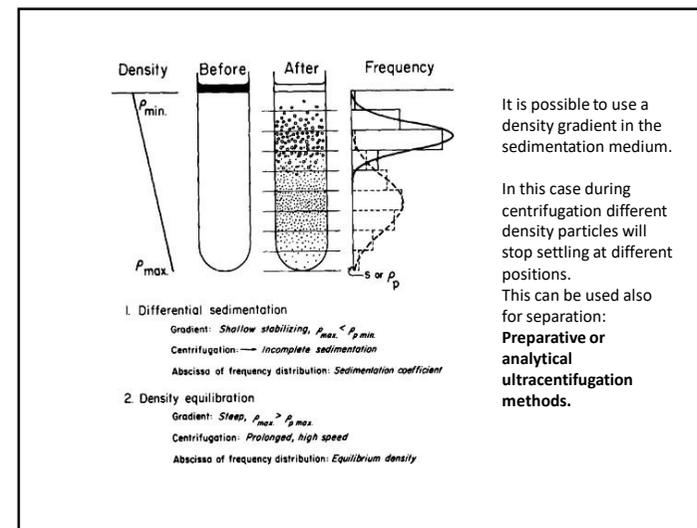
The only remaining unknown is the f : **form-factor**

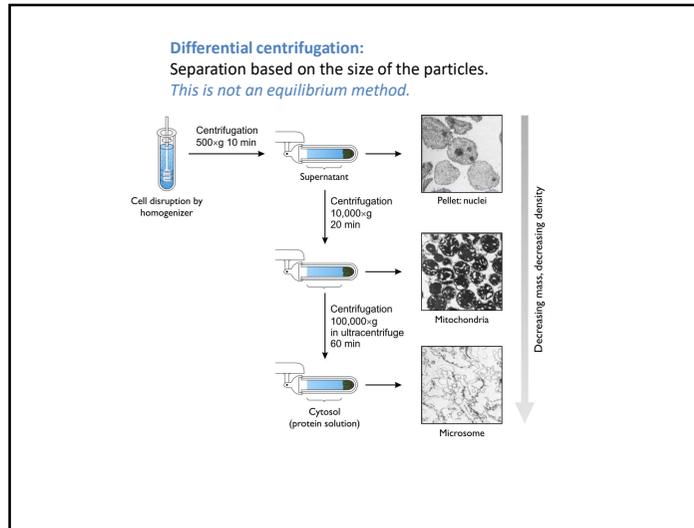
But this is also in the diffusion:

$$f = \frac{kT}{D}$$

where D is the diffusion coefficient.

So we need to measure diffusion, in order to get the particle size.





Sedimentation equilibrium method

Here we wait, until the sedimentation and the Brownian motion reach an equilibrium. (so there will be a concentration profile in the tube)
We spin with medium speed, so there is a sedimentation, but not a complete pellet formation

This means, in equilibrium the net drag force is 0.

In thermal equilibrium, the **Boltzmann distribution** will describe the position of the particles in any force field:

$$\frac{n_1}{n_2} = e^{-\frac{\Delta E}{kT}}$$

In the energy term, we take into account the work of the forces.
If 1 and 2 denote distances r_1 and r_2 from the center of rotation, then

$$\Delta E = \frac{m}{2} (r_1^2 - r_2^2) \omega^2 \left(1 - \frac{\rho_{fluid}}{\rho_{particle}} \right)$$

Extension material

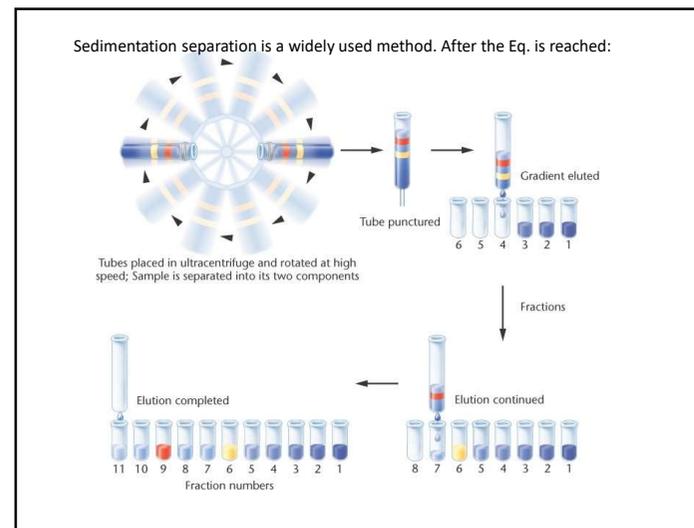
Substituting into the Boltzmann formula and taking the logarithm yields:

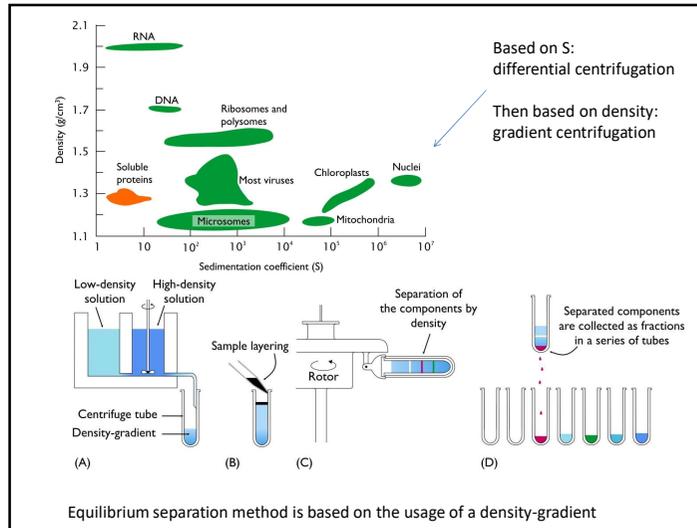
$$\ln \left(\frac{n_1}{n_2} \right) = \frac{m}{2kT} (r_1^2 - r_2^2) \omega^2 \left(1 - \frac{\rho_{fluid}}{\rho_{particle}} \right)$$

We can measure the concentrations (n_1, n_2) the densities, and we know the radii, so the mass can be calculated.

We do need the density, if that is unknown then at least 2 different solvents have to be used, so 2 independent equations can yield the 2 unknowns (m and density)

Extension material



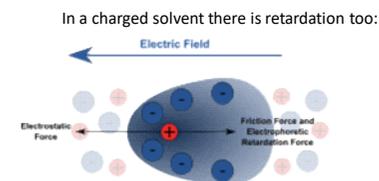
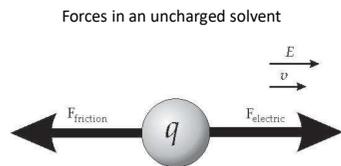


Electrophoretic methods

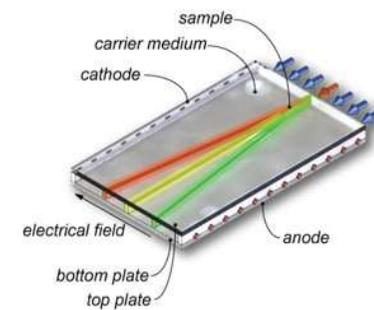
If a molecule is charged, and placed into an electric field, then a force will act on it.
This force (analogous to the sedimentation analysis) will cause a separation of the particles/molecules.
This is not an equilibrium method.

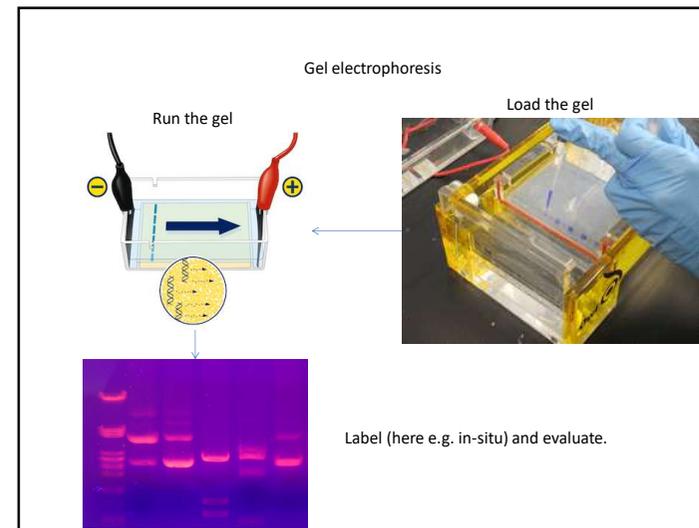
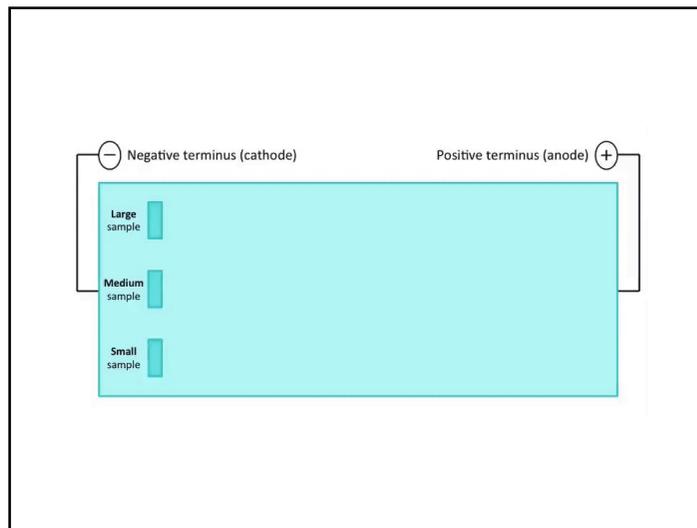
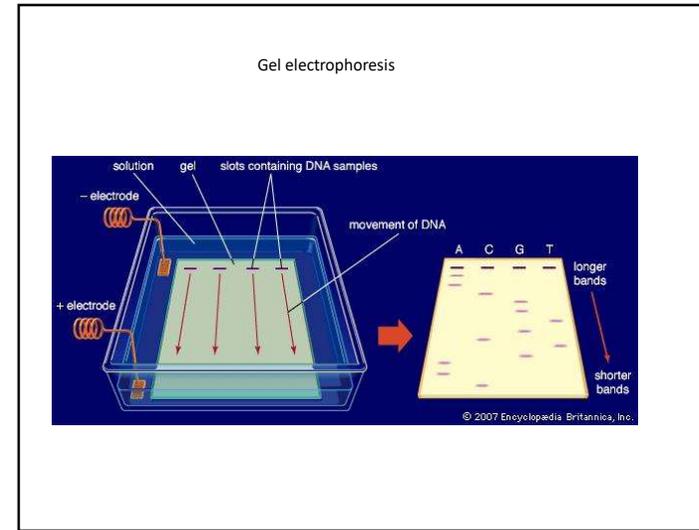
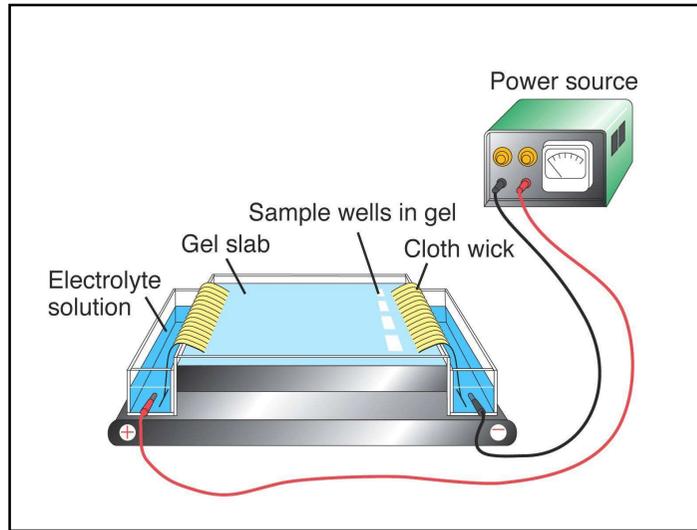
$$\mu_e = \frac{v}{E}$$

The **electrophoretic mobility** is defined by the velocity and the electric field creating that velocity.
This is specific for a given particle.



Free flow electrophoresis





Labeling in a gel is not easy.

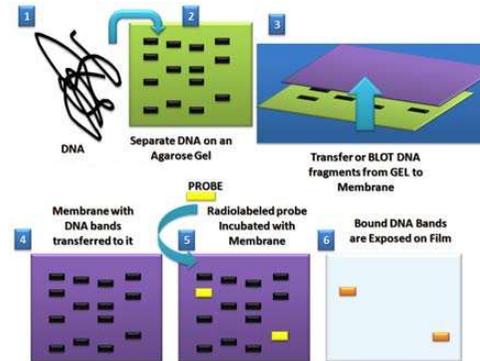
Blotting: one can transfer (and fix) the stripes on the gel onto a vinyl, or other membrane.
Visualization by staining is then done on the membrane.

This enables the use of complicated chemical/biochemical reactions.

Since the membrane has a higher density and viscosity than a gel, the diffusion is much less, so during the chemistry the bands will not "smear" as much.

Blotting is almost always done if the labeling takes considerable time (more than 1-2min)

Southern blot (Edwin Southern)

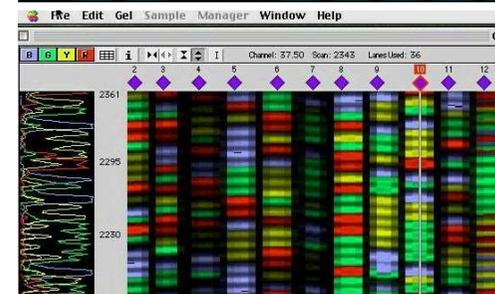
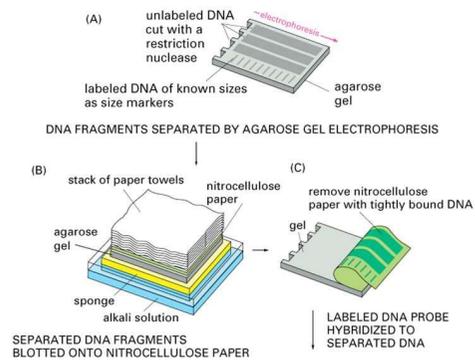


Radiolabeling:
High sensitivity!

Today: fluorescence
versions

Some blotting details...

Southern Blot (DNA)



Fluorescence is
much easier, and
can be
automated...

