Problem Solving

Consider a homogeneous isotropic molecular bar of circular cross-section, 20 nm in diameter, 2 micrometers long, with a Young's modulus of 1 GPa.

What is its bending spring constant?

$E = 10^9 \frac{N}{m^2}$

$L = 2 \cdot 10^{-6} m$

$D = 20 \cdot 10^{-9} m$

$R = 10 \cdot 10^{-9} m$

$\kappa_b = ?$

$$\kappa_b = \frac{4\pi}{3} \cdot \frac{E \cdot R^4}{L^3} =$$

$$= \frac{4\pi}{3} \cdot \frac{10^9 \frac{N}{m^2} \cdot (10 \cdot 10^{-9} m)^4}{(2 \cdot 10^{-6} m)^3} =$$

$$= \frac{\pi}{6} \cdot \frac{10^{-5} N}{m} =$$

$$= 0.52 \cdot 10^{-5} N/m = 0.52 \cdot 10^{-2} pN/nm$$

Consider a homogeneous isotropic molecular bar of circular cross-section, 20 nm in diameter, 2 micrometers long, with a Young's modulus of 1 GPa.

What is its tension spring constant?

$E = 10^9 \frac{N}{m^2}$

$L = 2 \cdot 10^{-6} m$

$D = 20 \cdot 10^{-9} m$

$R = 10 \cdot 10^{-9} m$

$\kappa = ?$

$$\kappa = \frac{E \cdot A}{L} =$$

$$= \frac{E \cdot \pi R^2}{L} =$$

$$= \frac{10^9 \frac{N}{m^2} \cdot \pi \cdot (10 \cdot 10^{-9} m)^2}{2 \cdot 10^{-6} m} =$$

$$= \frac{\pi}{2} \cdot \frac{10^{-1} N}{m} = 157\ pN/nm$$

Consider a homogeneous isotropic molecular bar of circular cross-section, 20 nm in diameter, 2 micrometers long, with a Young's modulus of 1 GPa.

What is the persistence length at 27 °C?

$E = 10^9 \frac{N}{m^2}$

$L = 2 \cdot 10^{-6} m$

$D = 20 \cdot 10^{-9} m$

$t = 27\ °C$

$R = 10 \cdot 10^{-9} m$

$T = 300\ K$

$L_P = ?$

$$L_p = \frac{E \cdot I}{k_B \cdot T} \quad ; \quad I = \frac{\pi R^4}{4}$$

$$L_p = \frac{10^9 \frac{N}{m^2} \cdot \frac{\pi}{4} \cdot (10 \cdot 10^{-9} m)^4}{1.38 \cdot 10^{-23} \frac{Nm}{K} \cdot 300 K} =$$

$$= \frac{\pi \cdot 10^{-2} m}{1.38 \cdot 12} = 1.9 \cdot 10^{-3} m$$

What is the smallest resolvable distance in a microscope that uses 520 nm light for imaging and has an aperture angle of 140 °?

$\lambda = 520\ nm$

$2\omega = 140\ °$

$\omega = 70\ °$

$\delta = ?$

$$\delta = 0.61 \cdot \frac{\lambda}{n \cdot \sin\omega} =$$

$$= 0.61 \cdot \frac{520\ nm}{1 \cdot \sin 70\ °} =$$

$$= 0.61 \cdot \frac{520\ nm}{0.94} = 337\ nm$$

How does the minimum distance that can be resolved by a microscope change if an immersion oil with a refractive index n = 1.5 is used?

$n = 1.5$

δ *without immersion oil*

δ_n *with immersion oil*

$\delta_n\ \ ?\ \ \delta$

$$\delta_n = 0.61 \cdot \frac{\lambda}{n \cdot \sin\omega} = \frac{\delta}{n} = \frac{\delta}{1.5}$$

What is the minimum resolvable distance in a microscope that uses 420 nm light for imaging and has a numerical aperture of 1.4?

$\lambda = 420\ nm$

$NA = 1.4$

$\delta = ?$

$$\delta = 0.61 \cdot \frac{\lambda}{NA} =$$

$$= 0.61 \cdot \frac{420\ nm}{1.4} =$$

$$= 0.61 \cdot 300\ nm = 183\ nm$$