Physical Foundations of Dental Materials Science

7.

Mechanical properties of materials 1. Elasticity

Keywords:
- 1 Stress-strain
- 2 Elasticity
- 3 Deformations

E-book chapter 14, 15.

Problems: 4.1, 4.3, 4.4, 4.5, 4.6, 4.9, 4.11, 4.16, 4.17, 4.18, 4.21, 4.22, 4.23, 4.24.

1

1

Mechanical propeties of materials

Stiffness, elasticity, toughness, hardness are the most importamt parameters when considering mastication, implantology or orthodontics.

Mechanical stress (force) is applied on material.

↓

The consequence is usuallay some kind of deformation.
(change in shape depends on the direction and point of application)

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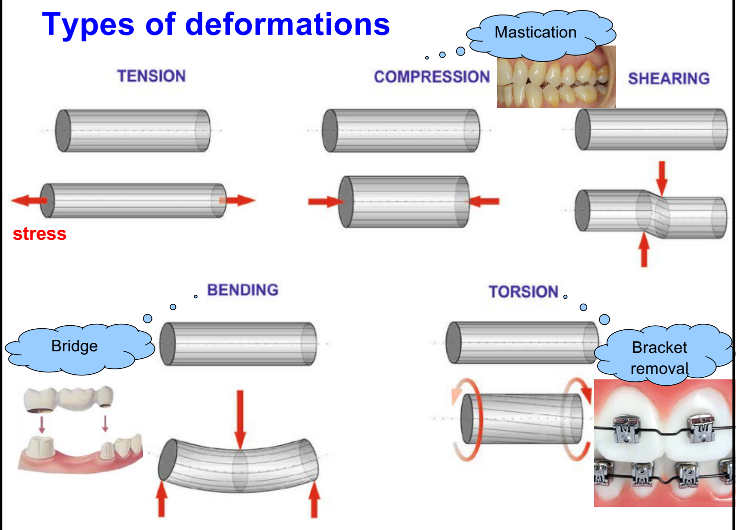
Depending on the materials mechanical properties, deformation may be reversible or irreversible.

2

Types of deformations



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elastic *adjective* able to resume its normal shape spontaneously after being stretched or compressed

rigid *adjective* unable to bend or be forced out of shape; not flexible

plastic *adjective* relating to the permanent deformation of a solid without fracture by the temporary application of force

strong *adjective* able to withstand force, pressure, or wear

weak *adjective* liable to break or give way under pressure; easily damaged

solid *adjective* firm and stable in shape; not liquid or fluid, strongly built or made of strong materials

tough *adjective* resistant to fractue or breaking

brittle *adjective* liable to break easily

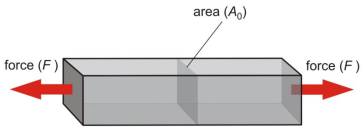
4

4

Basic consepts

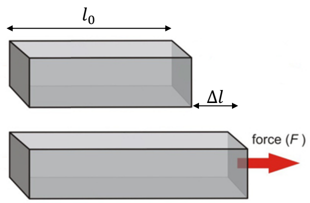
Stress

$\sigma = \frac{F}{A_0}$ Dimenson: $\left[\frac{N}{m^2} = Pa\right]$



Strain (deformation)

$\varepsilon = \frac{\Delta l}{l_0}$ Dimensionless: $\left[\frac{m}{m}\right]$



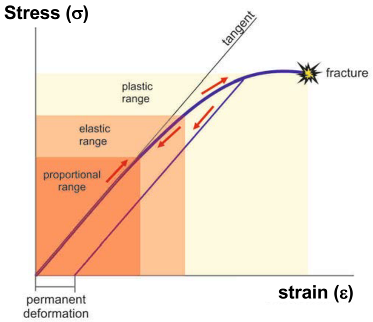
Strain is proprtional to stress!

$\sigma \backsim \varepsilon$

5

5

Stress-strain diagram



Elastic range
Range of reversible deformation. Unloaded length (l_0) recovers when released. Hysteresis may occour.

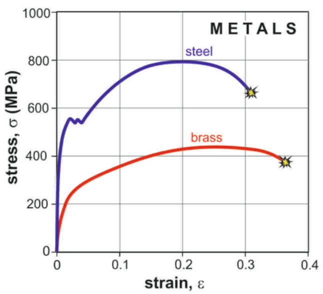
Proportional range
(part of elastic range)
Deformation is linearly proportional to the load. No hysteresis. *(see Hooke's law: Biophysics, Rasonance lab)*

Plastic range
After a critical stress value, object undergoes irreversible change of its structure. Unloaded length (l_0) does not recover. Permanent deformation of object.

Fracture
Desintegration of object.

6

Typical stress-strain curves
Metals



Short elastic range

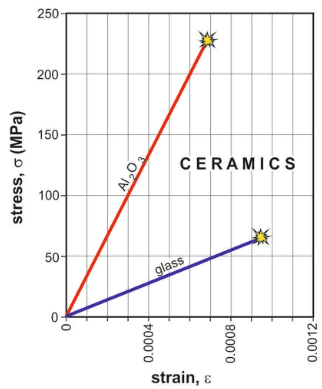
Serious plastic range

Metals are highly deformable
ε_{max}~0.3 (~30%)

7

7

Typical stress-strain curves
Ceramics



Extremly short elastic range
ε_{max}<0.001 (less than ~0.1%)

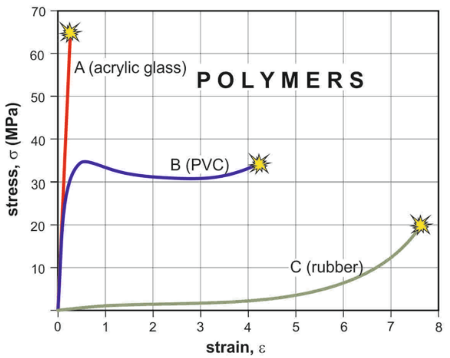
No plastic range!

Ceramics are brittle, they rupture after a short extension/compression….

8

8

Typical stress-strain curves
Polymers



Polymers show variouis behaviours

Typically they are deformable ε_{max} from 0.1 to 8

Acrylic glass
No plastic range - brittle

PVC
Large plastic range
Plastic polymer

Rubber
Elastic polymer
ε_{max} up to 7-8 (~700-800%)

9

9

Additional factors determining the stress-strain diagram

The main decisive is the material's structure (e.g. amorphous or crystalline)

1. Type of stress. (For example the behavior of ceramics is different in the case of tension or elongation.)

2. The shape, size commonly the geometry of the object. (It is more difficult to bend a thicker rod than a thin.)

3. The run-time of the stress: *stress rate*. (For example the glass is broken into pieces using a hammer but a bullet launched by a gun produces a small hole only.)
Mechanical properties – introduction

4. Temperature. (Some metals are very plastic and tough at room temperature but become more brittle cooling them.)
These conditions must be presented in the stress-strain diagram.

10

10

Hooke's law of elaticity

Strain is proprtional to stress!

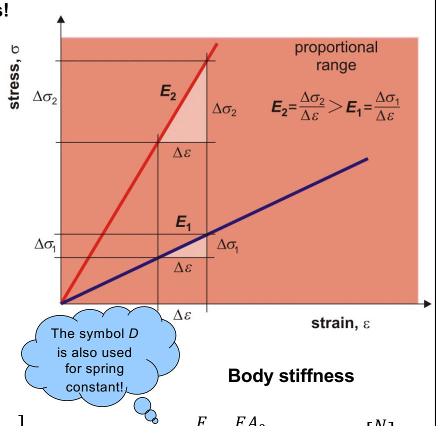
$$\sigma \backsim \varepsilon$$

$$\sigma = E\varepsilon$$

$$\frac{F}{A_0} = E\frac{\Delta l}{l_0}$$

$$F = \frac{EA_0}{l_0}\Delta l$$

$$F = k\Delta l$$



Proportionality coeffitients:

Young's modulus
(material stiffness)

$$E = \frac{\sigma}{\varepsilon} = \frac{F}{A_0}\frac{l_0}{\Delta l} \qquad E = \left[\frac{N}{m^2} = Pa\right]$$

Body stiffness

$$k = \frac{F}{\Delta l} = \frac{EA_0}{l_0} \qquad k = \left[\frac{N}{m}\right]$$

11

11

Quantification of stiffness

Young's modulus of various materials

GPa = 10^9 N/m^2

Spring constant
Easy to measure
Depends on size and shape of object

Young's modulus
Hard to measure (we need to know the object's dimensions)
Shape independent

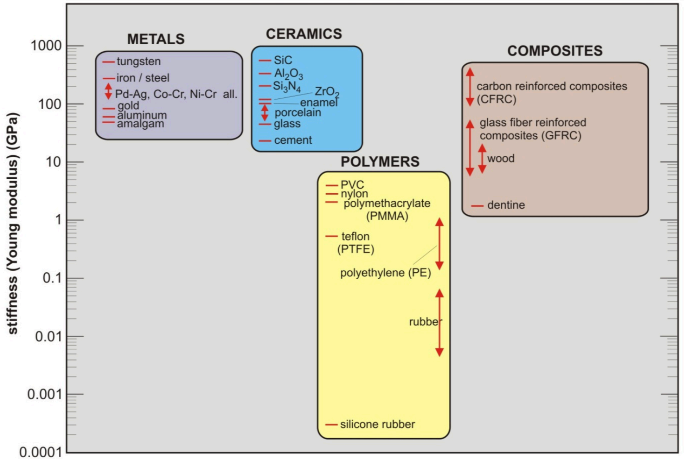
1/E – reciprocal of E is used to express compliance!

material	E (GPa)
dentine	≈ 15
enamel	≈ 100
silicon rubber	≈ 0.0003
steel	200-230
amalgam	50-60
glass	60-90
porcelain	60-110
gold	79
gold alloys	75-110
Pd-Ag alloys	100-120
titanium	110
titanium alloys	105-120
Co-Cr alloys	120-220
Ni-Cr alloys	140-190

12

12

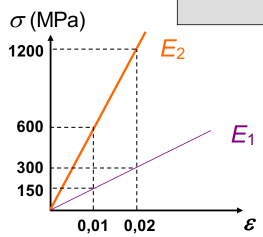
Stiffness

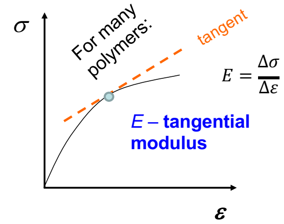


13

13

Example:





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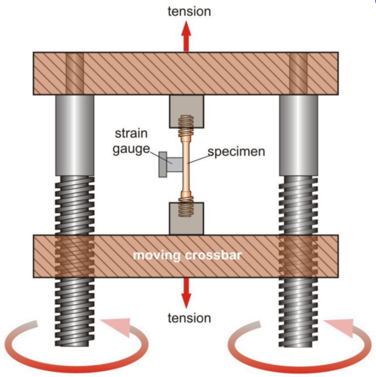
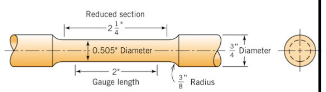
Testing



The result of shuch measurement is determined by:

- Type of stress (tension, ...)
- sample geometry



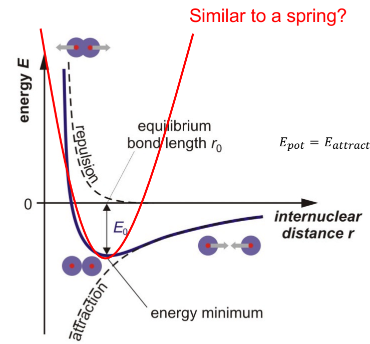
- Timecourse of stress
  - constant
  - changing
- temperature

15

15

Repetition

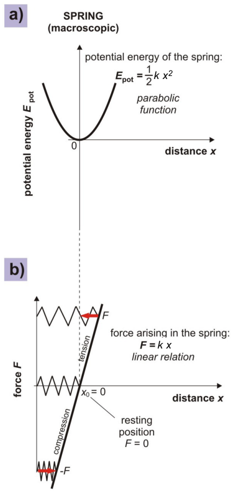
Potential energy of chemical bonds (Medical biophysics I/2)



$$E_{pot} = E_{attraction} + E_{repulsion} = -\frac{A}{r^n} + \frac{B}{r^m}$$

16

16



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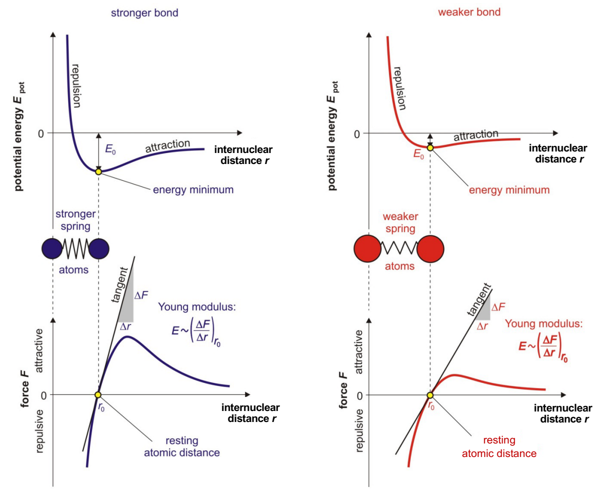

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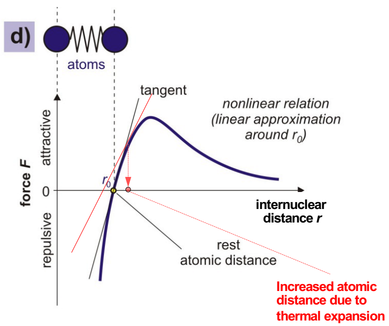
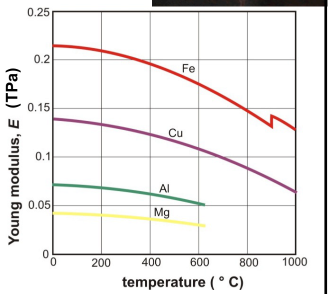
18

Young's modulus

Young's modulus decreases with temprature increasing



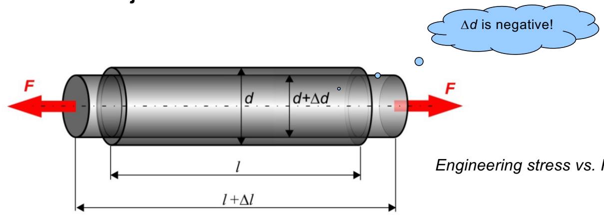


19

19

Changes in the cross-sectional dimension

Stretched objects become narrower or… ?



Engineering stress vs. Real stress

$$\frac{\Delta d}{d} = -\mu \frac{\Delta l}{l}$$

μ: Poisson's ratio (no dimension)
μ_{max} = 0.5 (*usually between 0.3-0.4*)

Auxetic materials have negative Poisson's number: They become thicker when stretched. (bullet proof vests, compression bandages)

http://www.youtube.com/watch?v=PLDbSWSm5i8

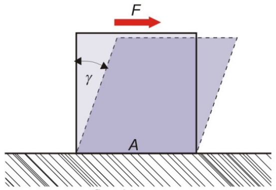
20

20

Deformations in other directions

Compression
Same as stretch, but signs of stress and strain are opposite, Hooke's law is valid.

Shear
Force is parallel to the surface,
Shear stress: $\sigma_{shear} = \frac{F_{shear}}{A}$
Strain is characterized by the γ angle



$\sigma_{shear} = G\gamma$ — *G: Shear modulus*, *γ: in radians!*

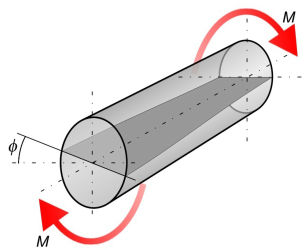
$G = \frac{E}{2(1+\mu)}$ — *Depends on Young's modulus and Poisson's ratio!*

21

21

Deformations in other directions

Torsion
Can be derived from shear. Torque (M) is applied on the object.
Angle of torsion (ϕ) is measured in radians!



$$M = G\frac{r^4\pi}{2l}\phi$$

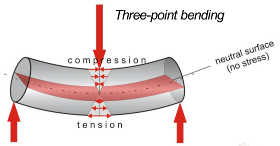
G: Shear modulus (contains E and μ)

r: radius
ϕ: angle of torsion
l: length

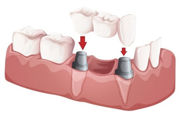
22

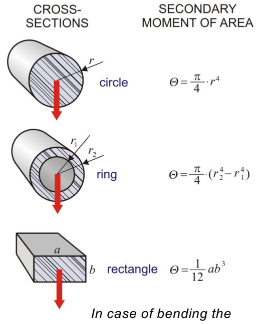
22

Deformations in other directions



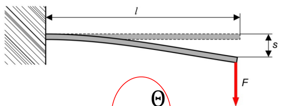
Bending
Compression on the side of force applied, tension on the opposite side of object. Neutral surface at the middle – does not deform





Cross-section	Secondary moment of area
circle	$\Theta = \frac{\pi}{4} \cdot r^4$
ring	$\Theta = \frac{\pi}{4} \cdot (r_2^4 - r_1^4)$
rectangle	$\Theta = \frac{1}{12} ab^3$

Cantilever deflection



$$F = 3E\frac{\Theta}{l^3}s$$

Body stiffness in case of deflection

G: Force
E: Young's modulus
Θ : secondary moment of area
s: deflection
l: length

In case of bending the force required depends stronger on the shape!

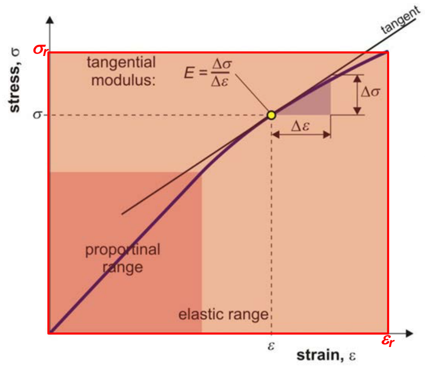
23

23

Elasticity beyond the proportional range

Elastic limit
End of the elastic range on the y-axis (σ_r) – maximum reversible strain

Elastic strain recovery
End of the elastic range on the y-axis (ε_r) – maximum reversible stress



Tangential Young's modulus

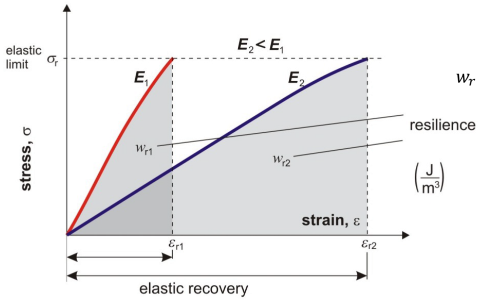
$$E = \frac{\Delta\sigma}{\Delta\varepsilon}$$

24

24

Resilience

Work done on the material per unit volume till the elastic limit.



$$w_r \approx \frac{1}{2}\sigma_r\ \varepsilon_r = \frac{1}{2}E\varepsilon_r^2 = \frac{1}{2E}\sigma_r^2$$

$$\sigma_r = E\ \varepsilon_r$$

$$w_r = \left[\frac{N}{m^2} = \frac{Nm}{m^3} = \frac{J}{m^3}\right]$$

25

25

Problems: 4.1, 4.3, 4.4, 4.5, 4.6, 4.9, 4.11, 4.16, 4.17, 4.18, 4.21, 4.22, 4.23, 4.24.

26

26