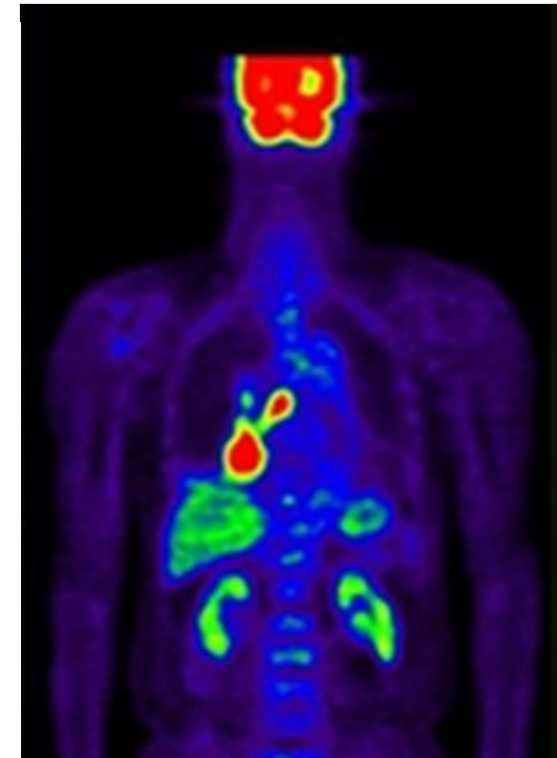
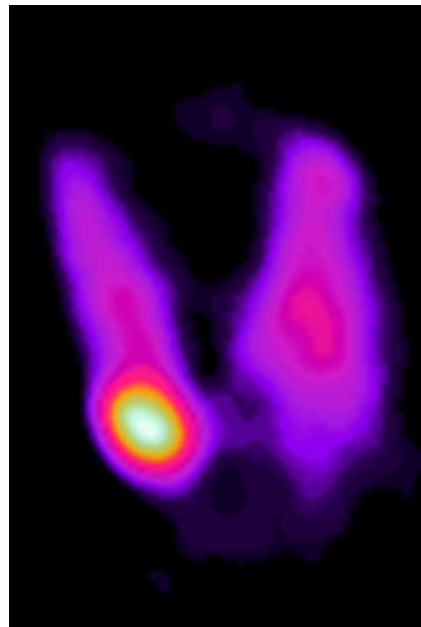
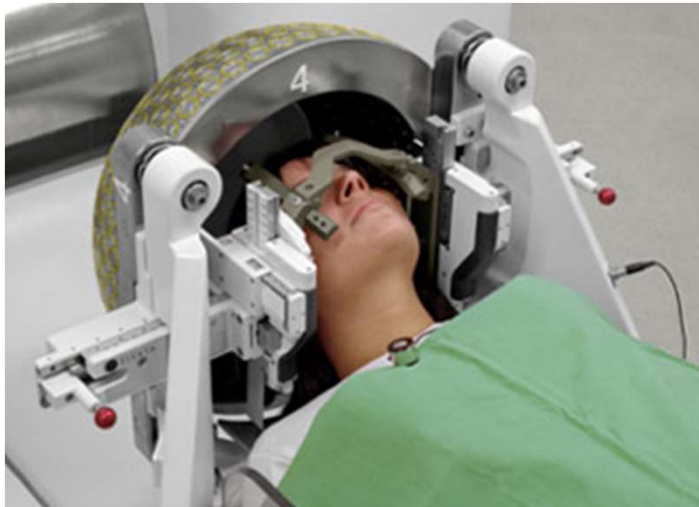


# The atomic nucleus. Radioactivity. Nuclear radiations

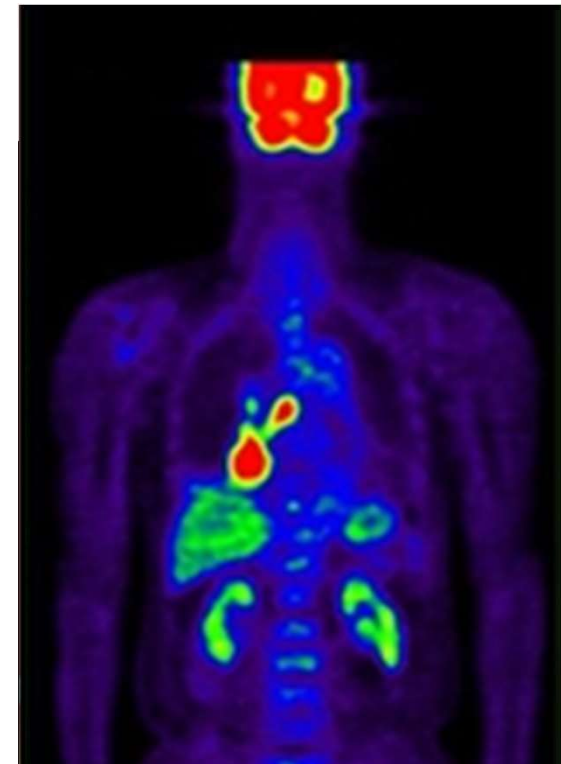
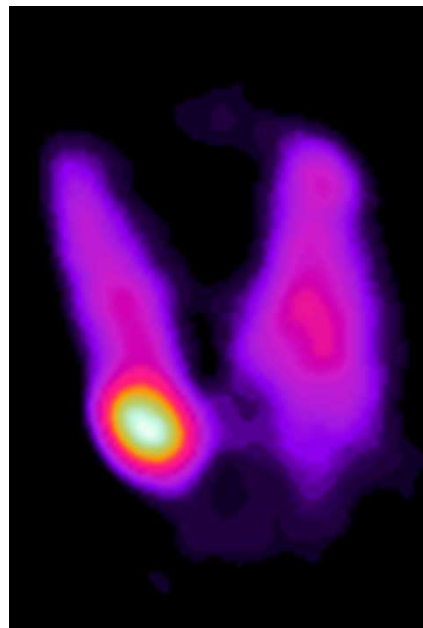
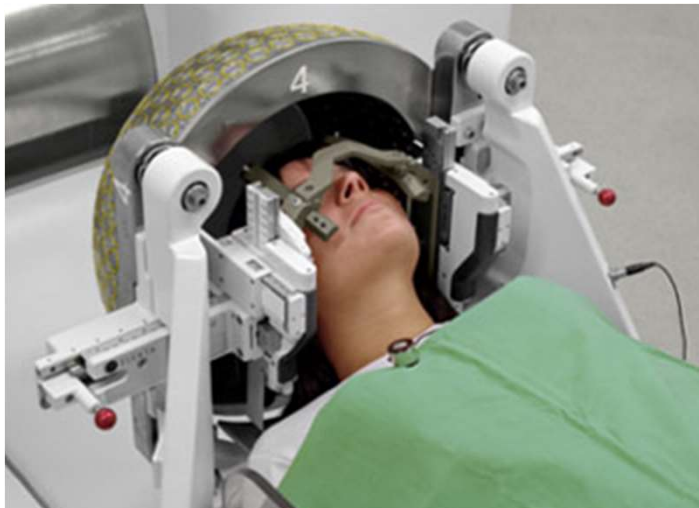
László Smeller






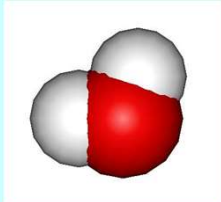

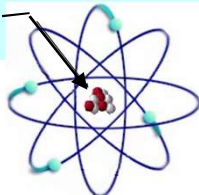
# Why?

Medical applications of the nuclear radiation:

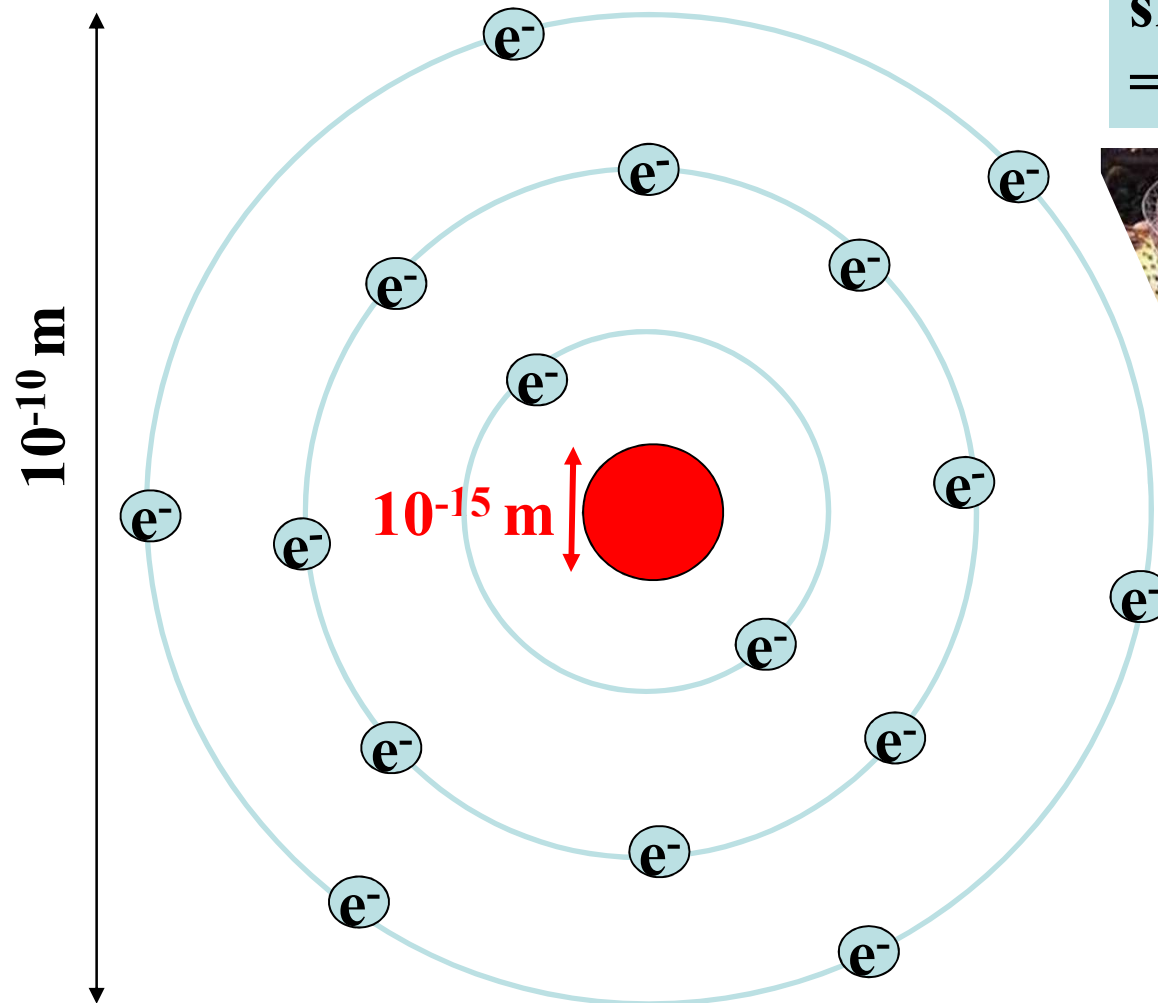
- Nuclear imaging
- Radiotherapy



# Length scale of the nature

m		
$10^0$	meter	men 
$10^{-3}$	millimeter	size of letters you can read
$10^{-6}$	micrometer	size of a cell (e.g. erythrocyte)  $\varnothing 7\mu\text{m}$
$10^{-9}$	nanometer	protein 
$10^{-10}$	– angstrom	diameter of an atom, bond length H atom $\varnothing \approx 1$ angstrom (Å) 
$10^{-12}$	picometer	wavelength of the X-ray 
$10^{-15}$	femtometer	size of the nucleus 

# The electrons and the nucleus

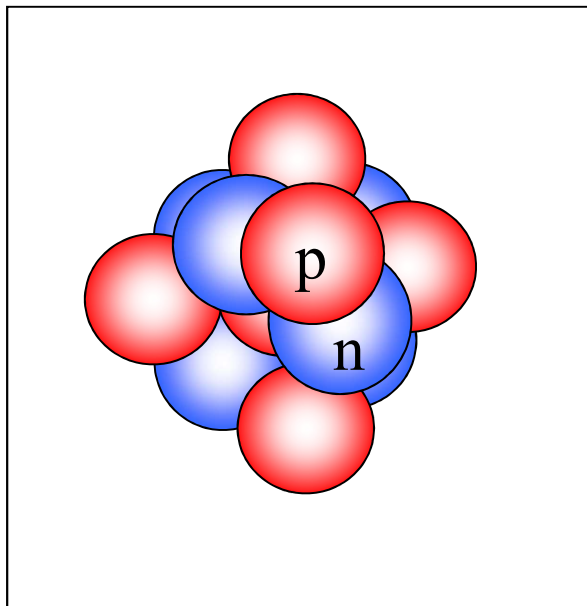


**Changes in the electron  
shell:  
=> chemical processes**



**Changes of the  
nucleus:  
=> radioactivity**

# Structure of the nucleus

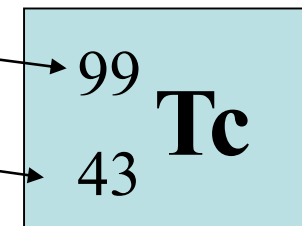


Elementary charge =  $1.6 \cdot 10^{-19} \text{ C}$

	charge	mass
<b>proton</b>	<b>+1 e</b>	<b>1 atomic mass unit</b>
<b>neutron</b>	<b>0</b>	<b>1 atomic mass unit</b>

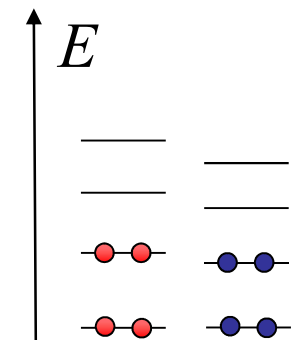
$A$  (mass number) = number of protons  
 + number of neutrons  
 $Z$  (atomic number) = number of protons

99 nucleon, 43 proton and 56 neutron



# Stability of the nucleus

- Coulomb force: destabilization  
(electrostatic repulsion between the protons)
- Nuclear force: very strong  
attractive force  
acts only on short range ( $\sim \text{fm}$ )  
independent on the charge
- Quantized energy levels for the nucleus.
- Typical binding energy is in the MeV range  
 $\text{eV} = 1.6 \cdot 10^{-19} \text{ J}$



# Isotopes

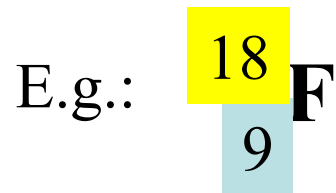
Variants of an element, which differ in  
- number of neutrons  
=> mass number

Remark:

The numbers of protons (=atomic numbers) are the same.



**György Hevesy**  
Nobel prize 1943



unstable  
(radioactive)



stable

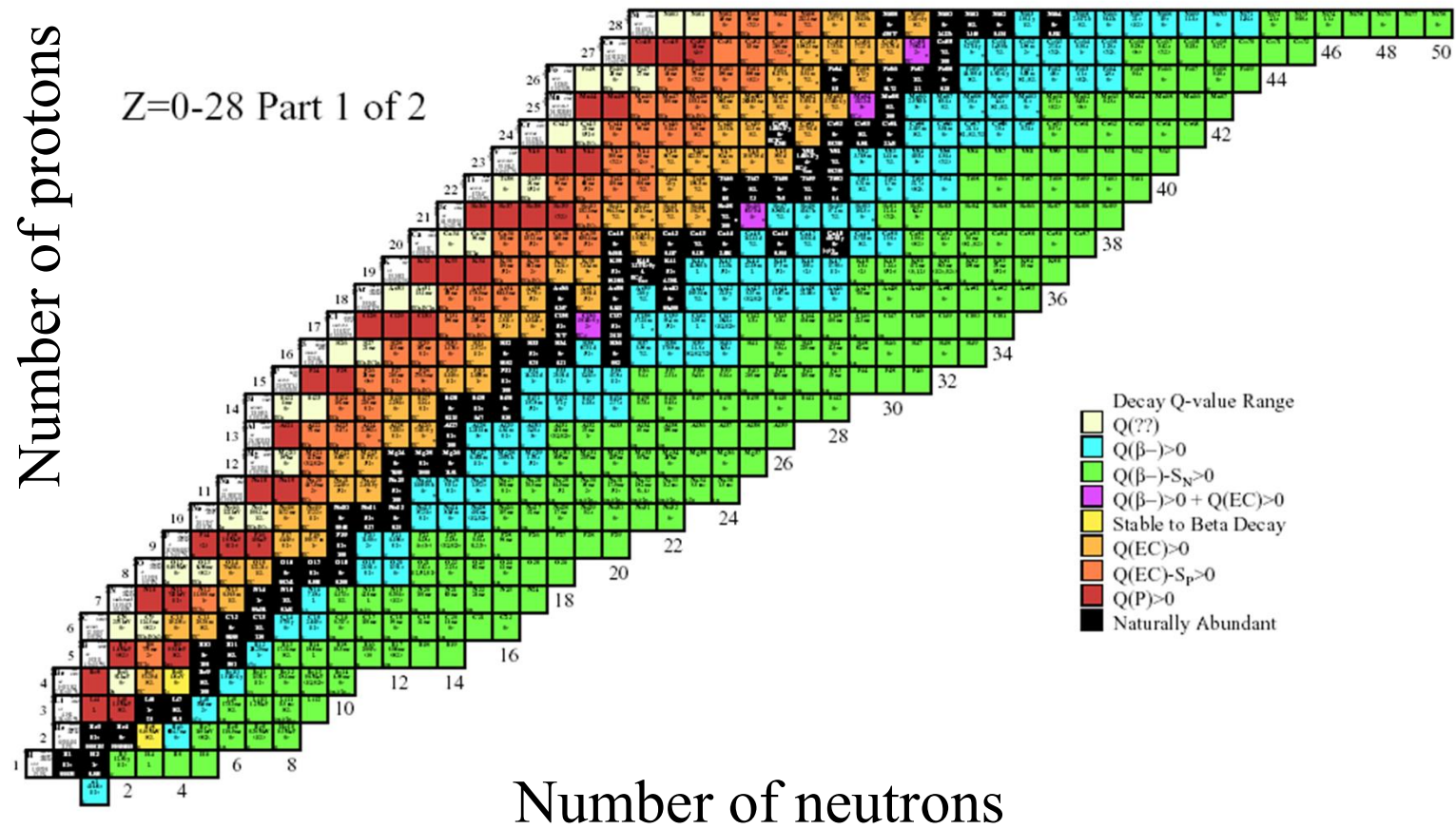


unstable  
(radioactive)

isotope  $\leftrightarrow$  radioactive isotope

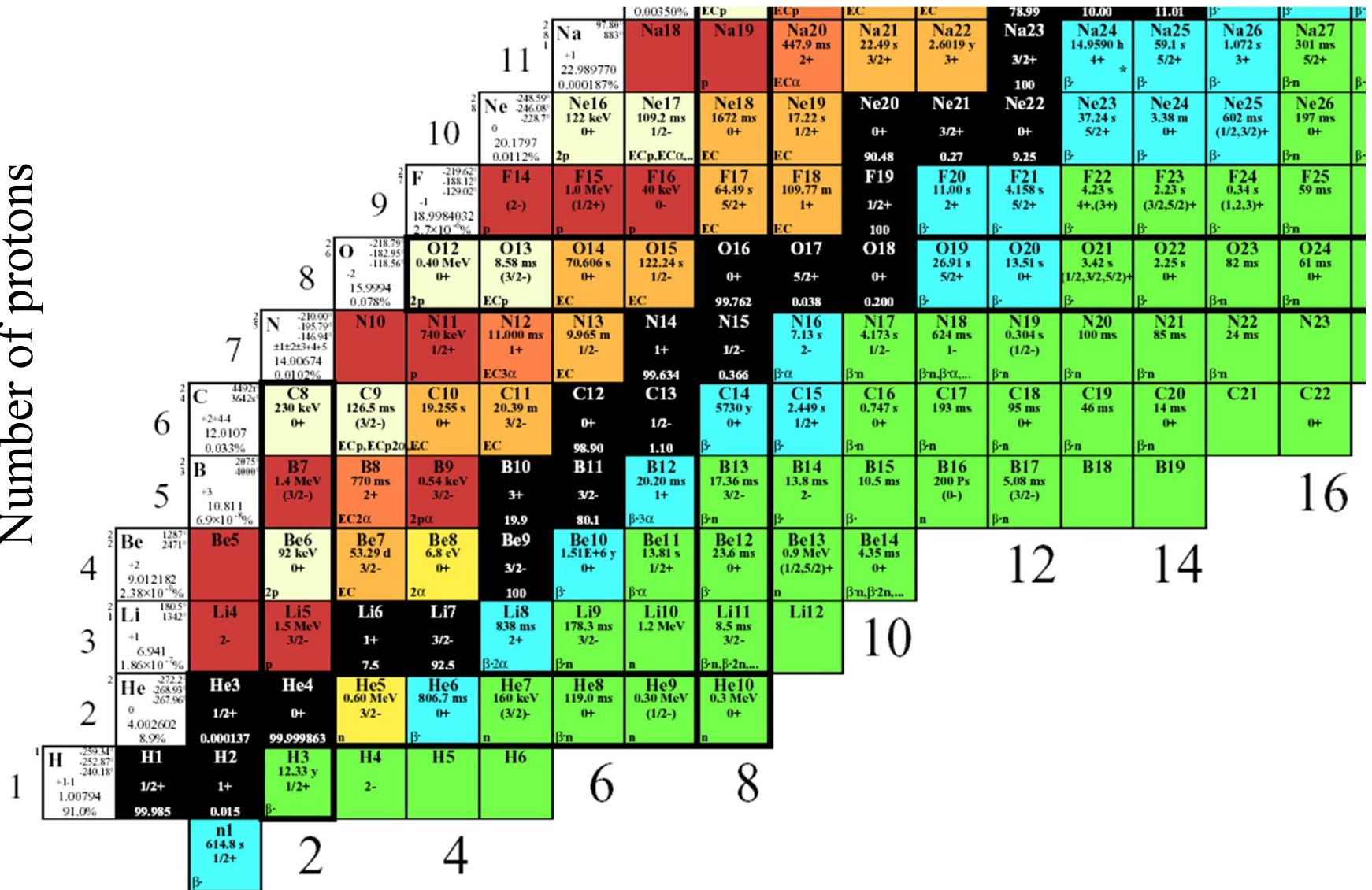


# Table of isotopes





Number of protons



Number of neutrons

# Radioactive decays and particles

$\alpha$  - decay

$\alpha$  - particle =  ${}^4_2\text{He}$  nucleus

$\beta$  - decay :  $\beta^-$   
 $\beta^+$

$\beta^-$  particle = electron

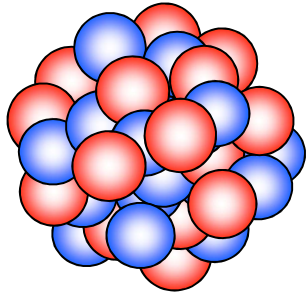
$\beta^+$  particle = positron

Isomeric transition

$\gamma$ -ray

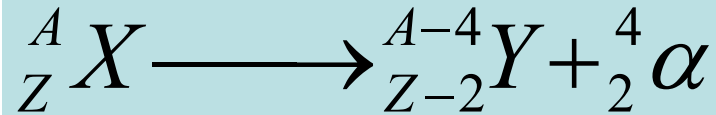
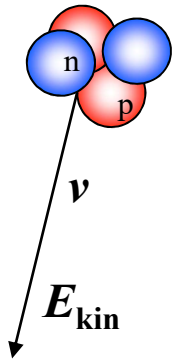
K-electron capture

characteristic x-ray photon



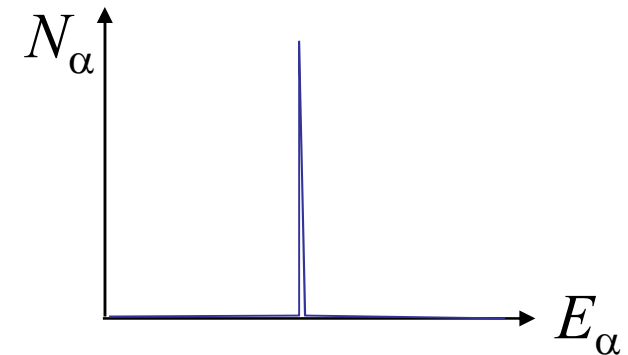
# $\alpha$ - decay

an  $\alpha$  particle ( ${}^4\text{He}$  nucleus) will be emitted  
typical for the heavy atoms

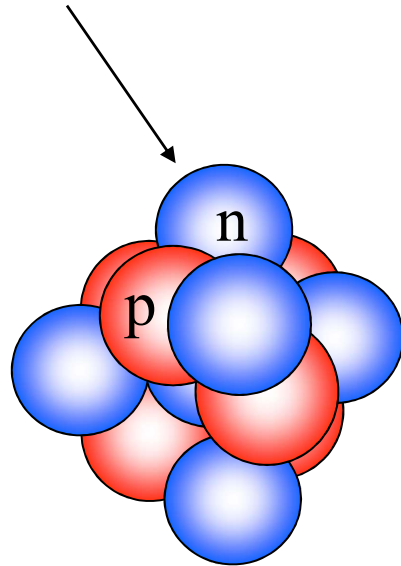


Line spectrum

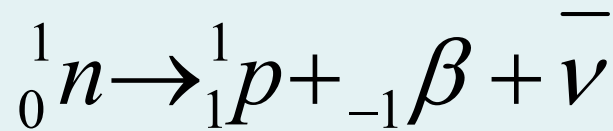
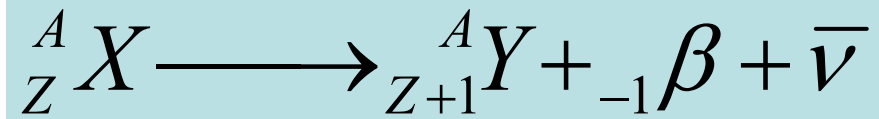
$E_\alpha \sim \text{MeV}$



neutron surplus

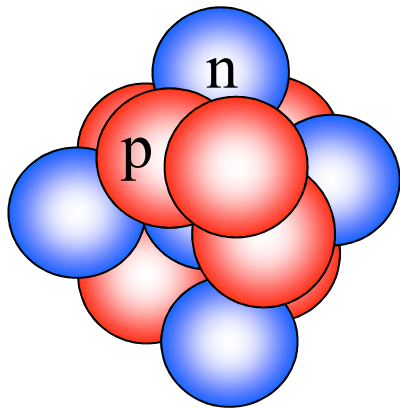


# $\beta^-$ - decay



remains in  
the nucleus

leave the  
nucleus

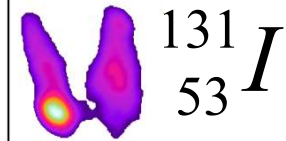
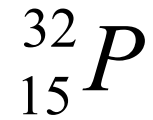
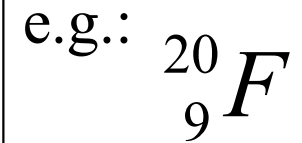
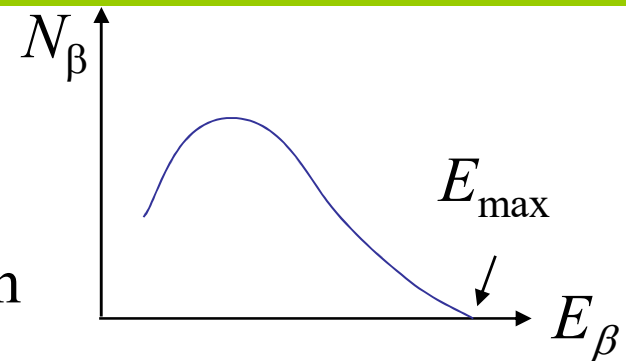


$\beta^-$ -ray

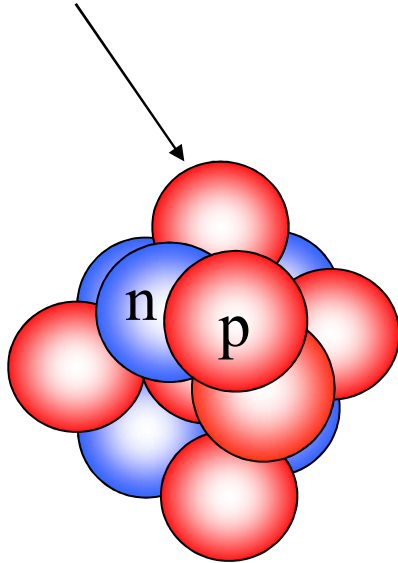


continuous  
energy spectrum

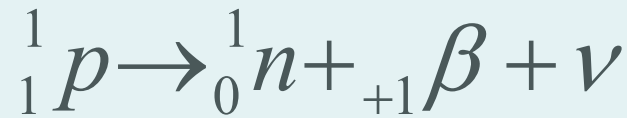
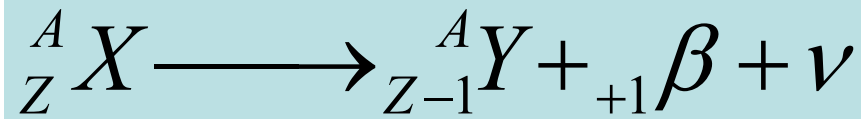
$$\beta^- = {}_{-1}\beta = e^-$$



proton surplus

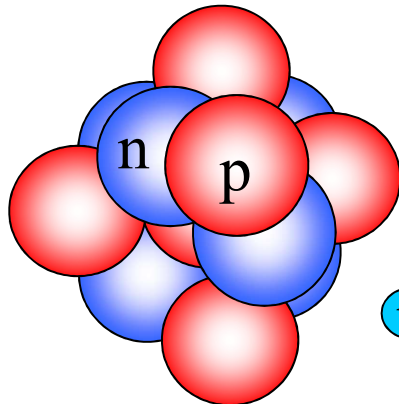


# $\beta^+$ - decay

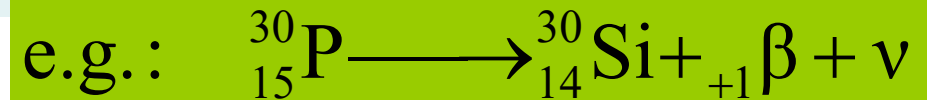


remains in  
the nucleus

leave the  
nucleus



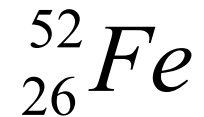
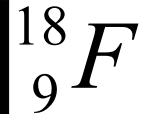
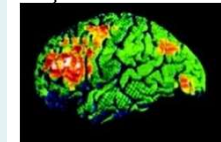
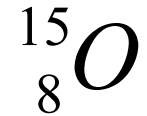
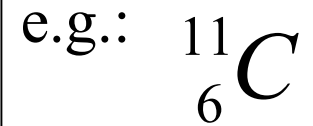
$\beta^+$ -ray

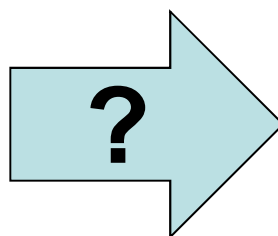


continous energy spectrum

These isotopes must be produced  
artificially (e.g. in cyclotron)

**Medial application: PET**



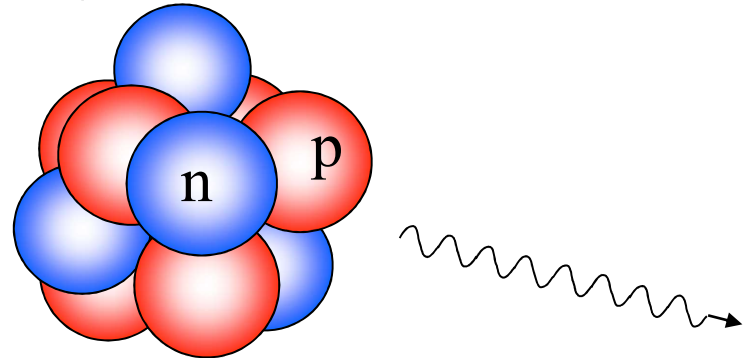
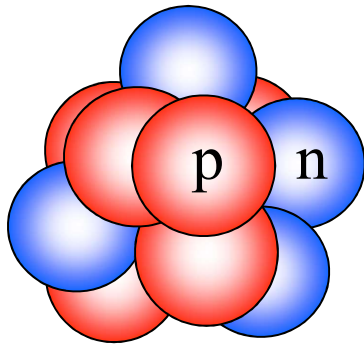
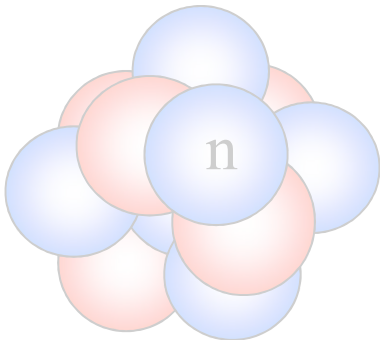
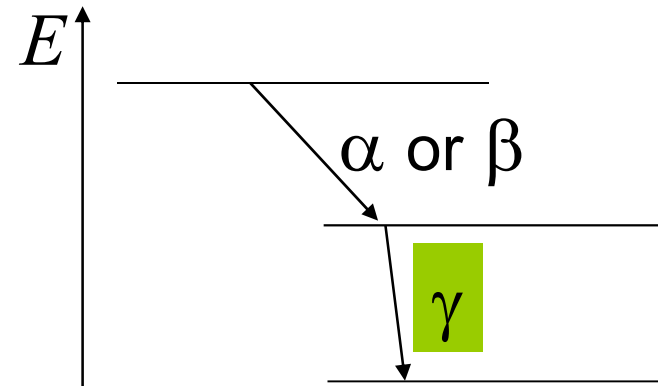




# Prompt $\gamma$ -radiation

The daughter nucleus might have an **energetically unfavoured** arrangement of nucleons.  
(excited state)

The surplus energy will normally be emitted immediately ( $< \text{ps}$ ) in form of the  $\gamma$  radiation



Atomic number, mass number are unchanged.

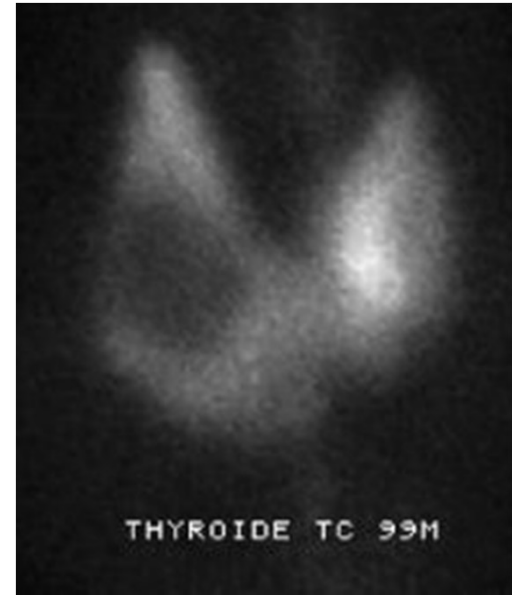


# Isomeric transition

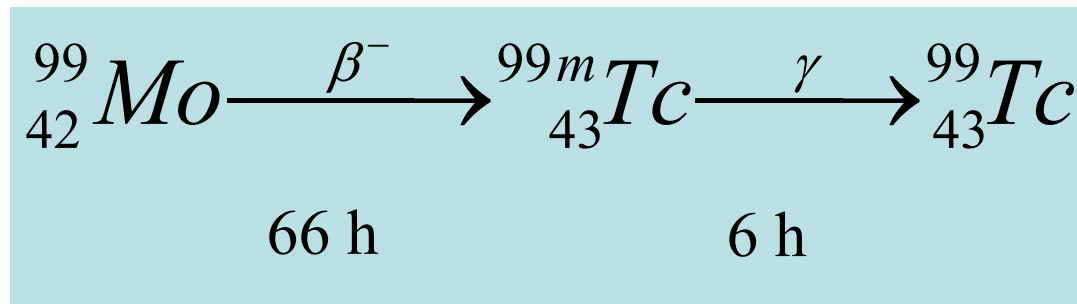
In some rare cases the excited state of the daughter nucleus is metastable, the  $\gamma$ -radiation will be emitted later.

The parent and daughter atoms can be separated: the daughter atom emits **only  $\gamma$ -radiation!**

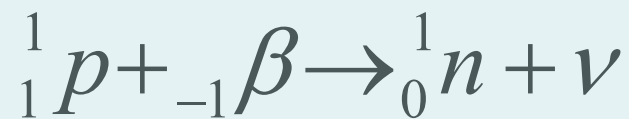
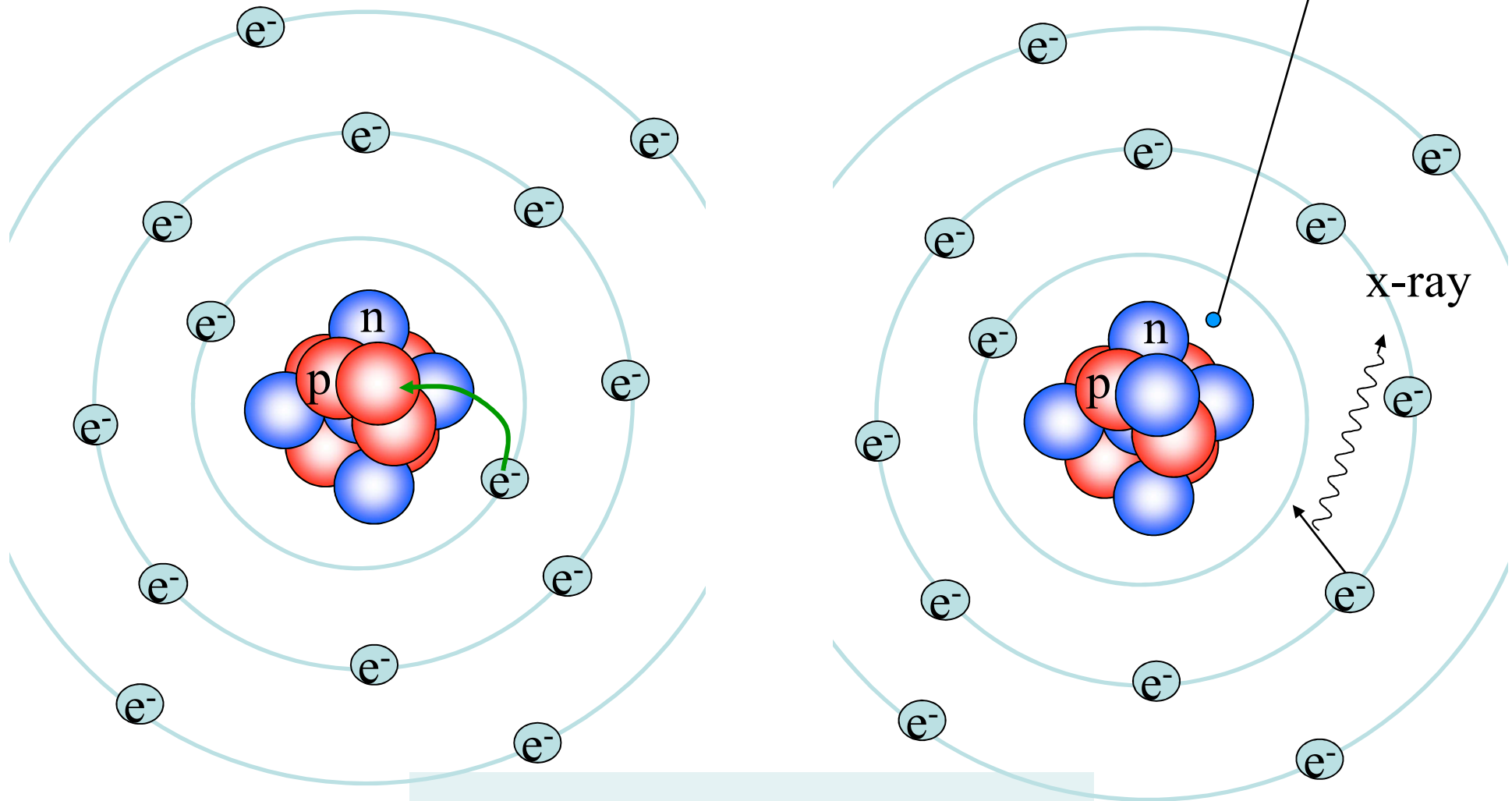
=> **Isotope diagnostics (nuclear imaging)**



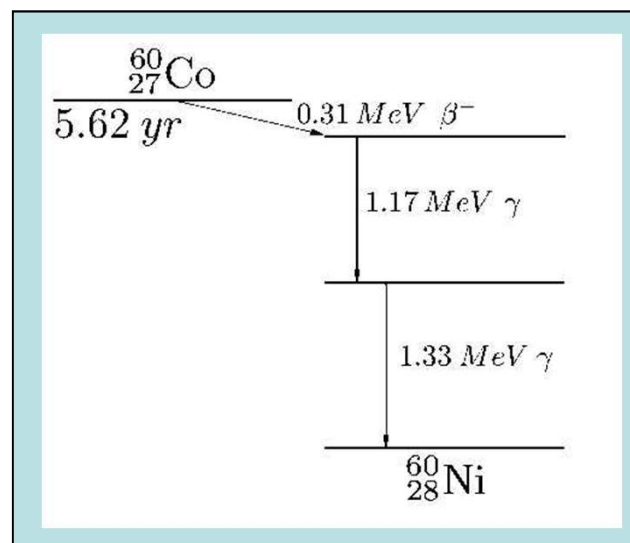
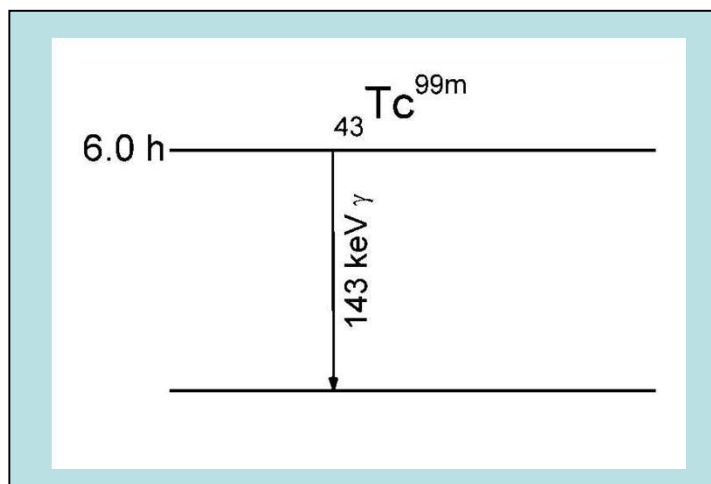
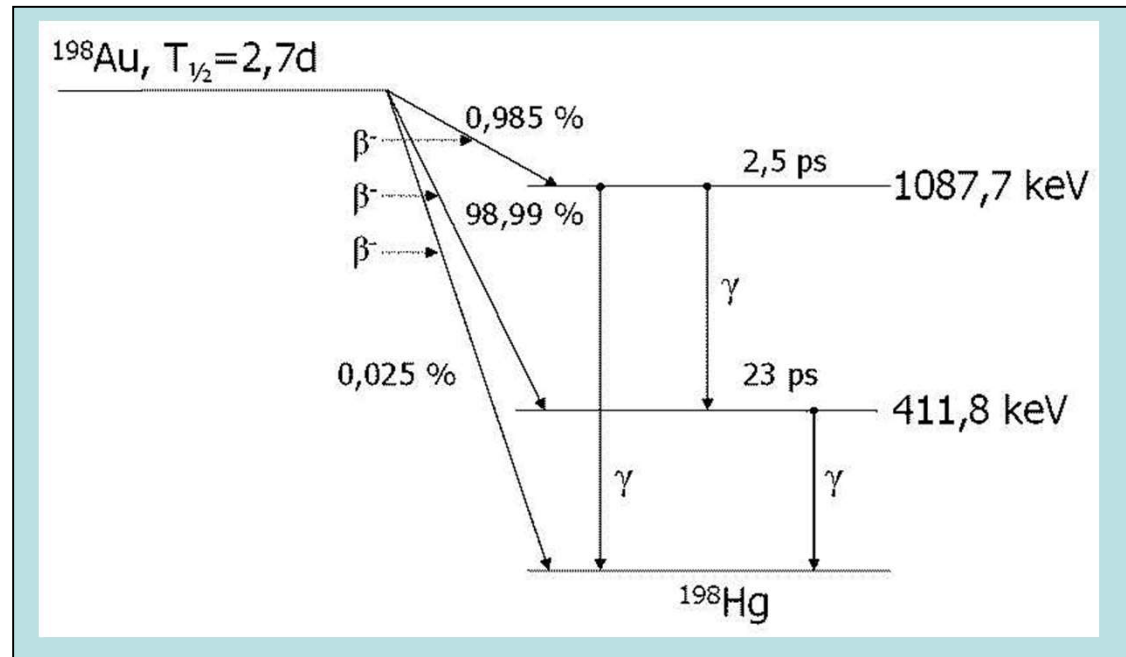
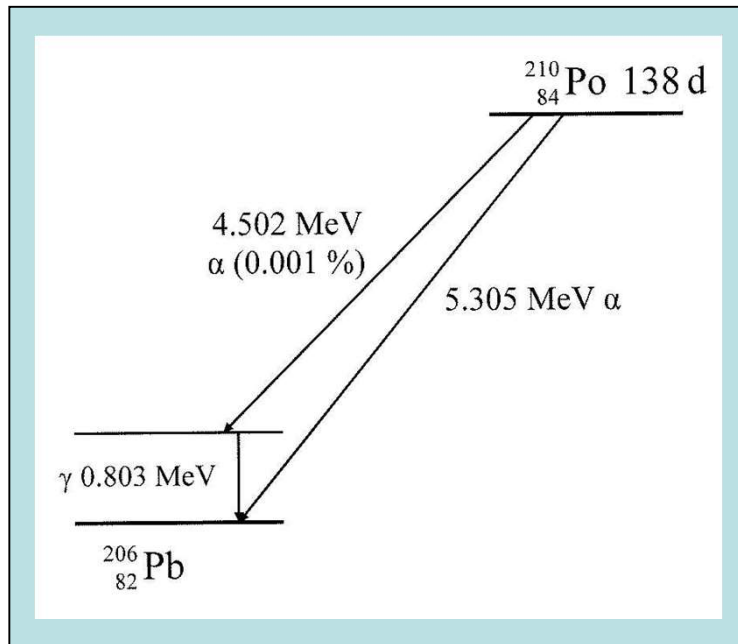
E.g.:  $^{99m}\text{Tc}$



# K-capture (inverse $\beta$ -decay)



# Some examples of the decay paths



# Characteristics of radioactive decays in general

activity                      characterizes the source

half life time              characterizes the speed of the decay\*

particle energy            characterizes the radiation\*

\*depends on the type of the isotope

# Activity ( $\Lambda$ )

$$\Lambda = \left| \frac{\Delta N}{\Delta t} \right|$$

$N$  = Number of undecayed atoms

$t$  = time

$-\Delta N$  = Number of decays during  $\Delta t$  time

Activity = number of decays in a unit time

unit: becquerel Bq

1 Bq = 1 decay/sec

old unit: Ci = 37 GBq

immeasurably  
small

level of natural  
activity

kBq, MBq, GBq, TBq, PBq

in vivo  
diagn.

work  
carefully  
with it!

activity applied  
in radiotherapy

# Law of radioactive decay

$$\Delta N = -\lambda N \Delta t$$

$N$ : Number of undecayed nuclei

( $-\Delta N$  = Number of decays during  $\Delta t$  time)

$\lambda$ : decay constant (probability of the decay [1/s])

$1/\lambda = \tau$  average lifetime

$$\frac{\Delta N}{\Delta t} = -\lambda N$$

Differential equation

solution:

$$N(t) = N_0 e^{-\lambda t}$$

Exponential decrease

number of undecayed atoms at  $t=0$

# An example

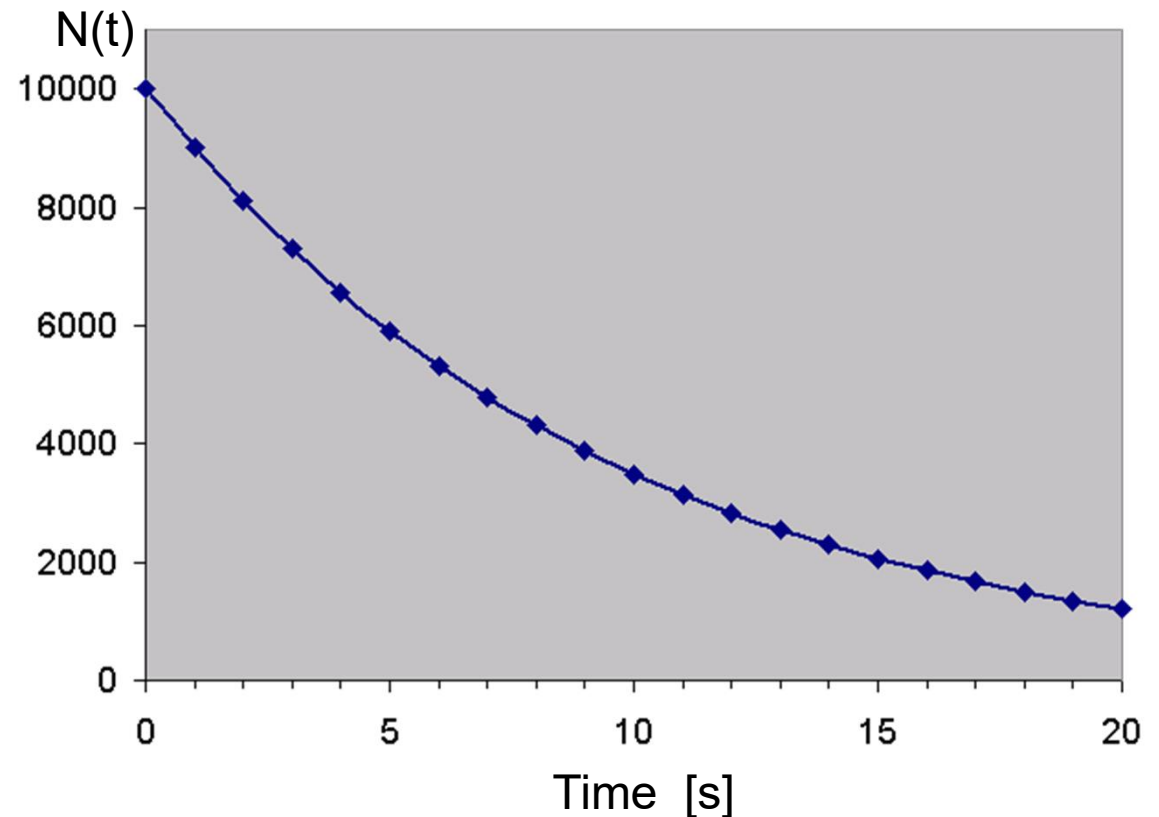
- At start:  $N_0=10000$  (number of atoms)  
 $\lambda=0.1 \text{ } ^1/\text{s}$  (decay probability)  
 $\Delta N = -\lambda N \Delta t$
- after 1 sec: 9000 (10000x0.1=1000 decayed)
- after 2 sec: 8100 (9000x0.1=900 decayed)
- after 3 sec: 7290 (8100x0.1=810 decayed)
- after 4 sec: 6561 (7290x0.1=729 decayed)
- ....



# An example

- At start:  $N_0=10000$  (number of atoms)  
 $\lambda=0.1 \text{ } ^1/\text{s}$  (decay probability)

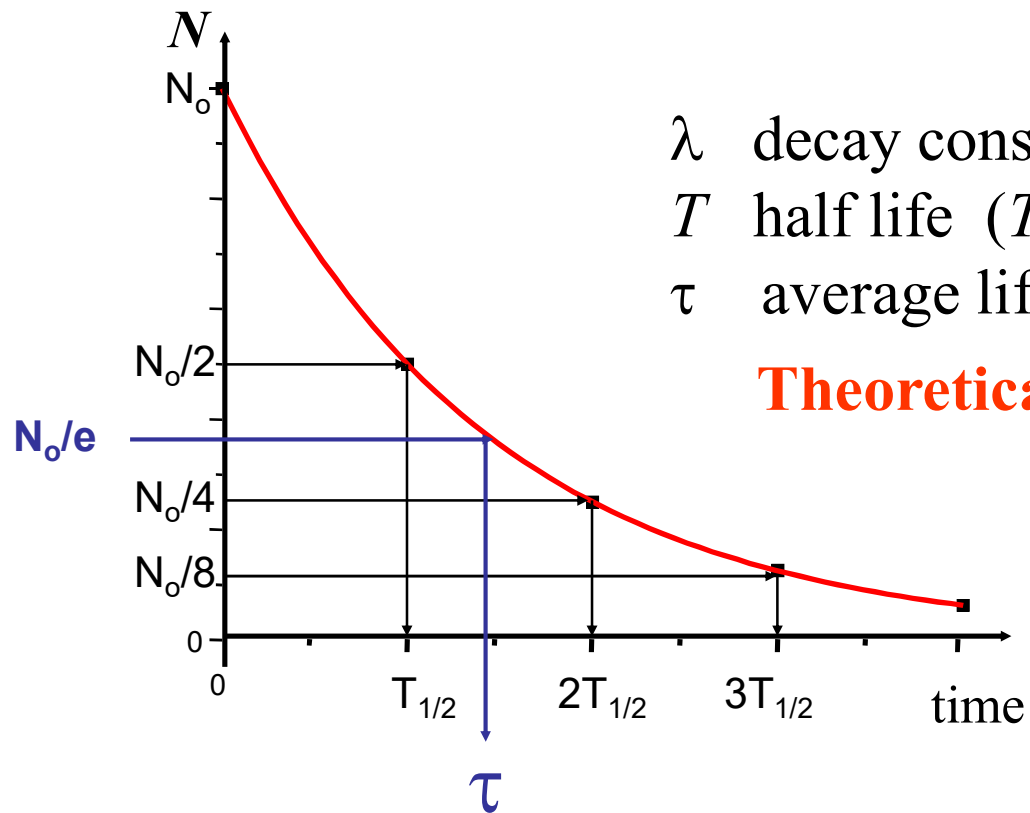
- after 1 s: 9000
- after 2 s: 8100
- after 3 s: 7290
- after 4 s: 6561
- ....



# Law of radioactive decay

$$N(t) = N_0 e^{-\lambda t} = N_0 2^{-\frac{t}{T}}$$

$$T = \frac{\ln 2}{\lambda} = \frac{0,693}{\lambda}$$



$\lambda$  decay constant (probability of the decay)

$T$  half life ( $T_{1/2}$ )

$\tau$  average lifetime

**Theoretically never decreases to zero !**

# Decrease of the activity as a function of time

$$A = \left| \frac{\Delta N}{\Delta t} \right| \quad \frac{\Delta N}{\Delta t} = -\lambda N$$

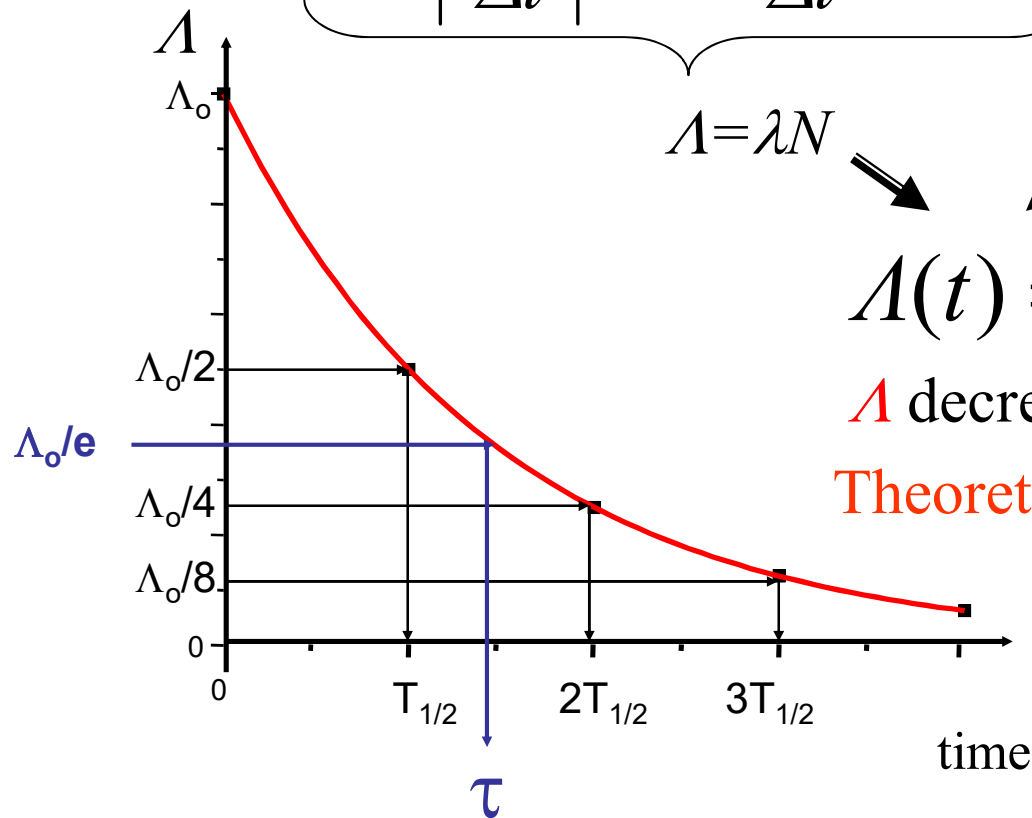
$$N(t) = N_0 e^{-\lambda t}$$

$$A = \lambda N$$

$$A(t) = A_0 e^{-\lambda t} = A_0 2^{-\frac{t}{T}}$$

$A$  decreases on the same way as  $N$ !

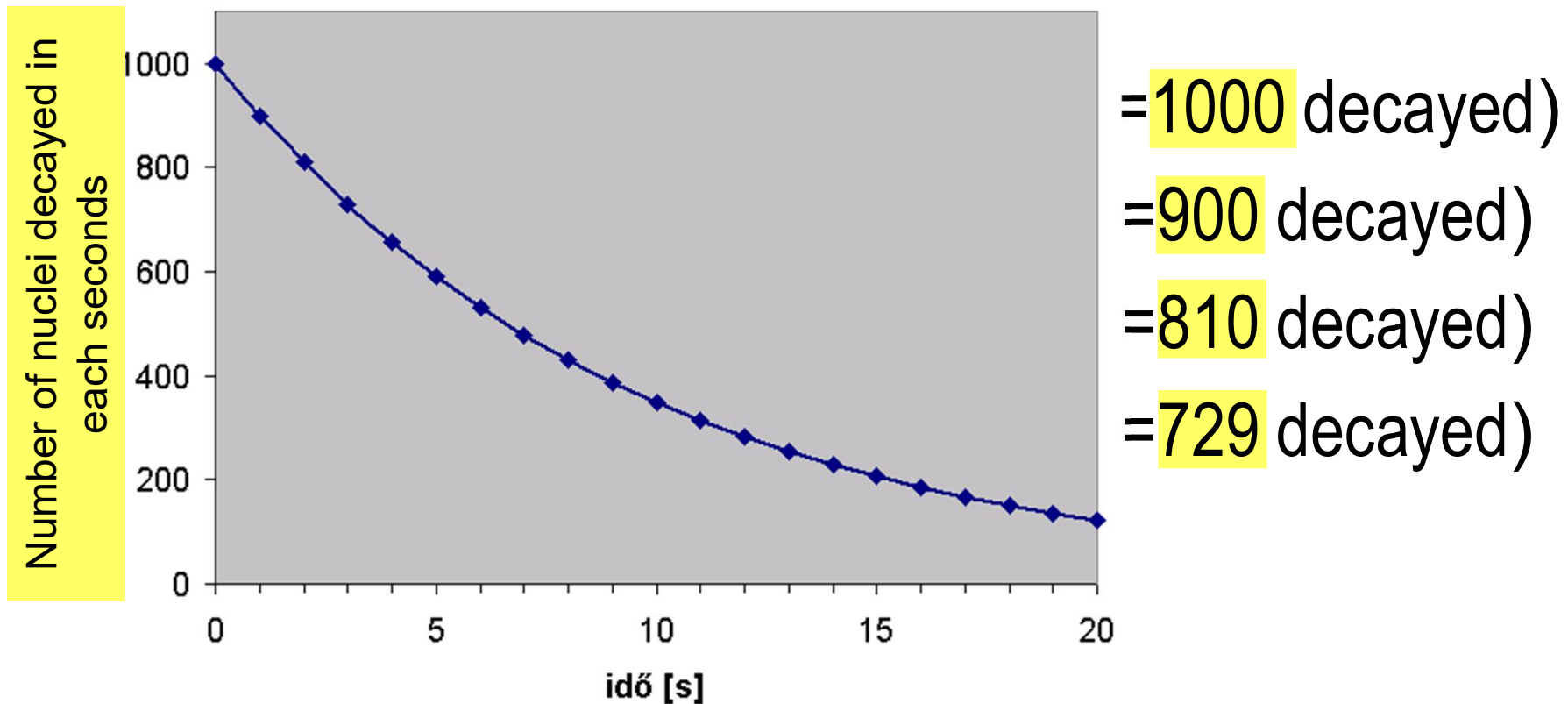
Theoretically never decreases to zero!



During about 10  $T$  the activity decreases to its 1/1000 (e.g GBq  $\rightarrow$  MBq)

# An example

- At start:  $N_0=10000$  (number of atoms)  
 $\lambda=0.1 \text{ } ^1/\text{s}$  (decay probability)



# Few examples for half life

$^{232}\text{Th}$	$1.4 \cdot 10^{10} \text{ y}$
-------------------	-------------------------------

$^{238}\text{U}$	$4.5 \cdot 10^9 \text{ y}$
------------------	----------------------------

$^{40}\text{K}$	$1.3 \cdot 10^9 \text{ y}$
-----------------	----------------------------

$^{14}\text{C}$	$5736 \text{ y}$
-----------------	------------------

$^{137}\text{Cs}$	$30 \text{ y}$
-------------------	----------------

$^3\text{H}$	$12.3 \text{ y}$
--------------	------------------

$^{60}\text{Co}$	$5.3 \text{ y}$
------------------	-----------------

$^{59}\text{Fe}$	$1.5 \text{ m}$
------------------	-----------------

$^{56}\text{Cr}$	$1 \text{ m (28 d)}$
------------------	----------------------

$^{131}\text{I}$	$8 \text{ d}$
------------------	---------------

$^{99\text{m}}\text{Tc}$	$6 \text{ h}$
--------------------------	---------------

$^{18}\text{F}$	$110 \text{ min}$
-----------------	-------------------

$^{11}\text{C}$	$20 \text{ min}$
-----------------	------------------

$^{15}\text{O}$	$2 \text{ min}$
-----------------	-----------------

$^{222}\text{Th}$	$2.8 \text{ ms}$
-------------------	------------------

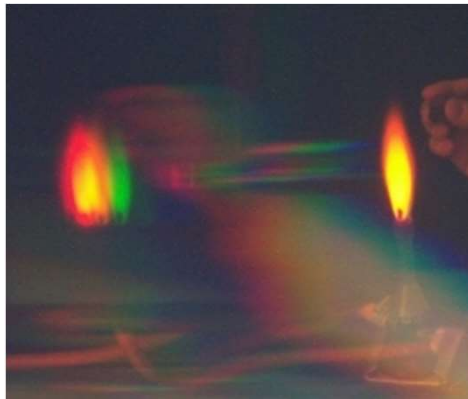
**Don't learn these numbers!**

# Typical energy levels in the microworld

Excitation of the  
outer electrons

eV (aJ)

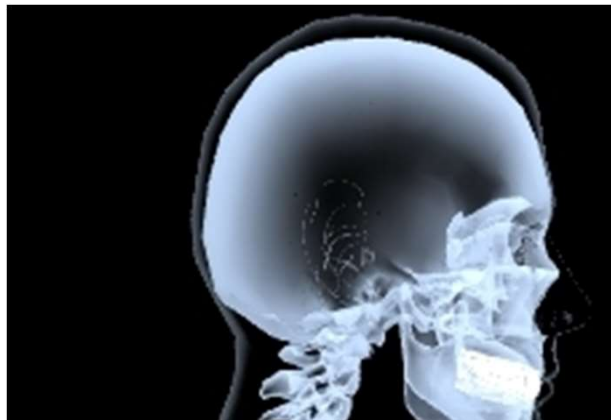
light



Electron transition  
between inner  
electrons

keV (fJ)

X-ray

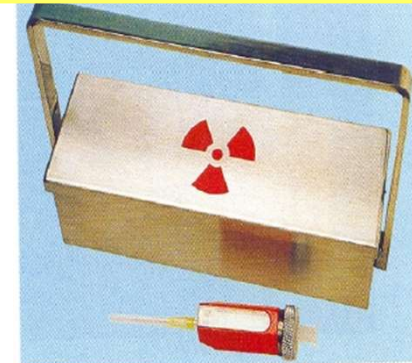


Transformation  
of the nucleus  
(decay)

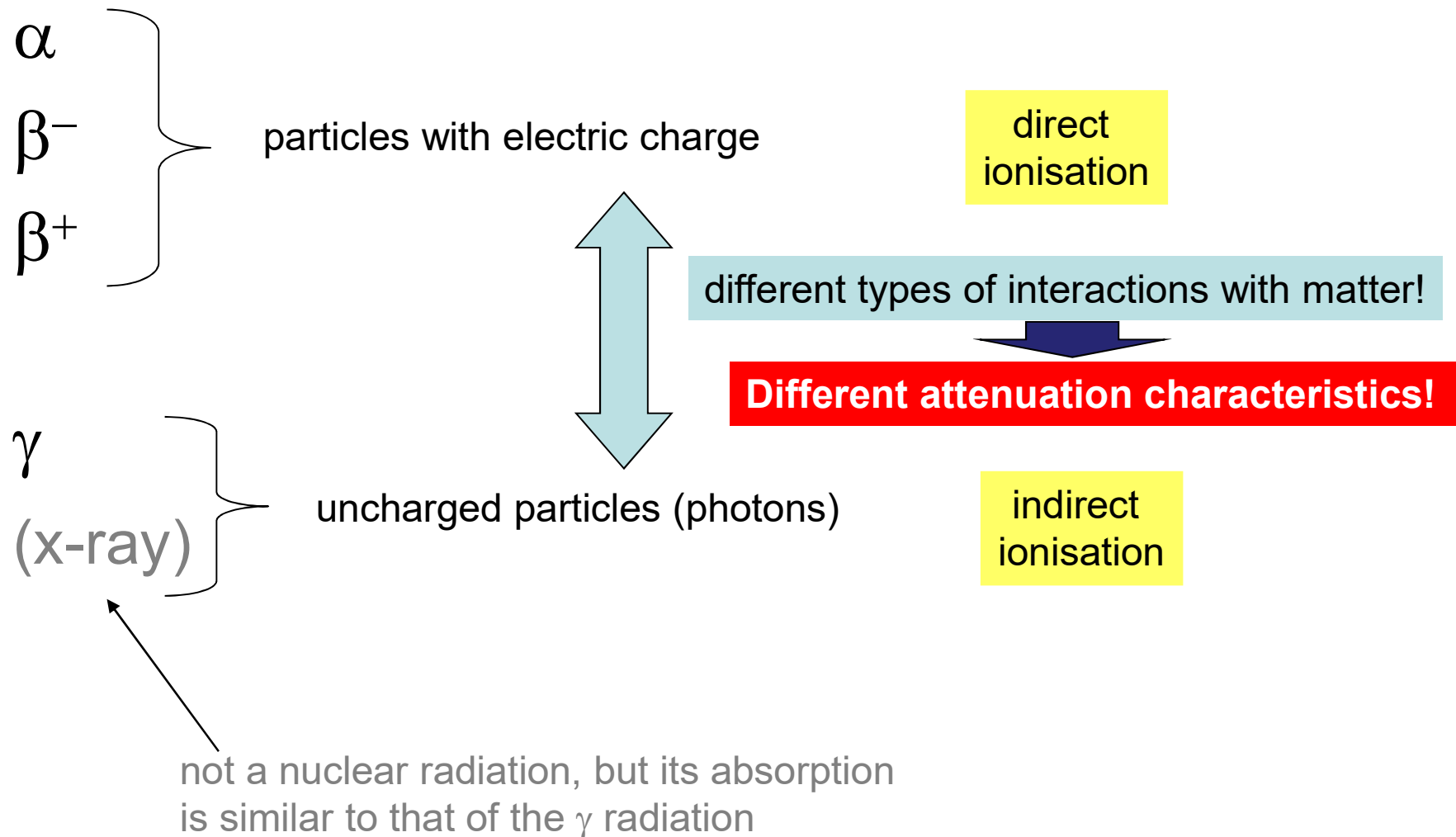
MeV (pJ)

Nuclear  
radiation

$\alpha$ ,  $\beta$ ,  $\gamma$



# Absorption of the nuclear radiation





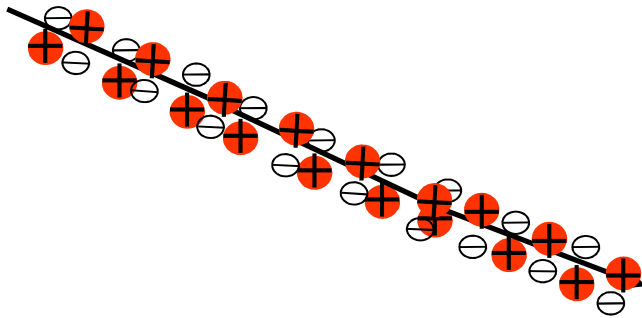
# Absorption of the charged particles

Ionizing during the path => continuous decrease of the particle energy  
The energy after a given path length decreases to the thermal value

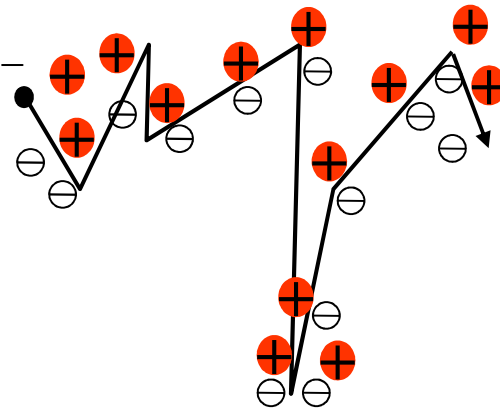


effective range

$\alpha$

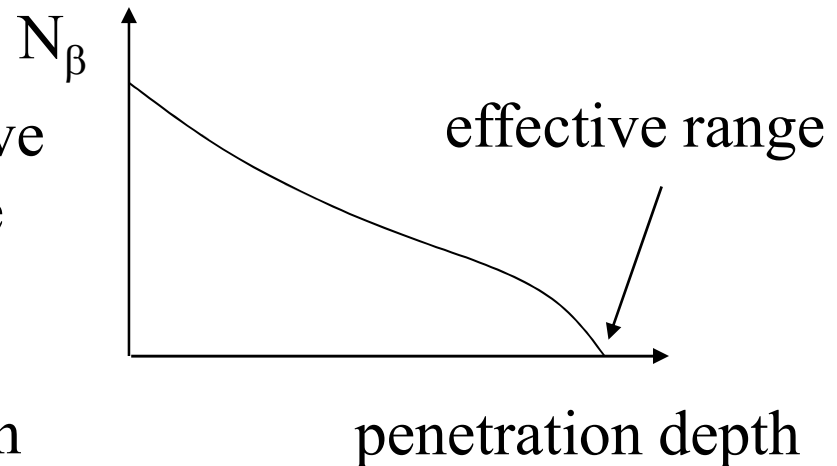
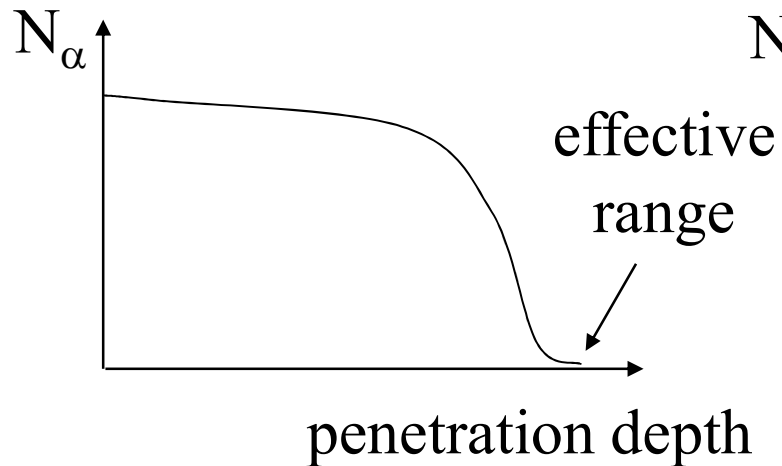


$\beta^-$



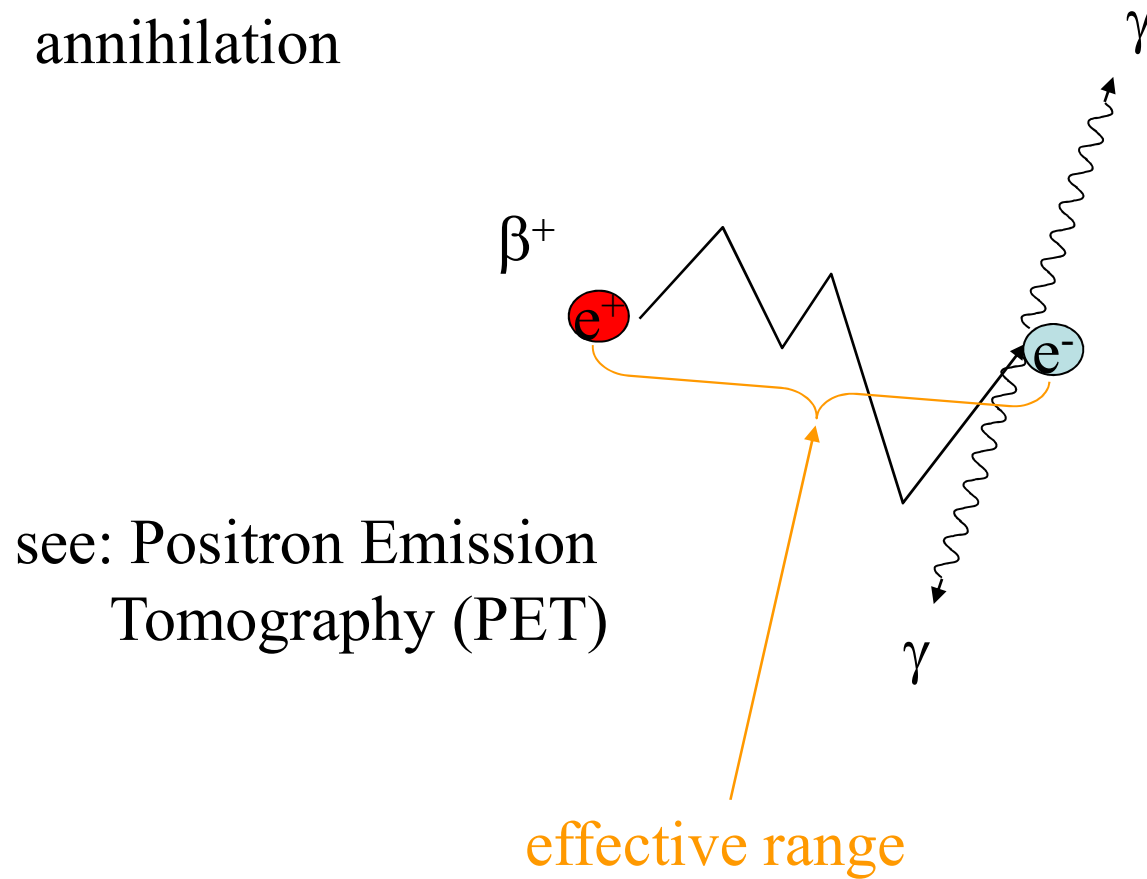
# Effective range

	$\alpha$ -particle	$\beta^-$ -particle
in air	<b>few cm</b>	<b><math>\sim</math> m</b>
in tissue	<b>0,01-0,1 mm</b>	<b><math>\sim</math> cm</b>



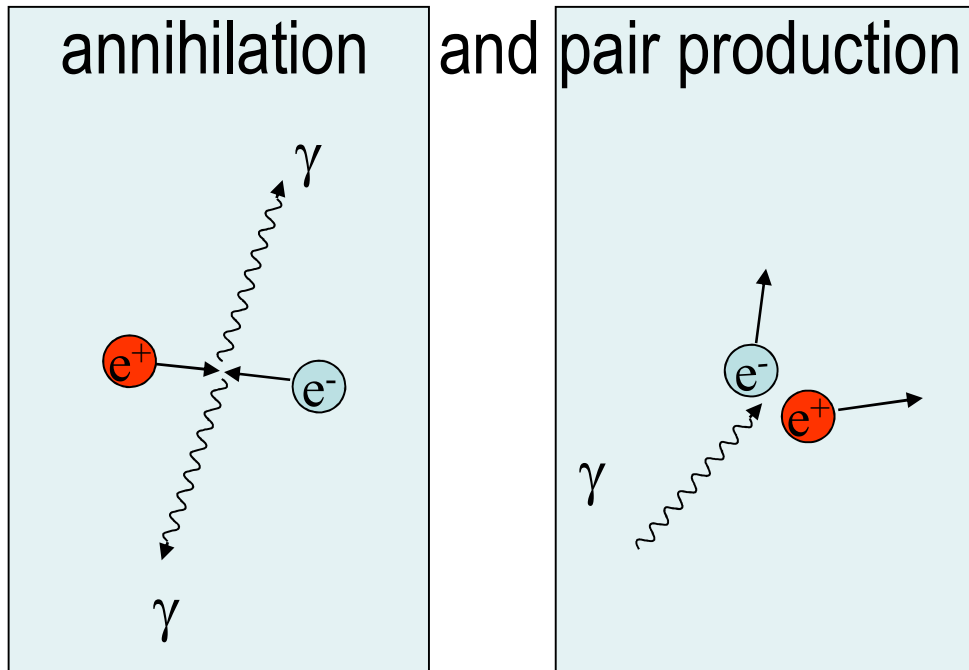
# $\beta^+$ -radiation

annihilation



# Electron and positron

- particle - antiparticle
- same mass,
- charge: same value, but different sign

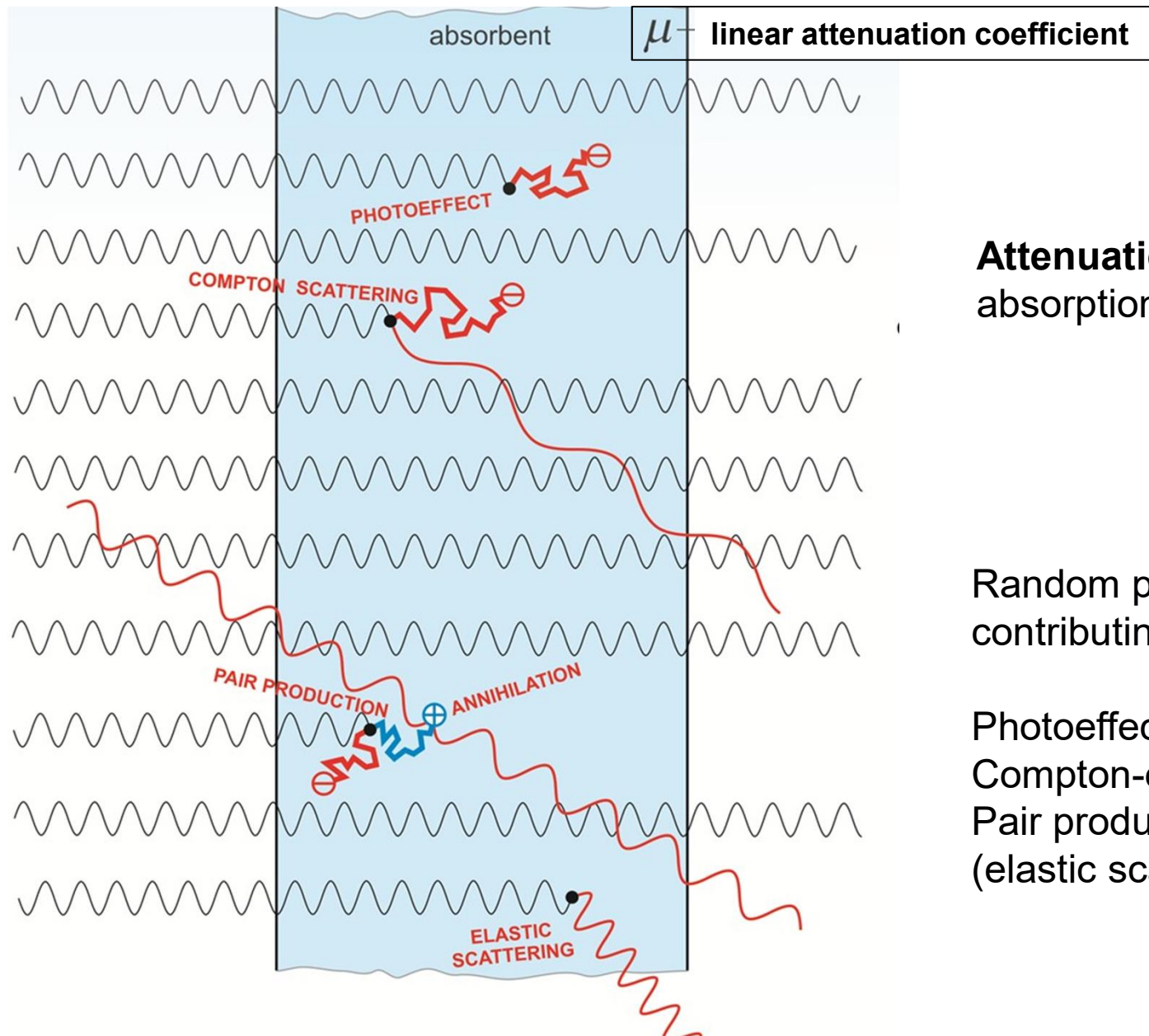


Einstein:  
mass-energy  
ekvivalence

$$E=mc^2$$

$$m_e c^2 = 511 \text{ keV} \approx 0.5 \text{ MeV}$$

# Absorption of the $\gamma$ -radiation (and x-ray)

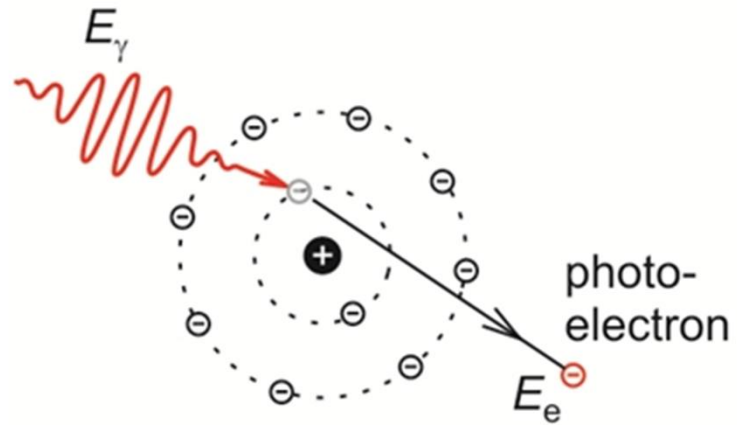


**Attenuation:**  
absorption + scattering

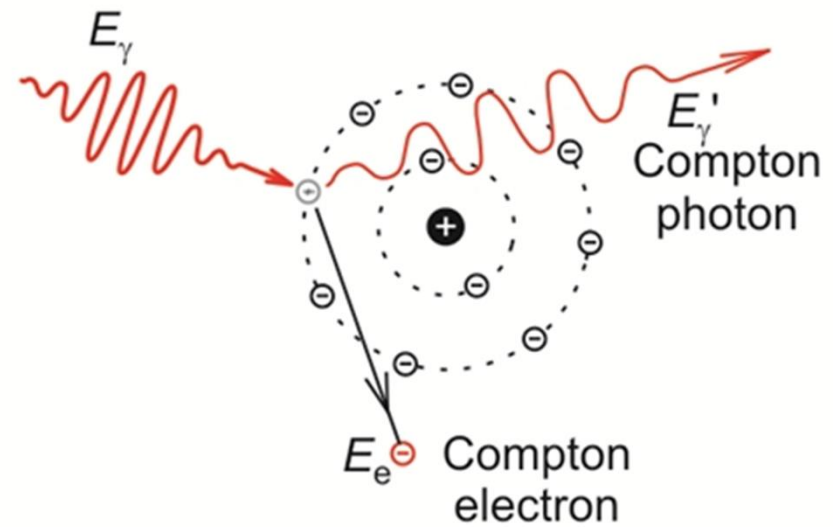
Random processes  
contributing:

Photoeffect,  
Compton-effect,  
Pair production,  
(elastic scattering)

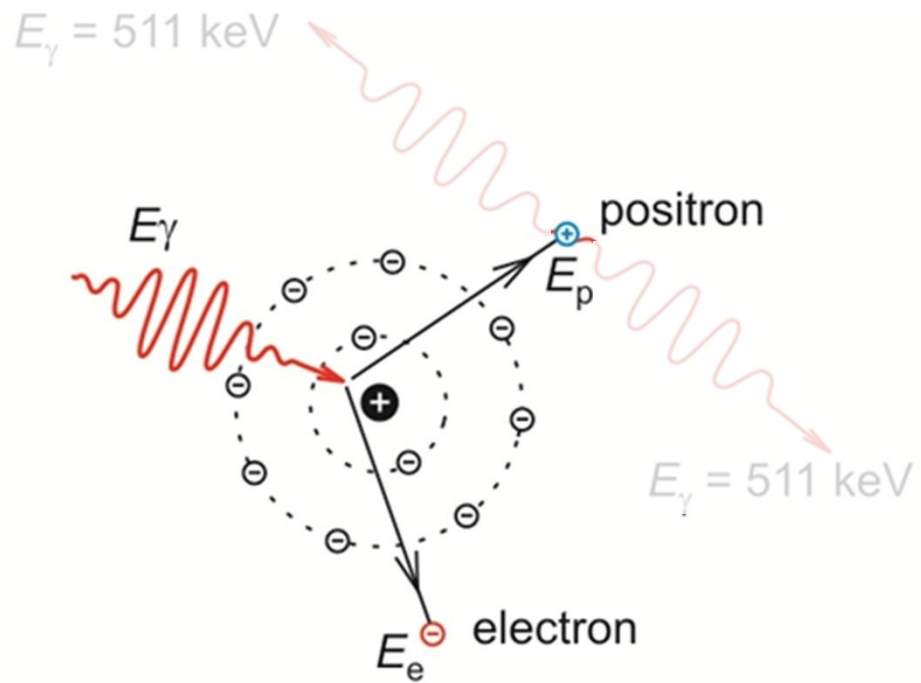
## Photoeffect



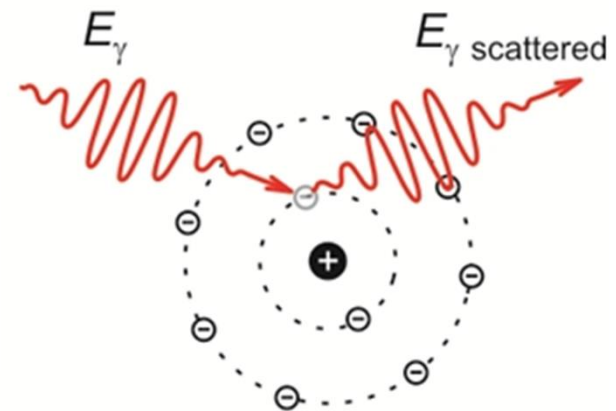
## Compton effect Compton-scattering



## Pair production

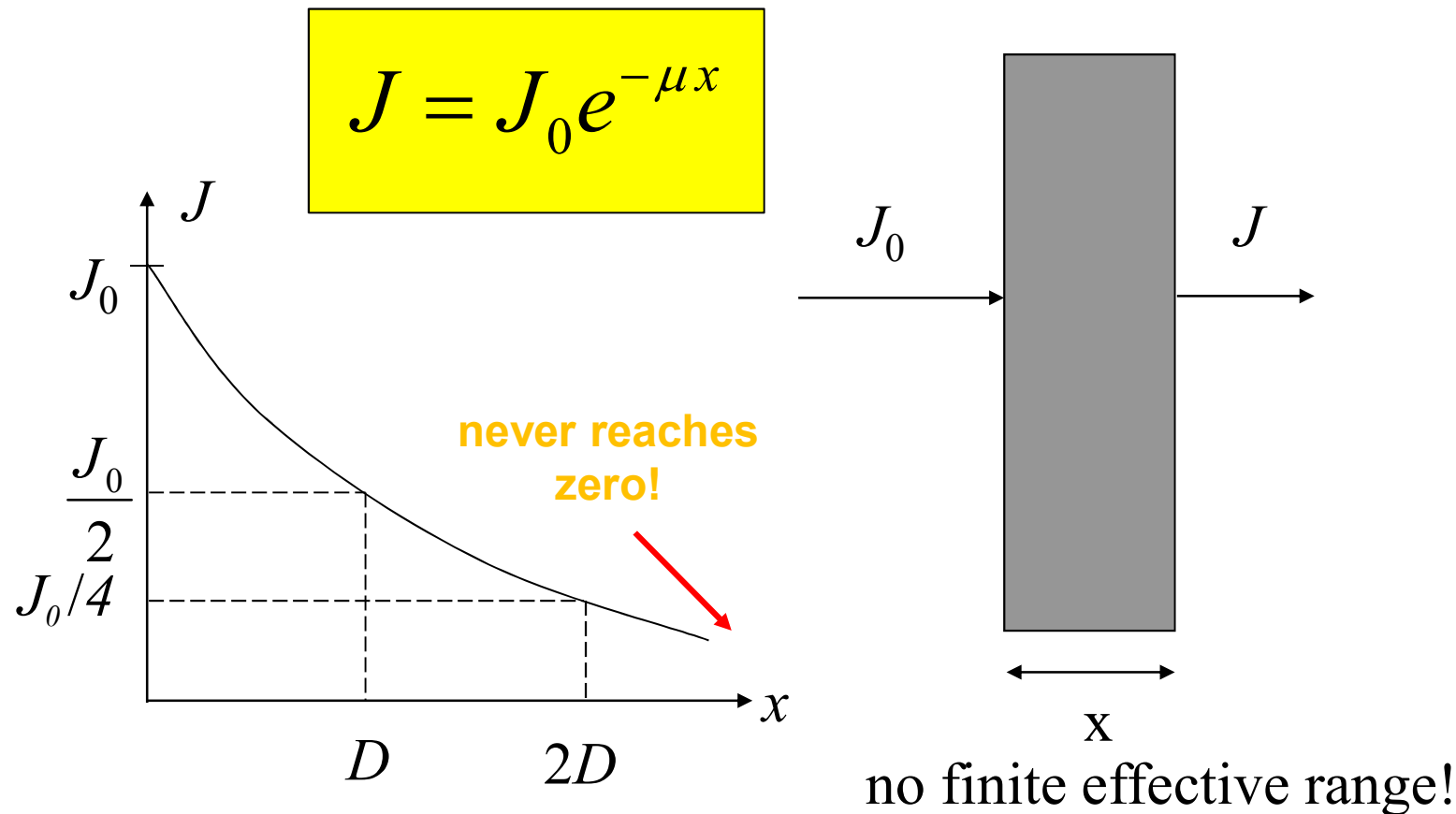


## elastic scattering

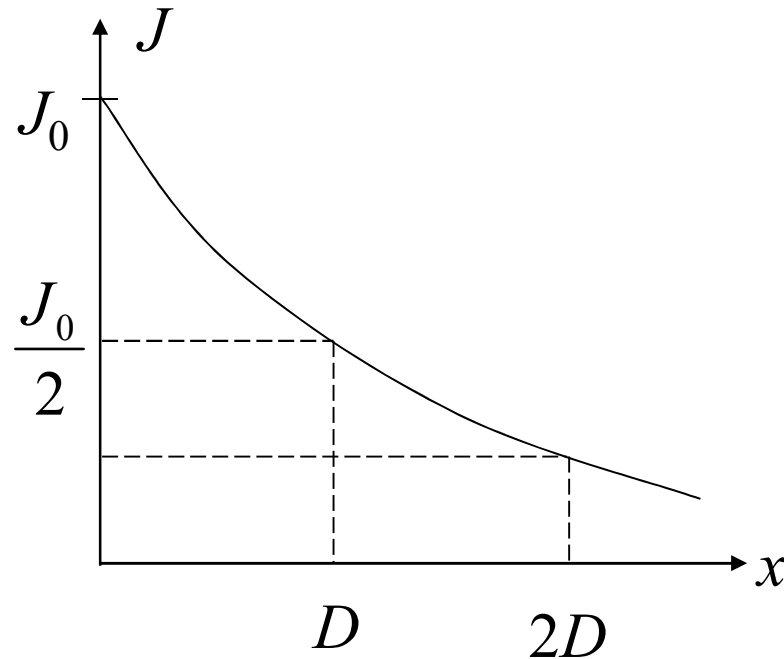




# Attenuation of the $\gamma$ -radiation and x-ray



few rules of thumb:  $x_{1/10} = 3,32 D$        $x_{1/1000} = 10 D$



$$J = J_0 e^{-\mu x}$$

$\mu$ : (linear) attenuation coefficient

its units are: 1/m, 1/cm

$$\delta = \frac{1}{\mu} \quad \text{„penetration depth”}$$

Intensity decreases to the e-th part (c.a. 37%)

$\mu$ (material, number of absorbing centers, energy of the radiation)  
 $= \mu(\text{material}, \rho, E_{\text{photon}}) \sim \rho$

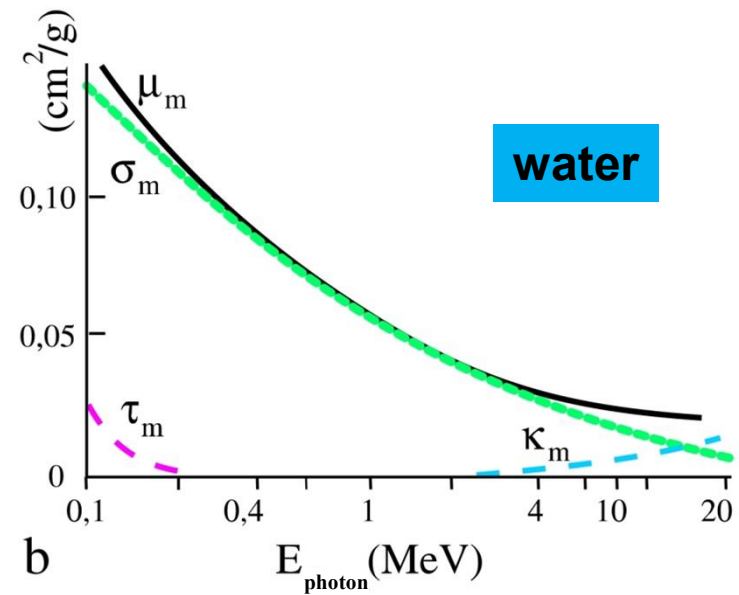
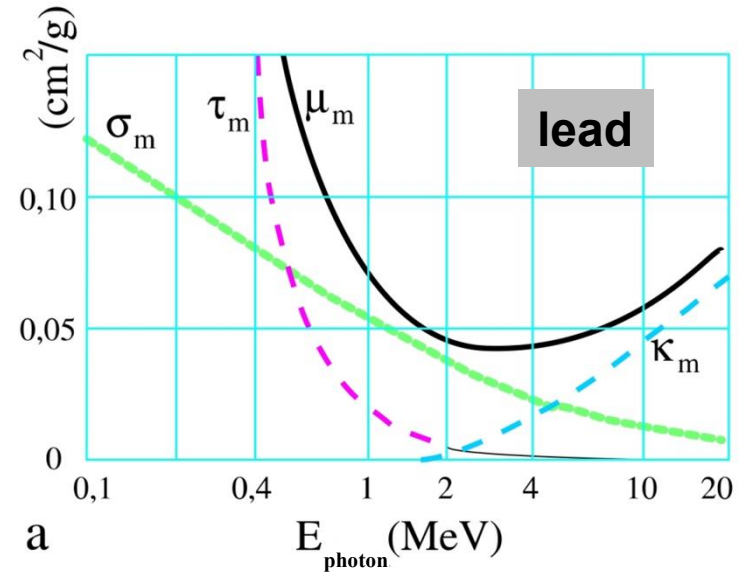
$$\mu_m = \frac{\mu}{\rho} \quad \text{mass attenuation coefficient}$$

mass attenuation coeff.

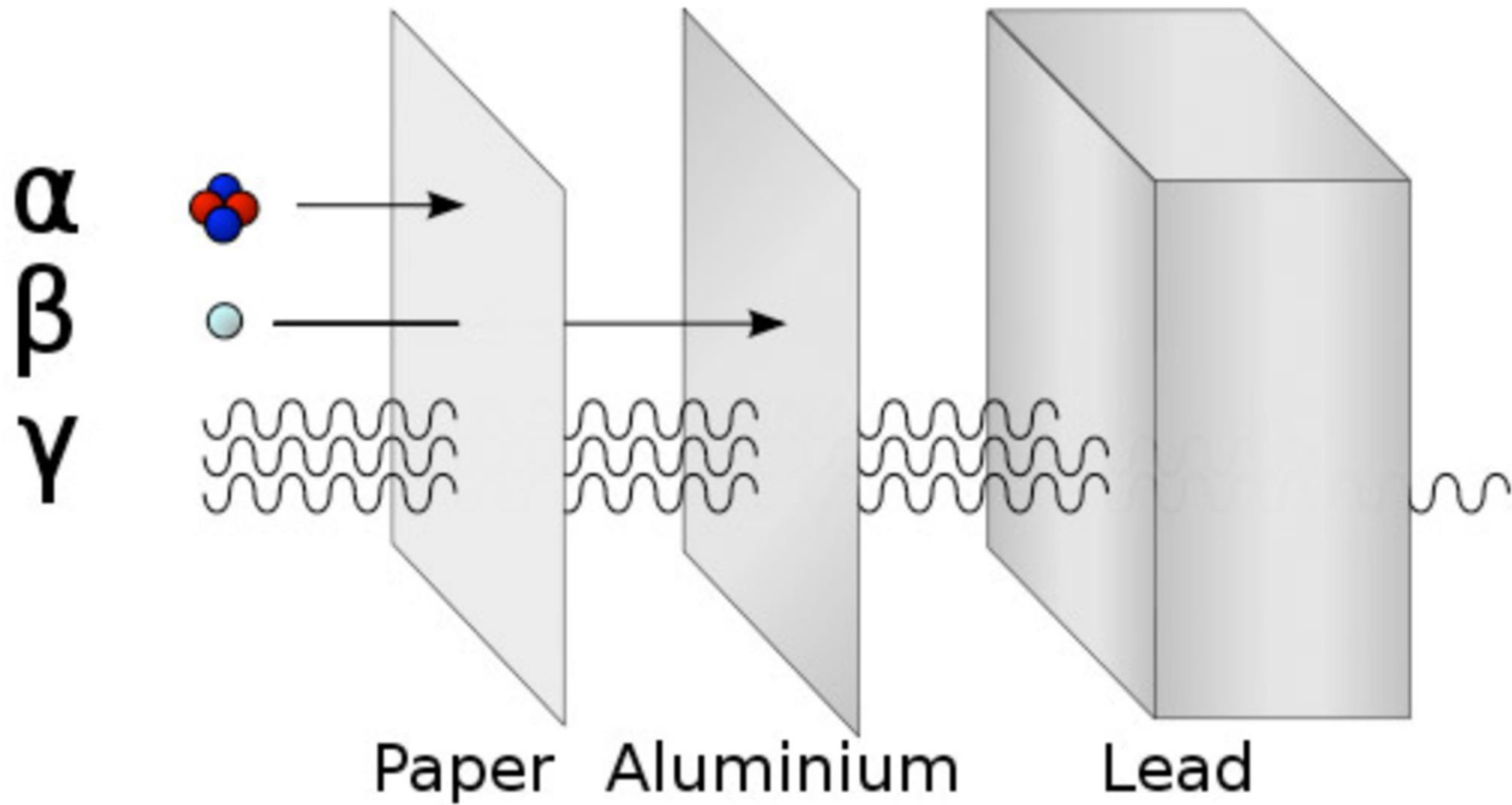
$$\mu_m = \frac{\mu}{\rho}$$

$$\mu_m = \tau_m + \sigma_m + \kappa_m$$

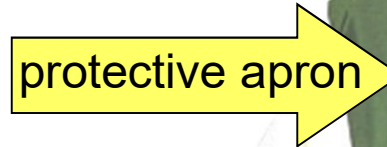
$$\tau_m = c\lambda^3 Z^3$$



# Summary of the absorption of $\alpha$ , $\beta$ and $\gamma$ radiation



# Applications (attenuation)



# Applications: isotopes and nuclear radiation

