

Medical Biophysics II.

5th lecture: Transport processes I.
Diffusion, Brownian motion, Osmosis
17th March 2021.
Dániel Veres

Diffusion?

Why?

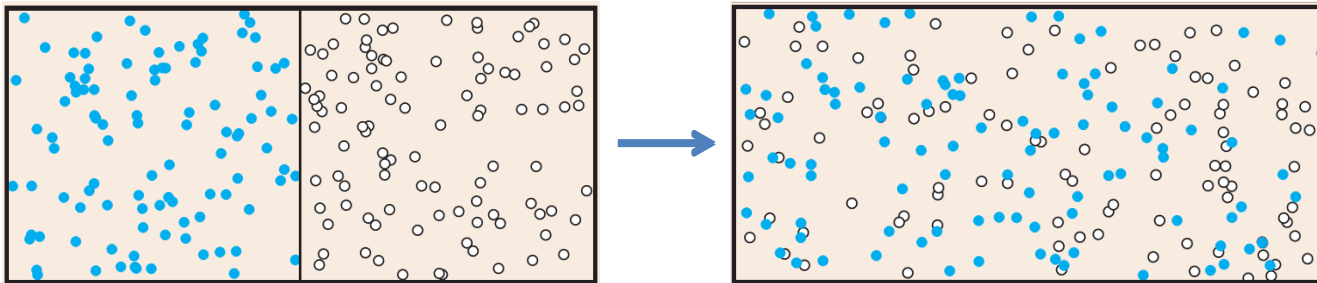
- physiology: cell function – ion diffusion...
- disorders: fibrosis, oedema, vasculitis, ascites...
- diagnostics: DWI MRI...
- therapy: dialysis, physiological saline....
- drug delivery: transdermal (liposomal), inhaled...

.....

Diffusion?

The change in the spatial distribution of particles because of **random thermal motion**.

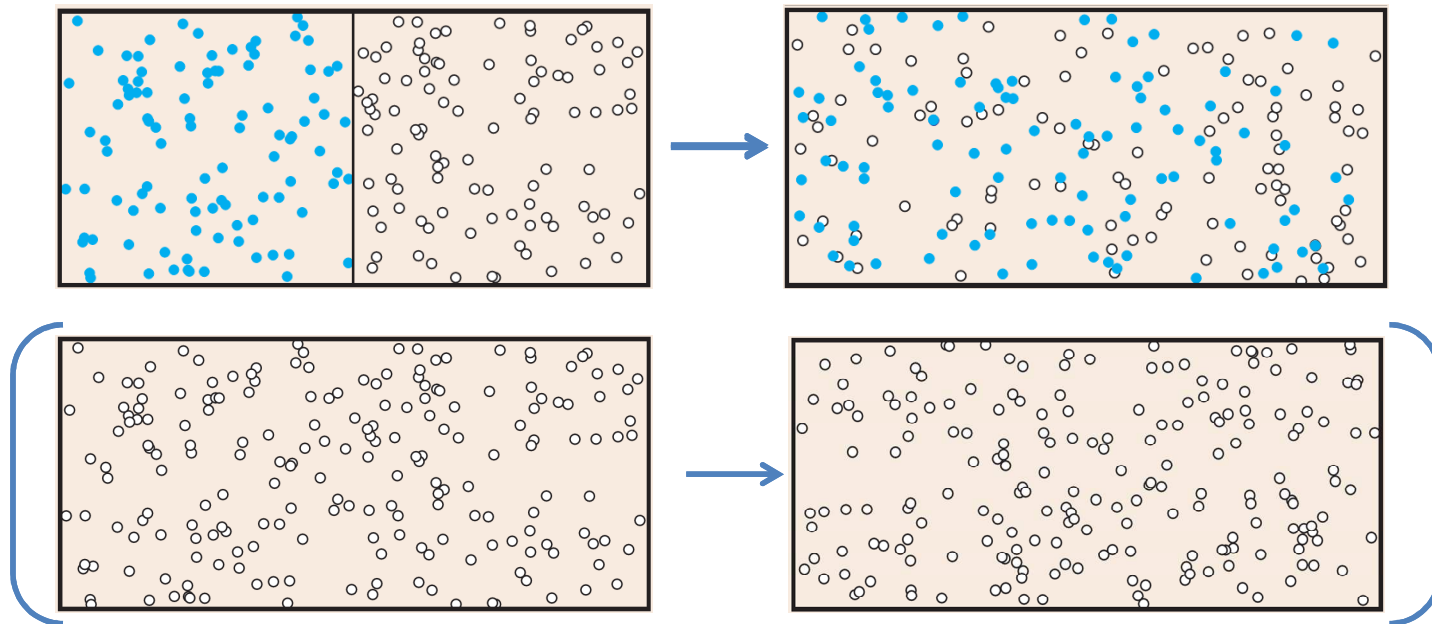
In microscopic level with **net matter transport**.



Diffusion?

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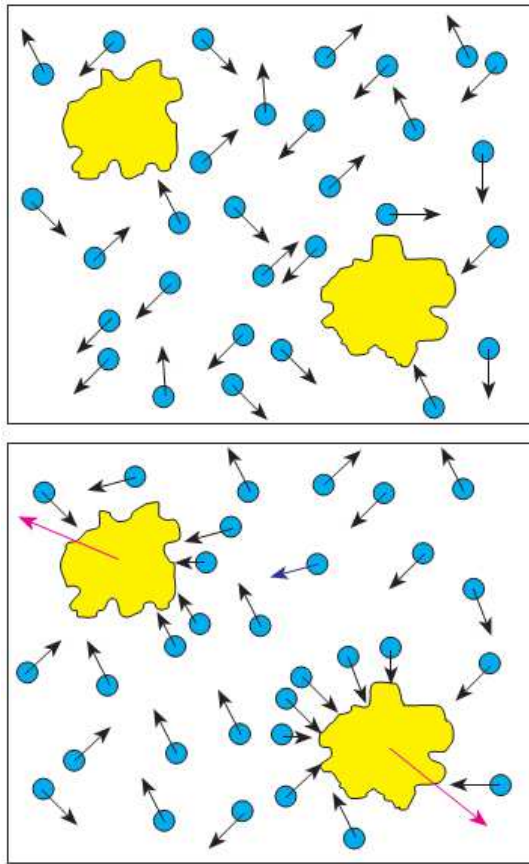
In microscopic level with **net matter transport**.



Relevant: NET transport of substance „A” in „B”.

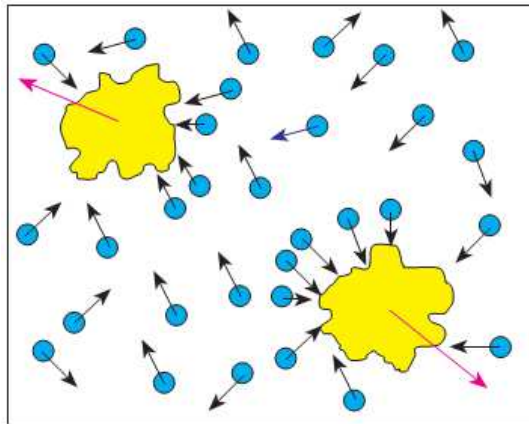
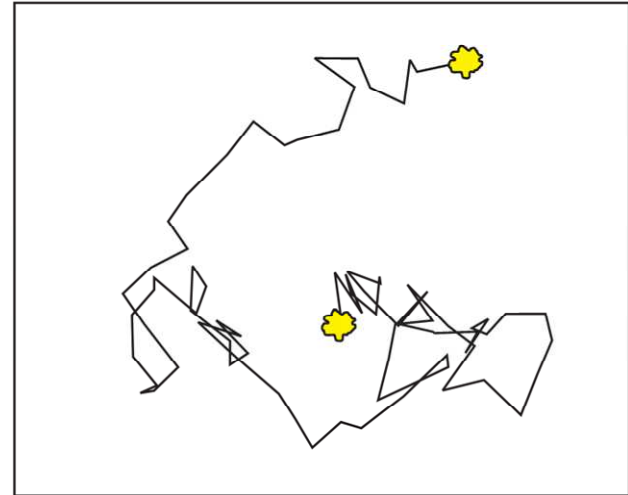
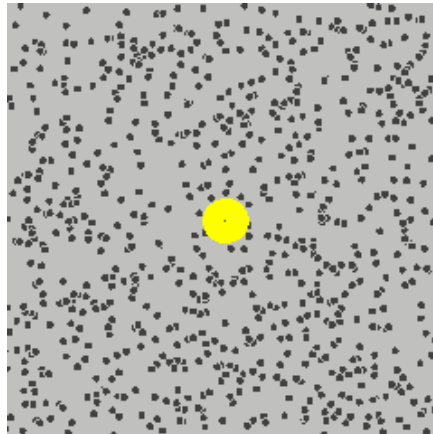
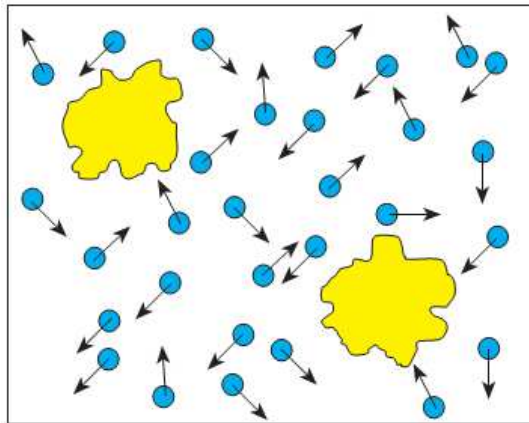
Brownian motion

The „random walk” of a particles resulting from their collision with other particles.



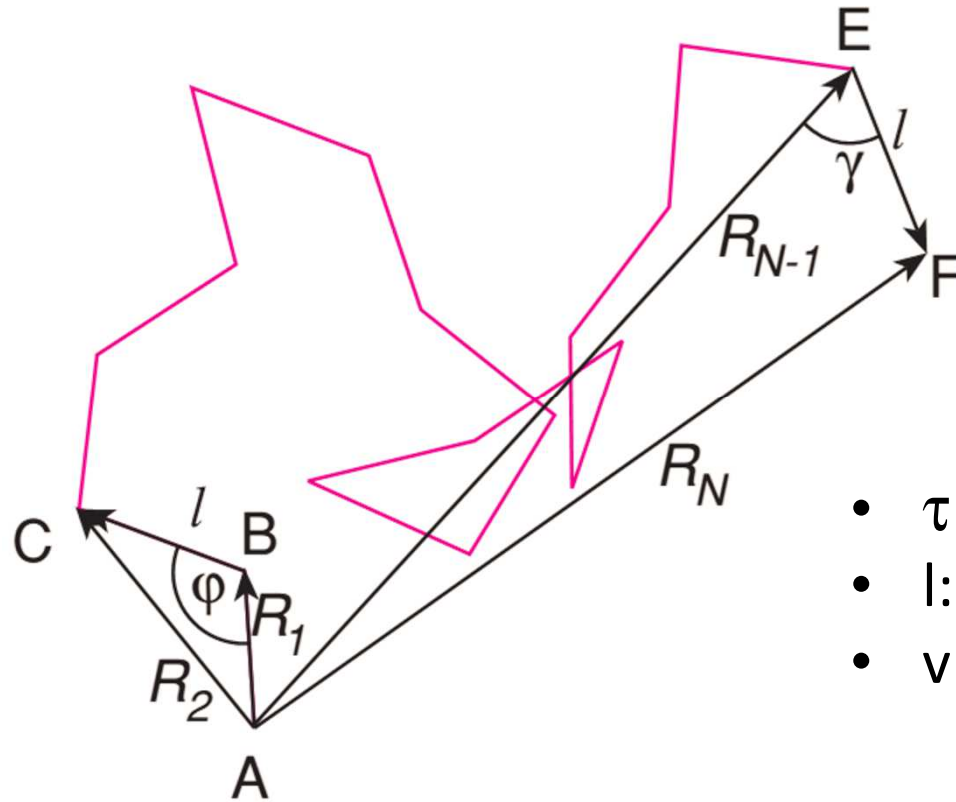
Brownian motion

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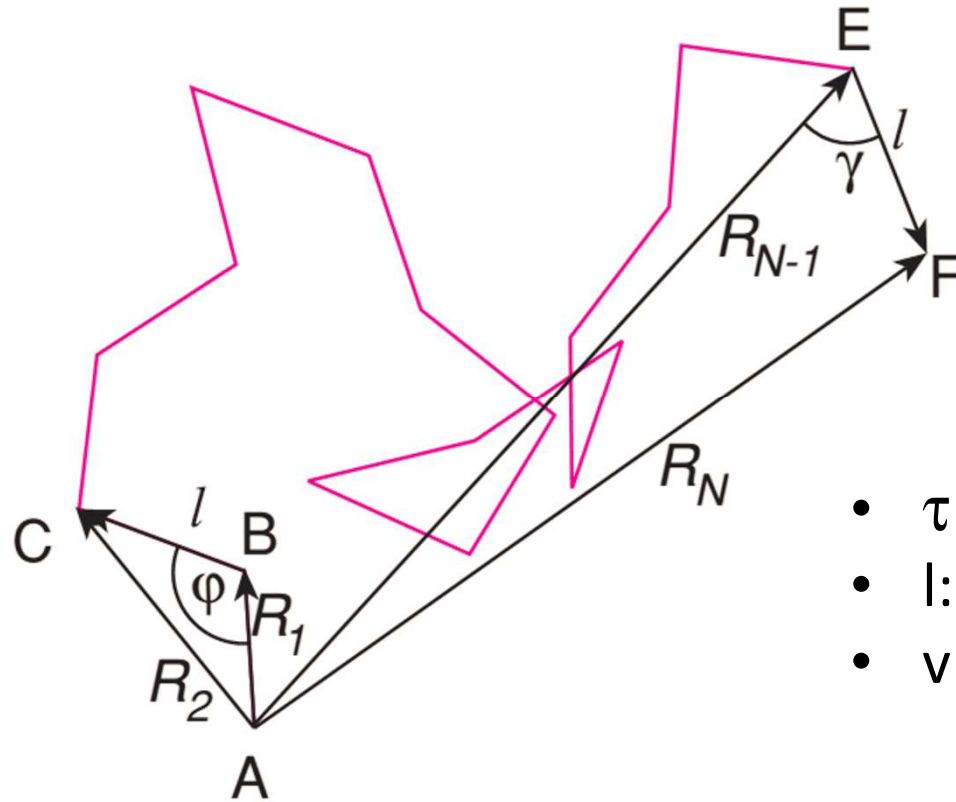
- τ : mean time between collisions
- l : mean free path
- v : mean speed of particles

How far reaches a particle?



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- l : mean free path
- v : mean speed of particles

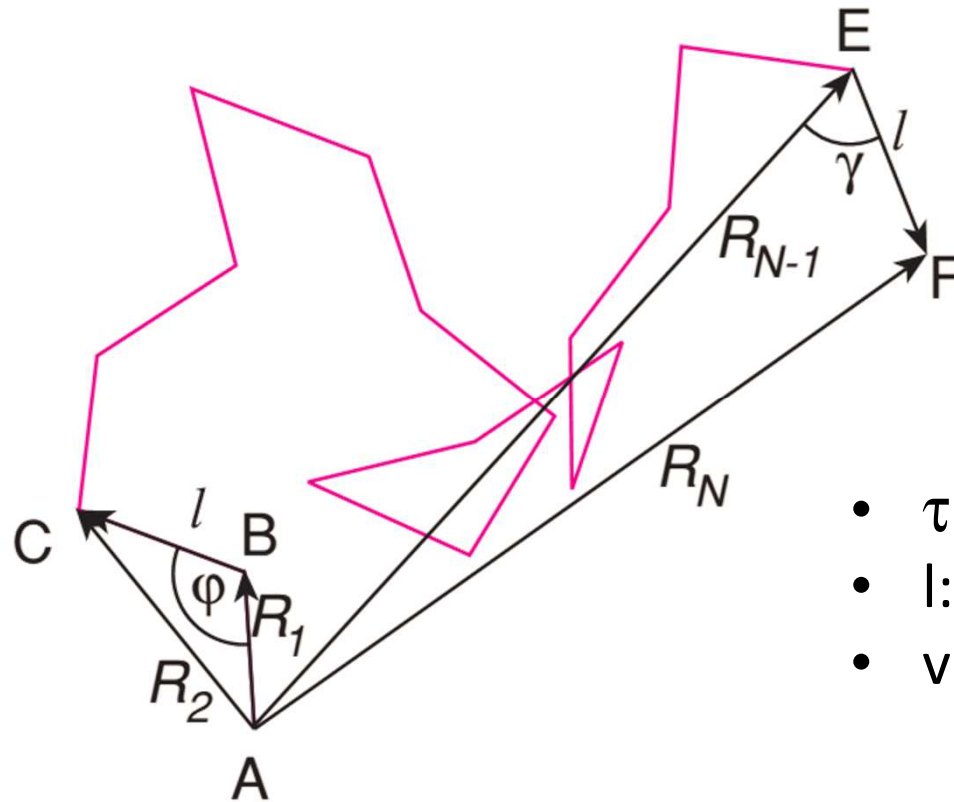
How far reaches a particle?



- τ : mean time between collisions
- l : mean free path
- v : mean speed of particles

One particle: $R_2^2 = R_1^2 + l^2 - 2 \cdot R_1 \cdot l \cdot \cos \varphi$

How far reaches a particle?

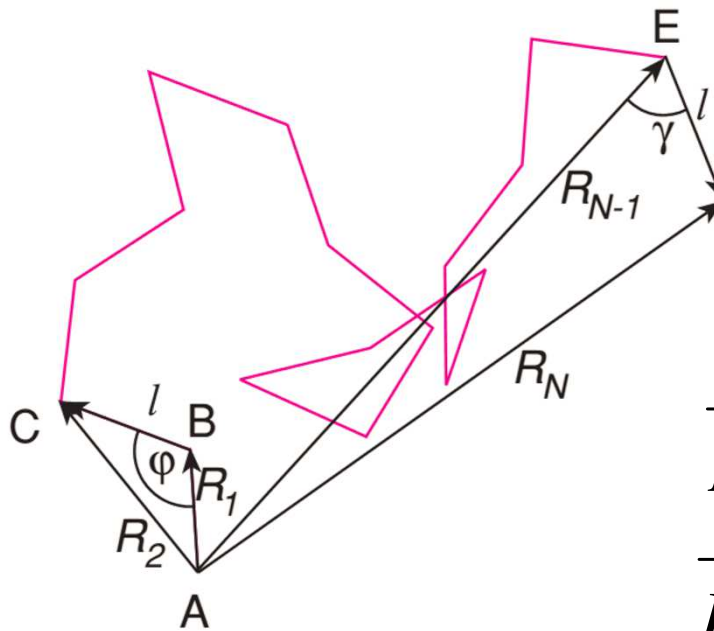


- τ : mean time between collisions
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- v : mean speed of particles

One particle: $R_2^2 = R_1^2 + l^2 - 2 \cdot R_1 \cdot l \cdot \cos \varphi$

A „mean“ particle:
(mean of n particles): $\overline{R_2^2} = \frac{1}{n} \cdot \sum_{i=1}^n \left(R_1^2 + l^2 - 2 \cdot R_1 \cdot l \cdot \cos \varphi_i \right)$

How far reaches a particle?



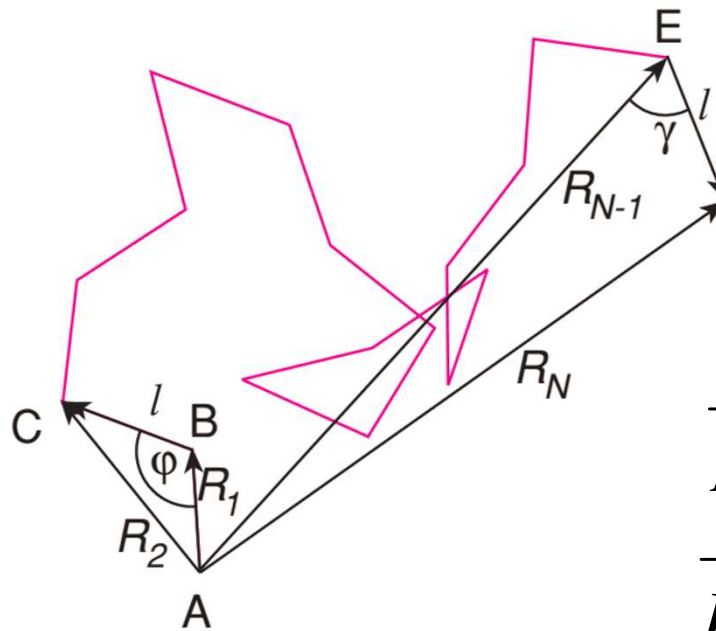
$$\overline{R_2^2} = \frac{1}{n} \cdot \sum_{i=1}^n (R_1^2 + l^2 - 2 \cdot R_1 \cdot l \cdot \cos \varphi_i)$$

$$\overline{R_2^2} = \frac{1}{n} \cdot n \cdot (R_1^2 + l^2) - 2 \cdot R_1 \cdot l \cdot \sum_{i=1}^n (\cos \varphi_i)$$

$$\overline{R_2^2} = R_1^2 + l^2 = l^2 + l^2 = 2 \cdot l^2$$

$$\overline{R_N^2} = N \cdot l^2$$

How far reaches a particle?



$$\overline{R_2^2} = \frac{1}{n} \cdot \sum_{i=1}^n \left(R_1^2 + l^2 - 2 \cdot R_1 \cdot l \cdot \cos \varphi_i \right)$$

$$\overline{R_2^2} = \frac{1}{n} \cdot n \cdot \left(R_1^2 + l^2 \right) - 2 \cdot R_1 \cdot l \cdot \sum_{i=1}^n \left(\cos \varphi_i \right)$$

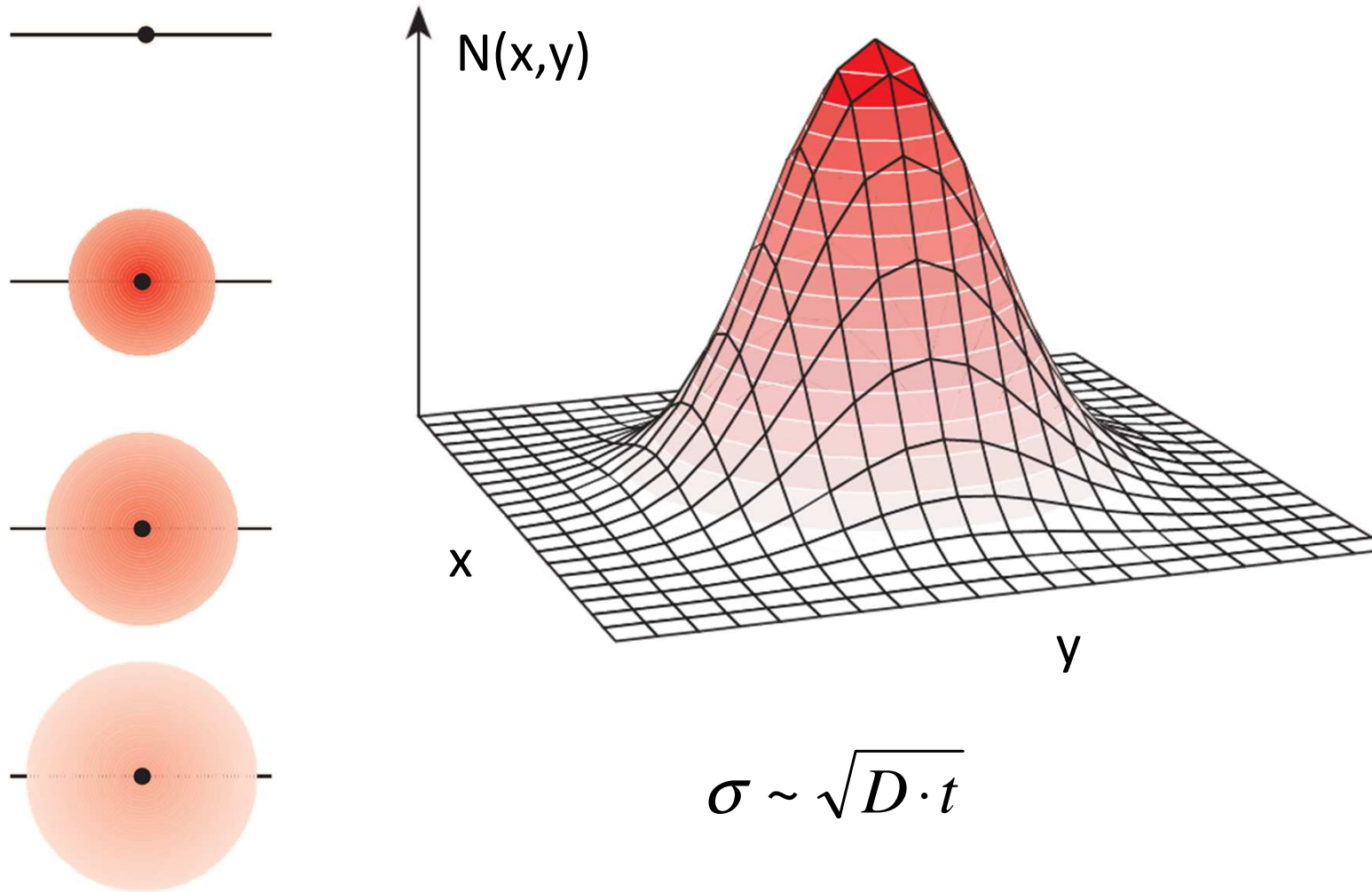
$$\overline{R_2^2} = R_1^2 + l^2 = l^2 + l^2 = 2 \cdot l^2$$

$$\overline{R_N^2} = N \cdot l^2$$

$$\overline{R_t} = \sqrt{N \cdot l^2} = \sqrt{\frac{t}{\tau} \cdot l \cdot l} = \sqrt{t \cdot v \cdot l} = \sqrt{3 \cdot D \cdot t}$$

$$\frac{v \cdot l}{3} = D$$

Experiment – 2D distribution



Matter transport - flow

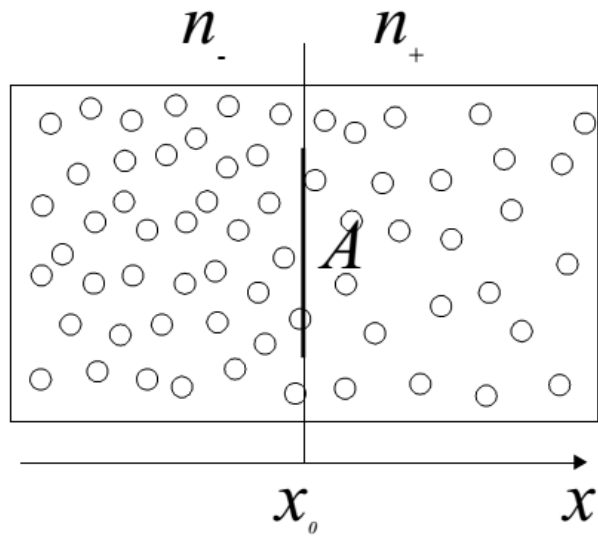
Particle flow rate: $I_N = \frac{\Delta N}{\Delta t}; \left[\frac{1}{s} \right]$

Particle flow density (flux): $J_N = \frac{\Delta I_N}{\Delta A}; \left[\frac{1}{m^2 \cdot s} \right]$

Matter flow rate: $I_v = \frac{\Delta v}{\Delta t}; \left[\frac{mol}{s} \right]$

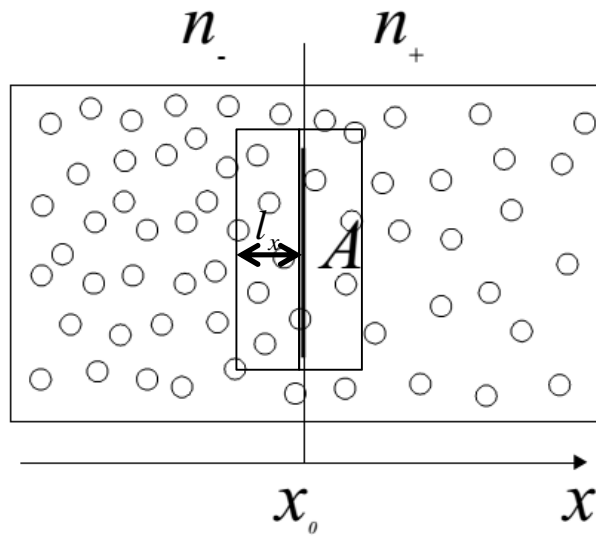
Matter flow density (flux): $J_v = \frac{\Delta I_v}{\Delta A}; \left[\frac{mol}{m^2 \cdot s} \right]$

Fick's first law



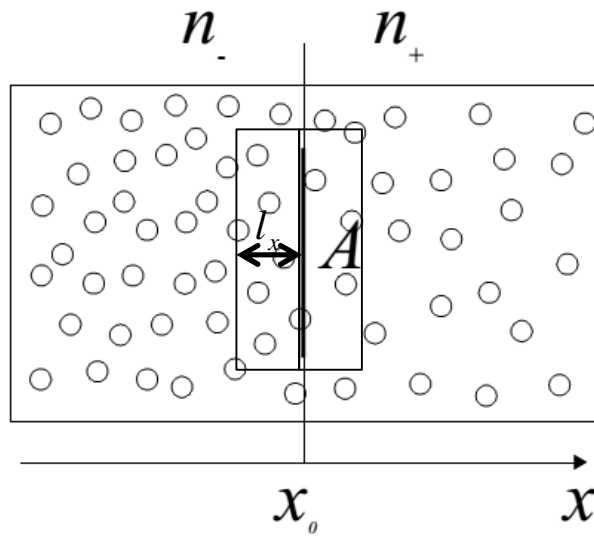
$$\Delta N = N_- - N_+$$

Fick's first law



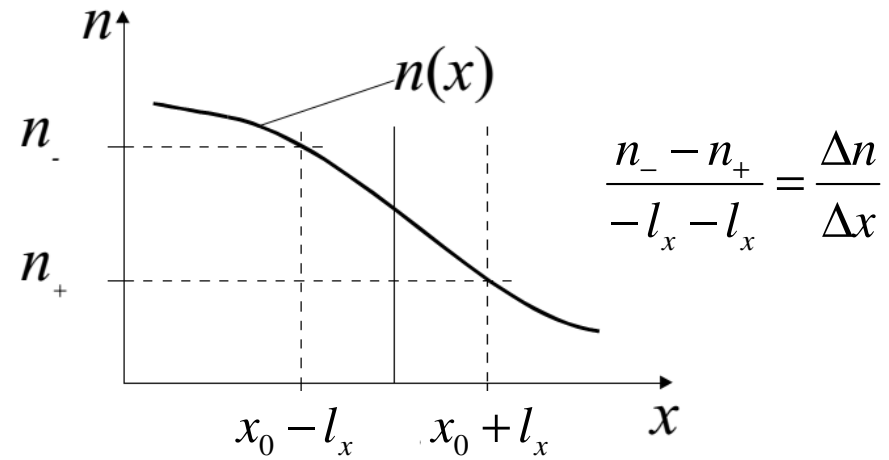
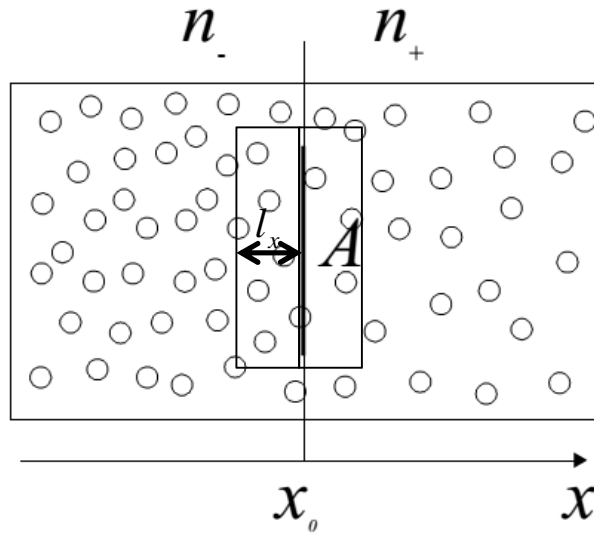
$$\Delta N = N_- - N_+ = \frac{1}{2} \cdot V_t \cdot n_- - \frac{1}{2} \cdot V_t \cdot n_+ = \frac{1}{2} \cdot V_t \cdot (n_- - n_+)$$

Fick's first law



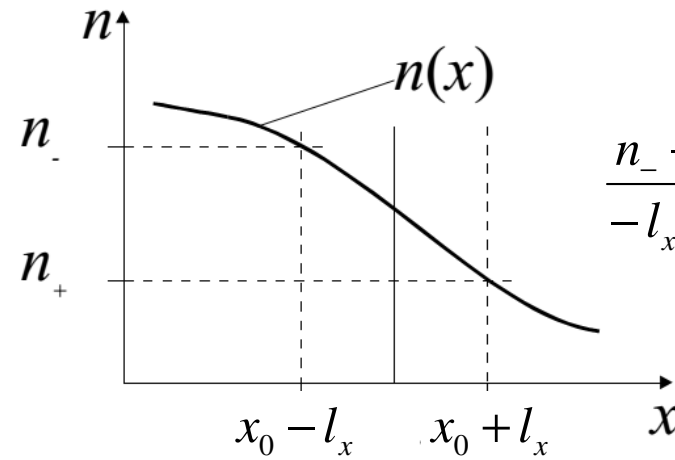
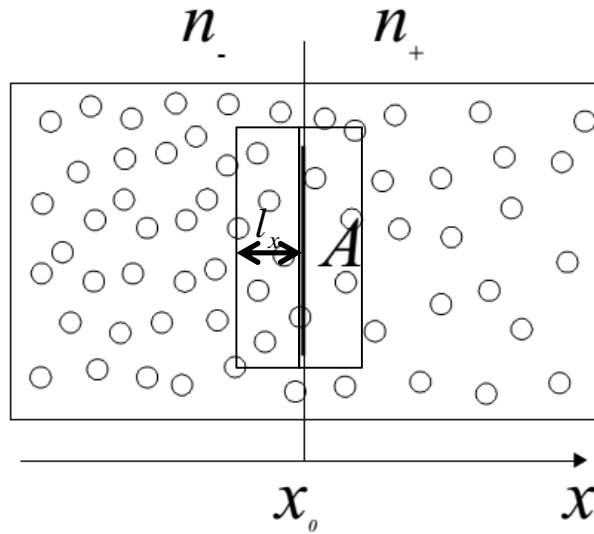
$$\Delta N = N_- - N_+ = \frac{1}{2} \cdot V_t \cdot (n_- - n_+) = \frac{1}{2} \cdot \overbrace{v_x \cdot \Delta t}^{l_x} \cdot A \cdot (n_- - n_+)$$

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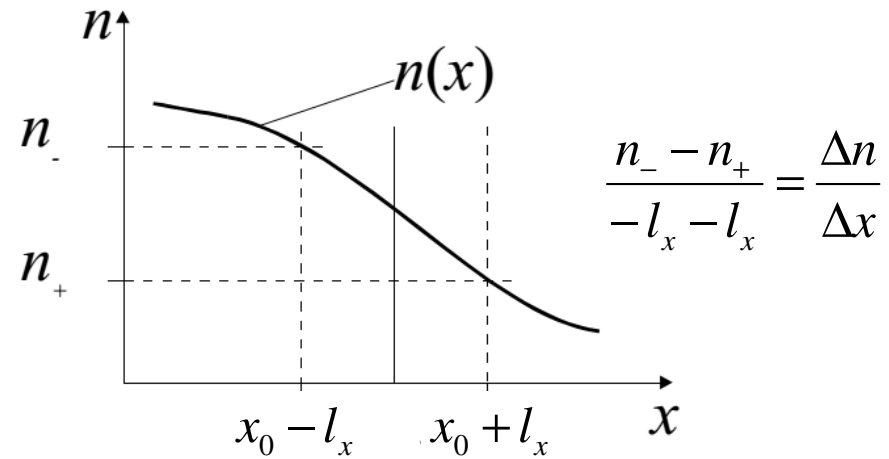
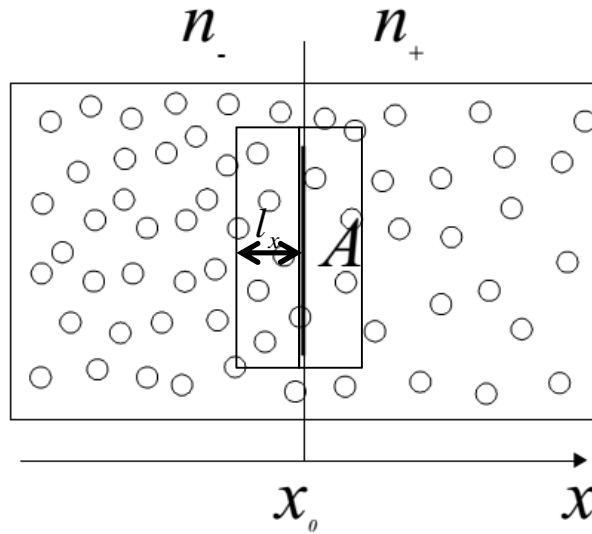


$$\frac{n_- - n_+}{-l_x - l_x} = \frac{\Delta n}{\Delta x}$$

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$$\Delta N = \frac{1}{2} \cdot v_x \cdot \Delta t \cdot A \cdot 2 \cdot l_x \cdot -\frac{\Delta n}{\Delta x}$$

Fick's first law



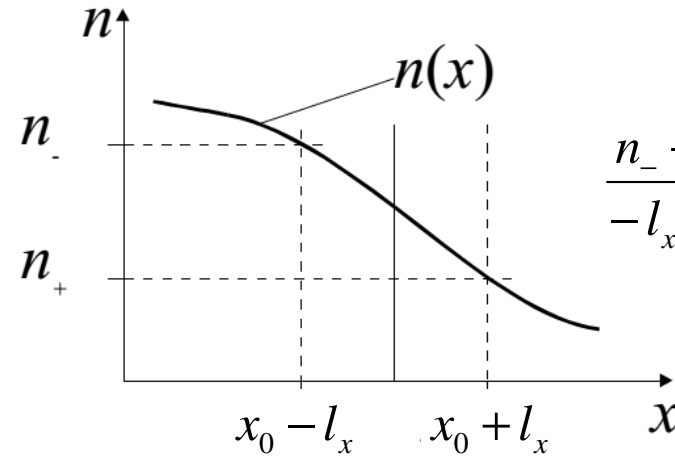
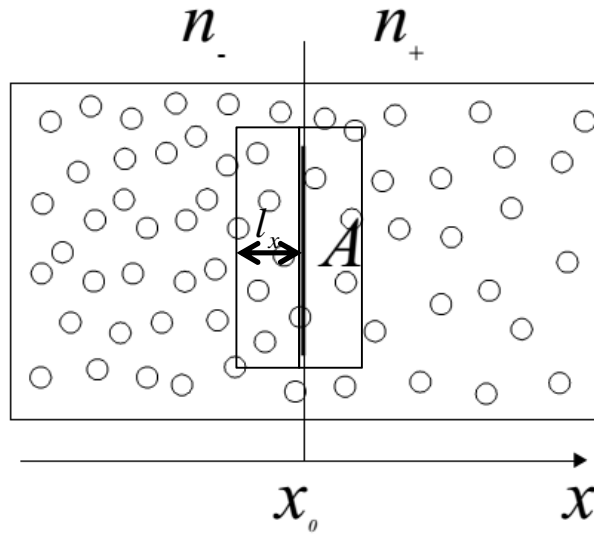
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$$J_{Nx} = \frac{1}{2} \cdot v_x \cdot 2 \cdot l_x \cdot -\frac{\Delta n}{\Delta x} = -D \cdot \frac{\Delta n}{\Delta x}$$

$$v_x \cdot l = D$$

Fick's first law



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$$v_x \cdot l = D$$

$$J_v = -D \cdot \frac{\Delta c}{\Delta x}$$

BUT!: Δc is NOT the „driving force“!

Diffusion coefficient

D gives the amount of matter diffused across a unit area in a unit time in a case of unit concentration gradient.

$$D = \frac{v \cdot l}{3}; \left[\frac{m^2}{s} \right]$$

$$D = u \cdot k \cdot T$$

Einstein-Stokes
(spheres)

$$D = \frac{k \cdot T}{6 \cdot \pi \cdot \eta \cdot r}$$

BUT!

Not directly proportional with T!

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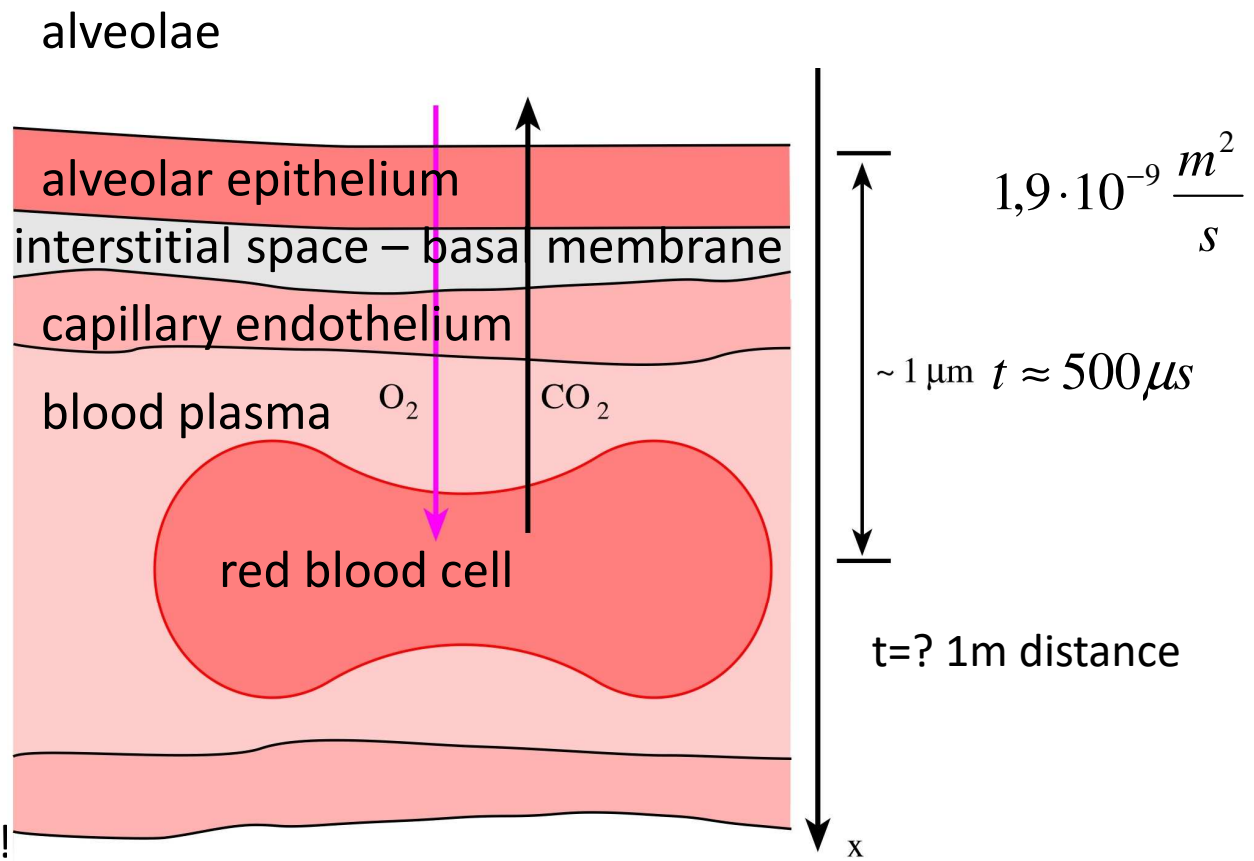
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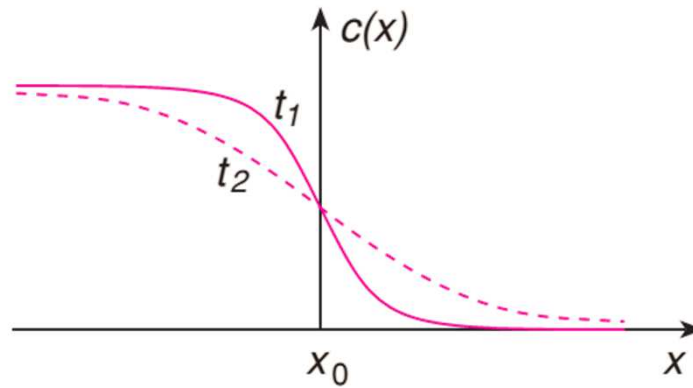
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Fick's second law

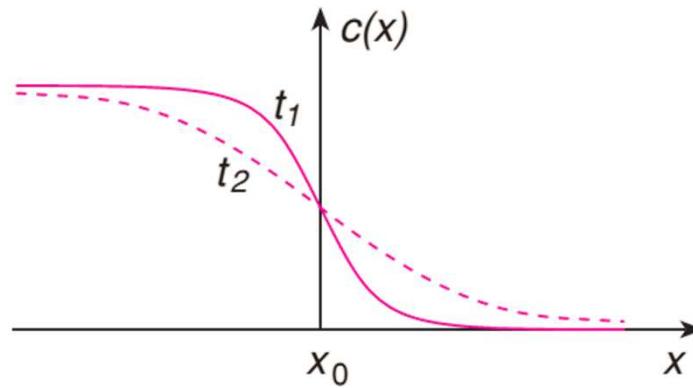
Fick II: change of the concentration gradient in time



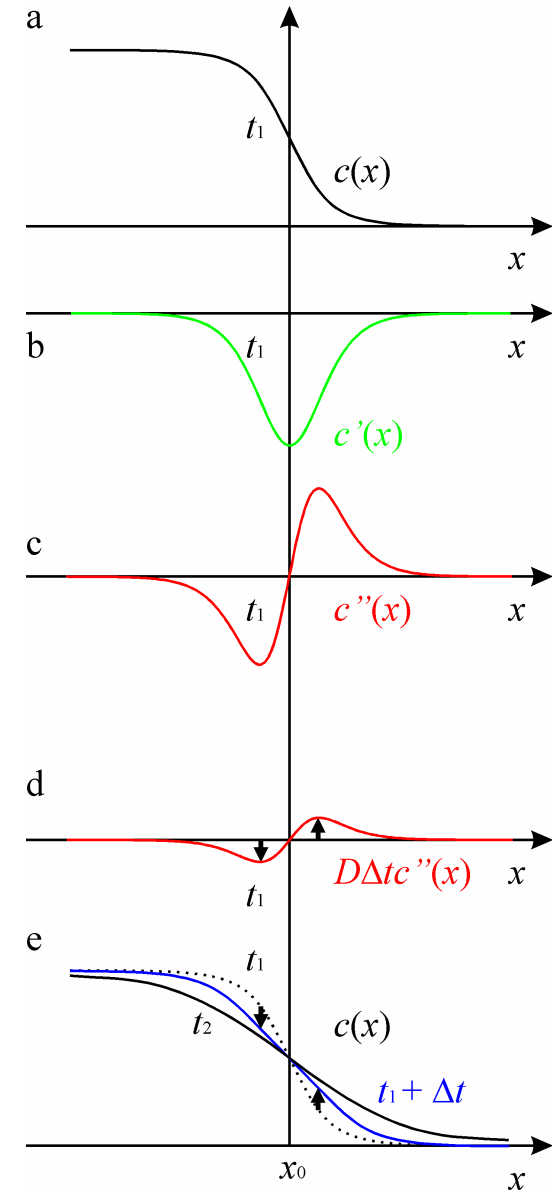
$$c(t + \Delta t) = c(t) + D \cdot \Delta t \cdot \frac{\Delta \left(\frac{\Delta c}{\Delta x} \right)}{\Delta x}$$

Fick's second law

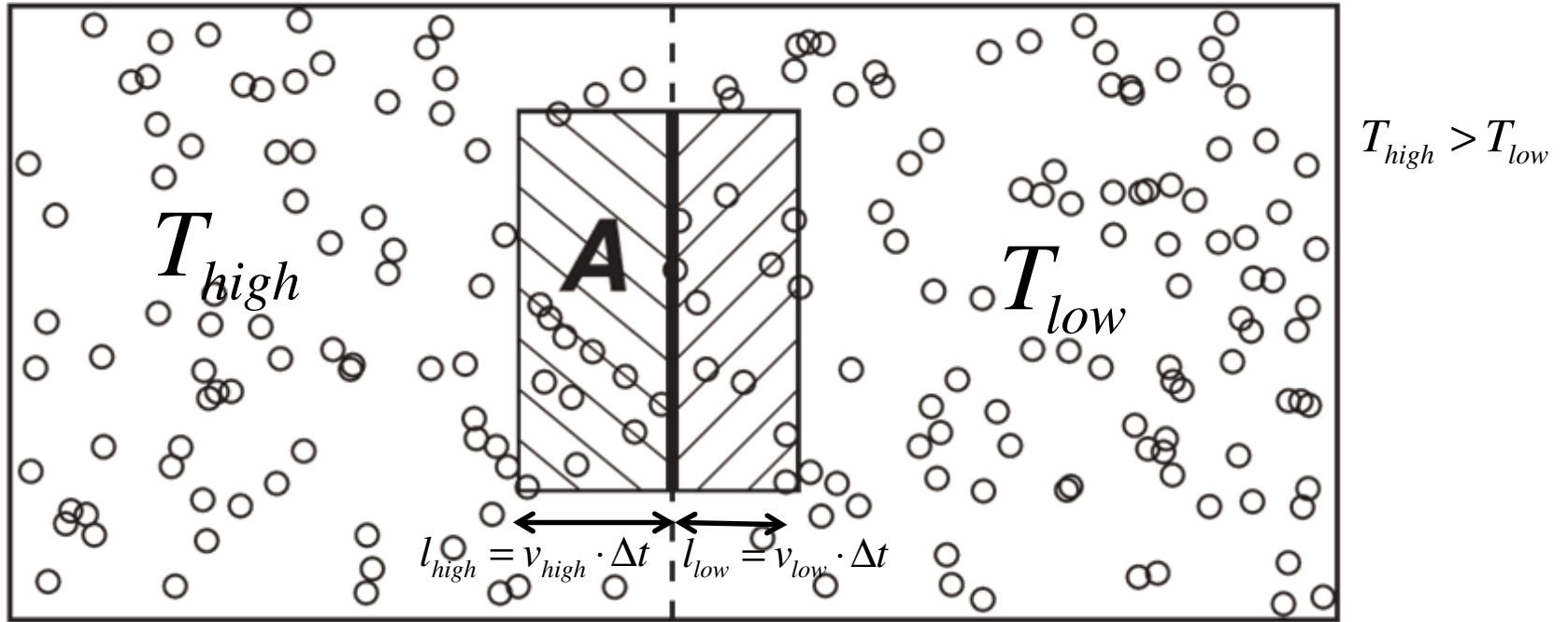
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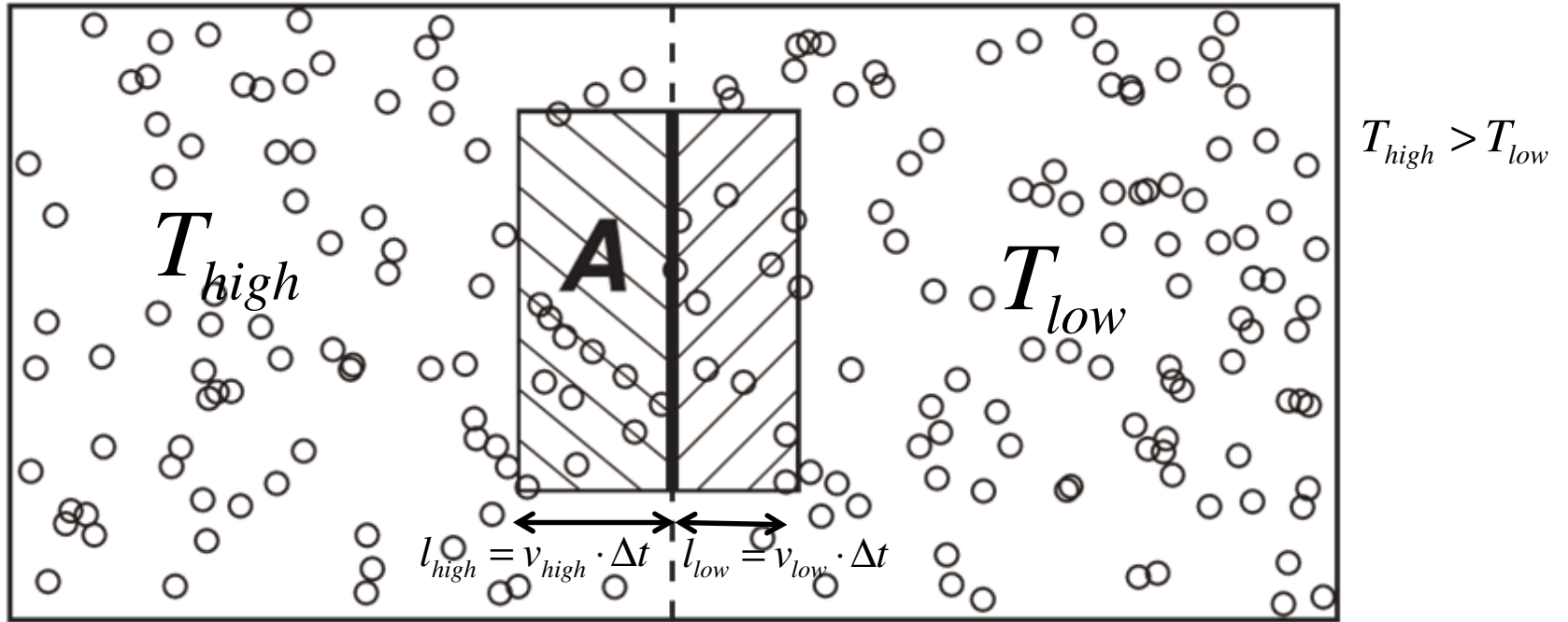


Thermodiffusion



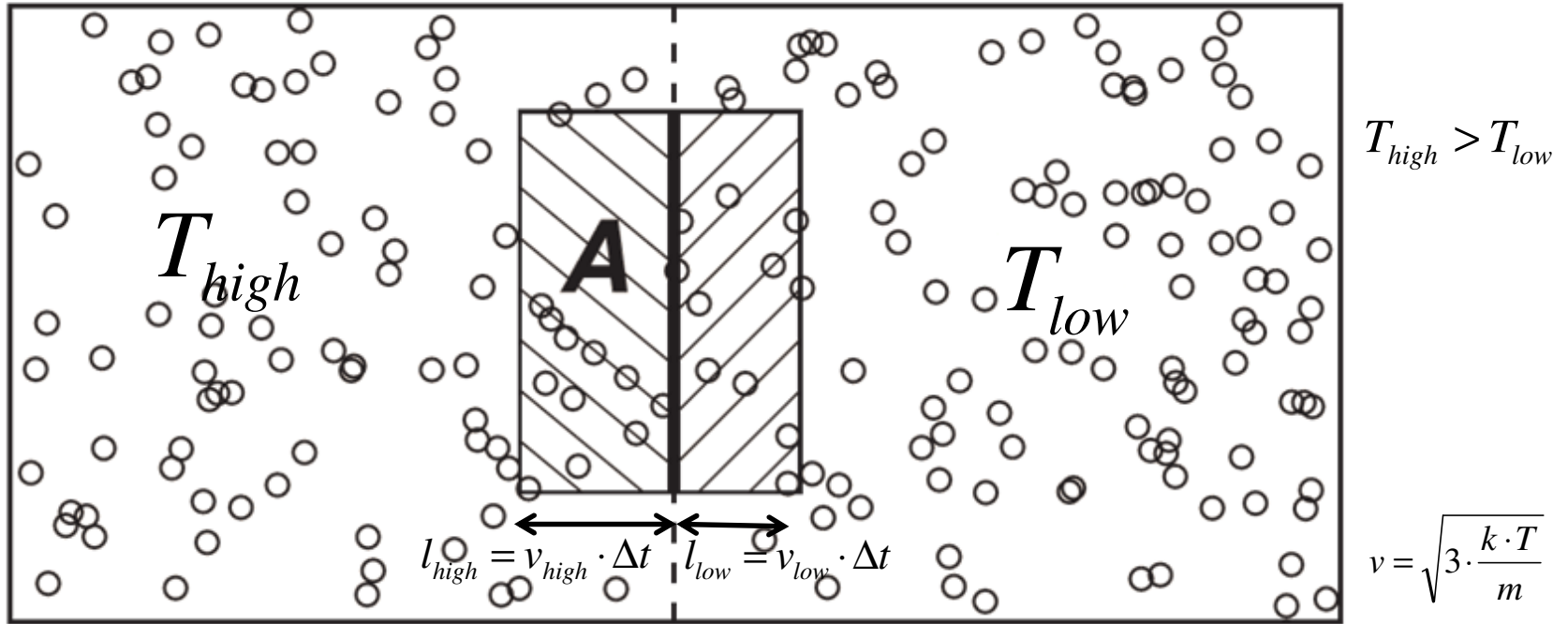
$$\Delta N = N_{high} - N_{low} = \frac{1}{2} \cdot n \cdot \Delta t \cdot A \cdot v_{high} - \frac{1}{2} \cdot n \cdot \Delta t \cdot A \cdot v_{low}$$

Thermodiffusion



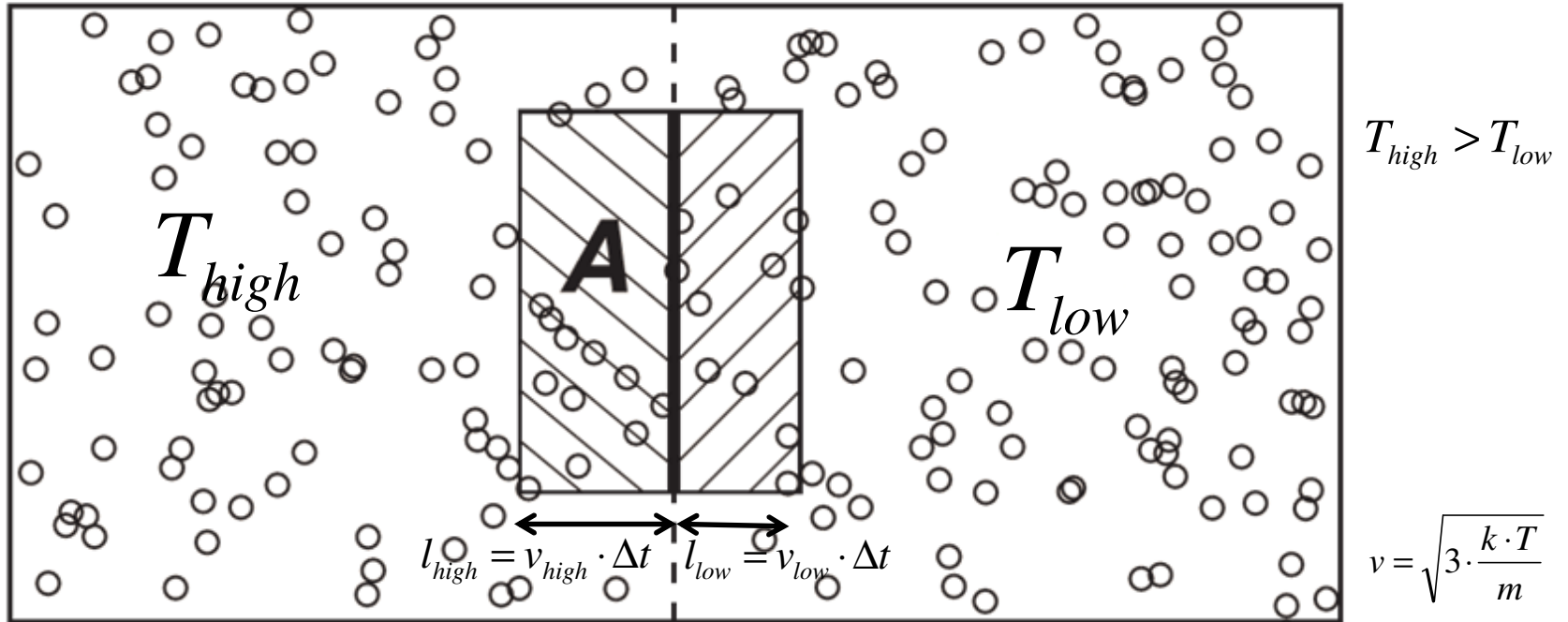
$$\Delta N = N_{high} - N_{low} = \frac{1}{2} \cdot n \cdot \Delta t \cdot A \cdot (v_{high} - v_{low}) = \frac{1}{2} \cdot n \cdot \Delta t \cdot A \cdot (v_{high} - v_{low}) \cdot \frac{(v_{high} + v_{low})}{(v_{high} + v_{low})}$$

Thermodiffusion



$$\frac{1}{2} \cdot n \cdot \Delta t \cdot A \cdot \frac{(v_{high}^2 - v_{low}^2)}{(v_{high} + v_{low})} = \frac{1}{2} \cdot n \cdot \Delta t \cdot A \cdot \frac{\left(\frac{3 \cdot k \cdot T_{high}}{m} - \frac{3 \cdot k \cdot T_{low}}{m}\right)}{2 \cdot v_{mean}}$$

Thermodiffusion

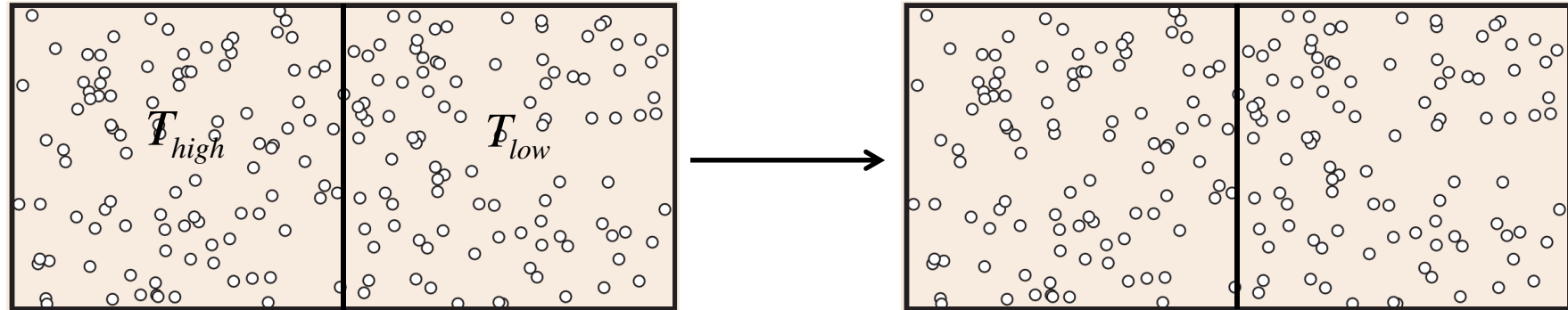


$$\frac{1}{2} \cdot n \cdot \Delta t \cdot A \cdot \frac{(v_{high}^2 - v_{low}^2)}{(v_{high} + v_{low})} = \frac{1}{2} \cdot n \cdot \Delta t \cdot A \cdot \frac{(\frac{3 \cdot k \cdot T_{high}}{m} - \frac{3 \cdot k \cdot T_{low}}{m})}{2 \cdot v_{mean}}$$

$$\frac{1}{2} \cdot \left(\frac{n \cdot \Delta t \cdot A \cdot 3 \cdot k}{m \cdot 2 \cdot v_{mean}} \right) \cdot (T_{high} - T_{low})$$

$$J_v = -L_T \cdot \frac{\Delta T}{\Delta x} \quad (\text{Ludwig-Soret effect})$$

Heat conduction



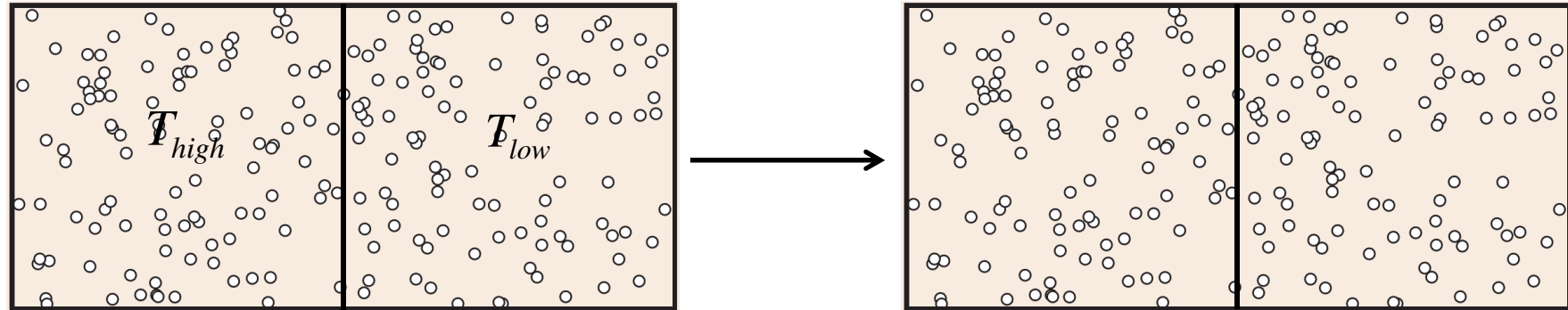
$$T_{high} > T_{low}$$

$$\Delta N = N_{high} - N_{low} = 0$$

$$N_{high} = N_{low}$$

$$\bar{\epsilon} = \frac{3}{2} \cdot k \cdot T$$

Heat conduction



$$T_{high} > T_{low}$$

$$\Delta N = N_{high} - N_{low} = 0$$

$$N_{high} = N_{low}$$

$$\bar{\varepsilon} = \frac{3}{2} \cdot k \cdot T$$

Energy flow density

$$J_v = \frac{\Delta E}{A \cdot \Delta t} = \frac{N_{high} \cdot \frac{3}{2} \cdot k \cdot (T_{high} - T_{low})}{A \cdot \Delta t} = -\lambda \cdot \frac{\Delta T}{\Delta x}$$

(Fourier law for heat conduction)

Generalization

Onsager-relation: $J_{ext.} = L_{cond} * X_{int_grad}$

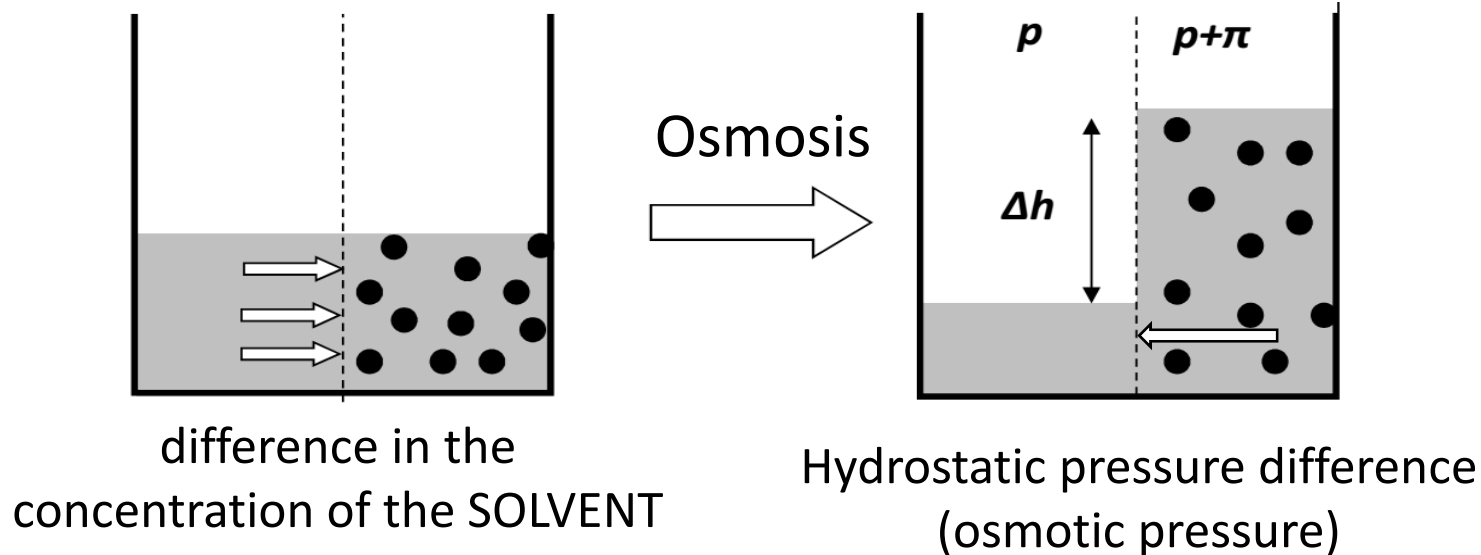
$J_{ext.}$: flow density of extensive quantity (eg. J_{matter})

X_{int_grad} : gradient of intensive quantity (eg. $\frac{\Delta c}{\Delta x}$)

L_{cond} : conductivity coefficient (eg. D)

Osmosis

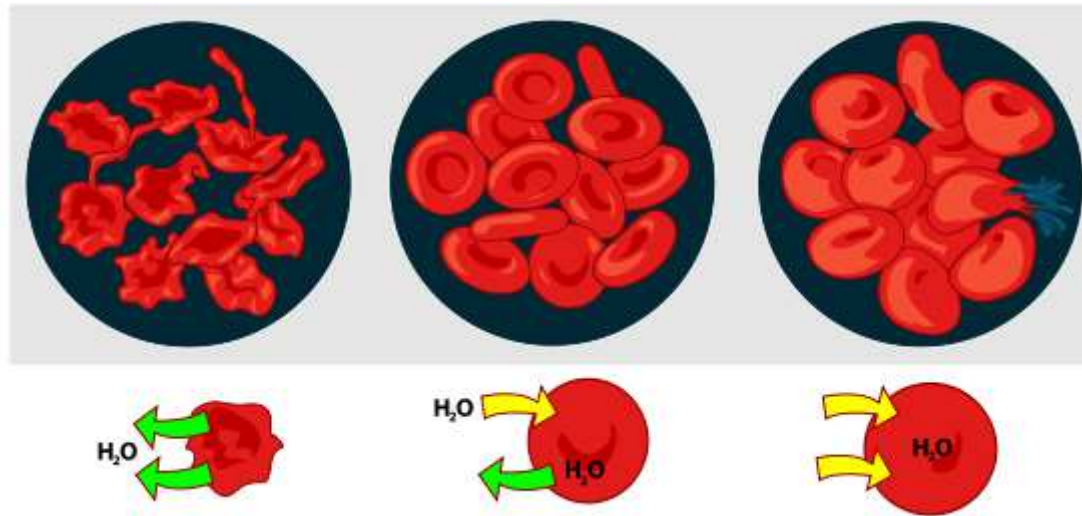
One-way diffusion of the SOLVENT.



$$p_{osm} = \pi = c_{solute} \cdot R \cdot T \quad (\text{Van 't Hoff law})$$

Osmotic concentration (equivalent osmotic pressure, „ozmolarity”, „ozmolality”):
The concentration of a solution that keeps balance with a heterogeneous solution.
Derived units: *mOsm*(/L), mmol/L, mmol/kg

Medical practice



Osmotic concentration of the blood: about 300mOsm/L

Isosmotic („Physiological”, „isotonic”, „normal”) solutions:

Physiological/Normal/Isotonic saline: 0,9% (w/v) NaCl

d5W: 5% (w/v) glucose

Ringer, Ringer's lactate