Transport processes-1
fluid flow

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transport = translocation of materials

biological significance: material flow among or inside cells
(breathing, blood circulation, transport across membranes,
metabolism)

distinct transport mechanisms:

– particles of the quantity in question are translocated collectively, such as in a fluid flow

– the detected macroscopic translocation is the result of individual motions of particles, such as diffusion topics covered:

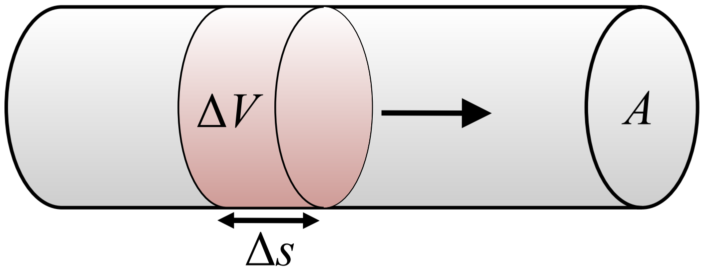
– types and laws of flow of fluids and gases

– physical properties of blood determining flow properties

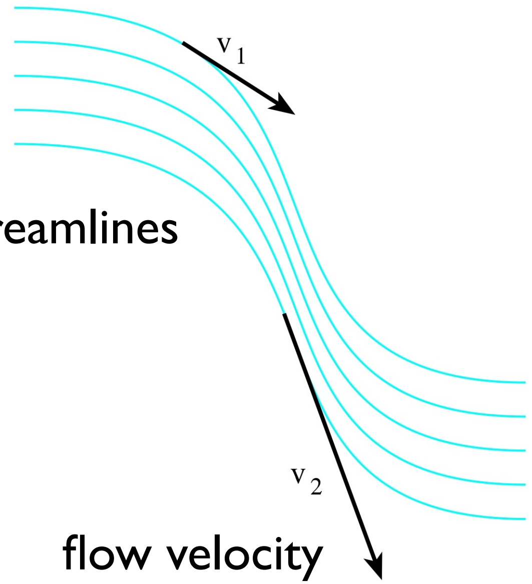
Flow in rigid-wall tubes

Volumetric flow rate (I_V):

$$I_V = \frac{\Delta V}{\Delta t} = A\frac{\Delta s}{\Delta t} = A\overline{v}$$



Average velocity: $\overline{v} = \frac{I_V}{A}$



(direction of tangent = direction of velocity
density of streamlines = magnitude of velocity)

Measuring flow rate

ultrasound methods (Doppler-examination):

f frequency

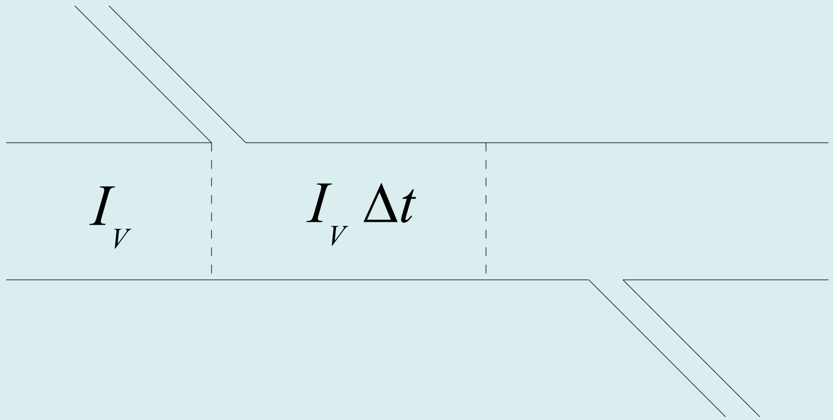
υ velocity of the object

c ultrasound velocity

$$f = f_0\left(1 \pm \frac{\upsilon}{c}\right)$$

dilution techniques: ν amount of dye (mol); c concentration of dye

$$\frac{\Delta\nu}{\Delta t} = M$$



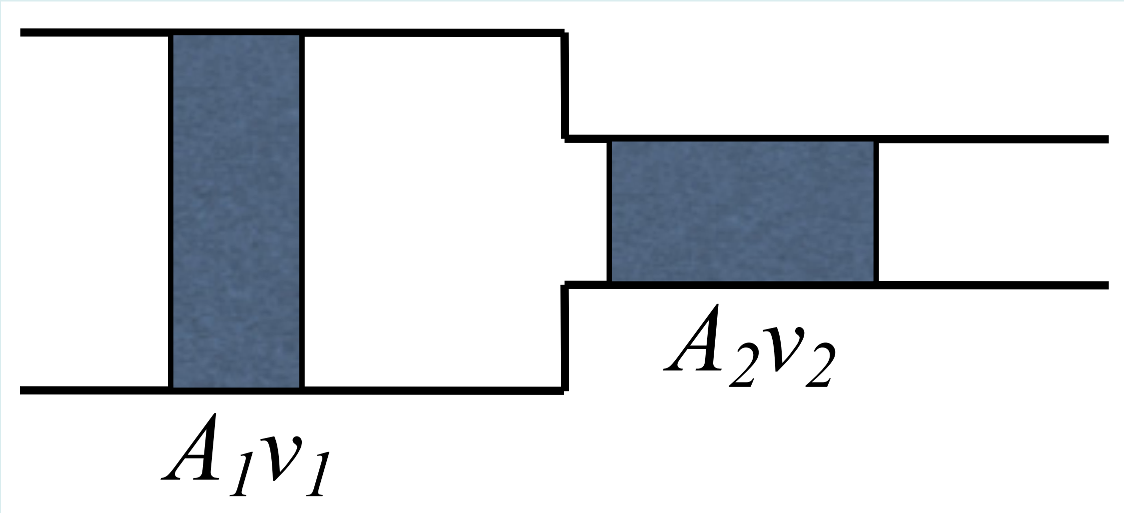
$$c = \frac{\Delta\nu}{\Delta V} = \frac{\Delta\nu}{I_V \Delta t}$$

Stacionary flow

parameters of flow are constant in time

Continuity principle: in a stacionary flow the volumetric flow rate is constant

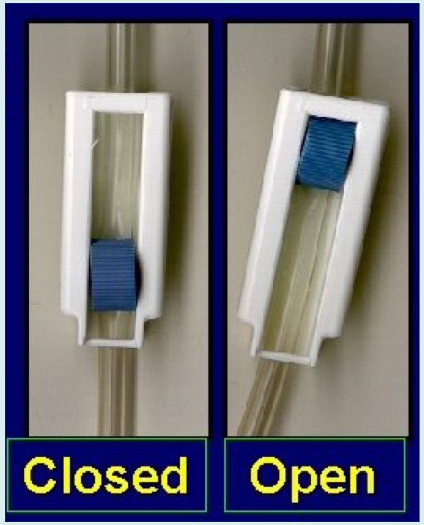
e.g. law of mass balance for incompressible fluids (gases?)



$$A_1v_1 = A_2v_2 = \text{constant}$$

A = cross-sectional area

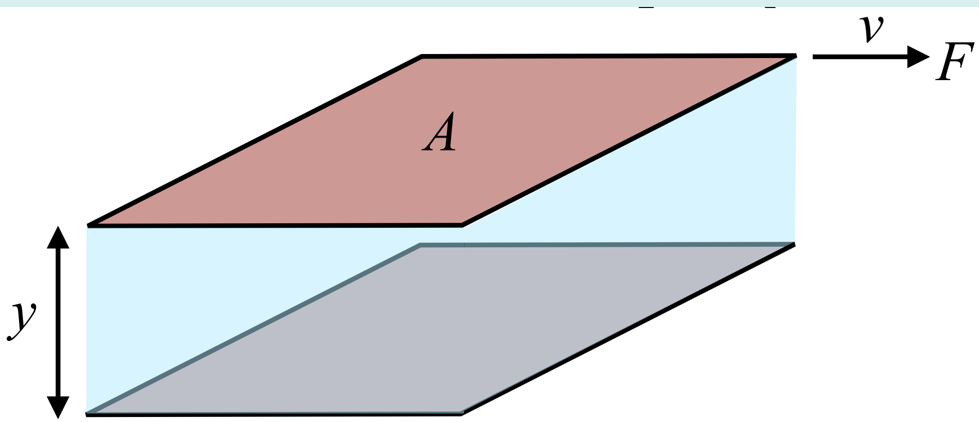
v = flow velocity

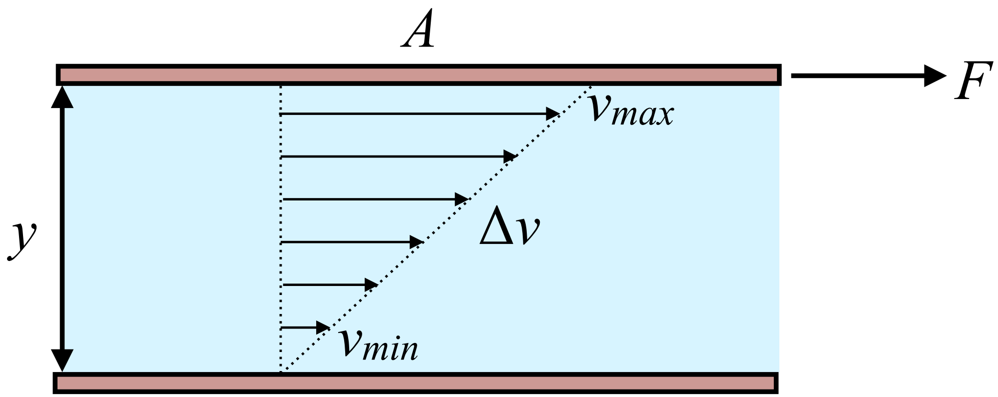


compressibility:

$$\kappa = -\frac{1}{V}\frac{\Delta V}{\Delta p}$$

Viscosity – internal friction





F	= shear force
A	= area of fluid layer
η	= viscosity
v	= flow velocity
y	= distance between fluid layers
F/A	= shear stress (τ)
$\Delta v/\Delta y$	= velocity gradient (D)

$$\frac{F}{A} = \eta \frac{\Delta v}{\Delta y}$$ (Newton's friction law)

$$\eta = \frac{\tau}{D}$$

Units of viscosity: $1 Pas = 1 \frac{Ns}{m^2} = 10 P (poise)$

Viscosity of distilled water (25 ˚C): 1 mPas (1 centipoise)

Viscosity – internal friction

substance	η (mPa·s) 20 °C
air	(101 kPa) 0.019
water	1
ethanol	1.2
blood (37 °C)	2–8
glycerine	1490
honey	2000–14000

viscosity of gases increases with rising temperature $\eta \sim T$

viscosity of fluids decreases with rising temperature

$$\eta \sim e^{\frac{E}{kT}}$$

$$\frac{F}{A} = \eta \frac{\Delta v}{\Delta y}$$

Fluids obeying the above equation are called **newtonian fluids**.

Types of flow based on viscosity

1. Ideal

frictionless, incompressible

ρ = constant, η = 0

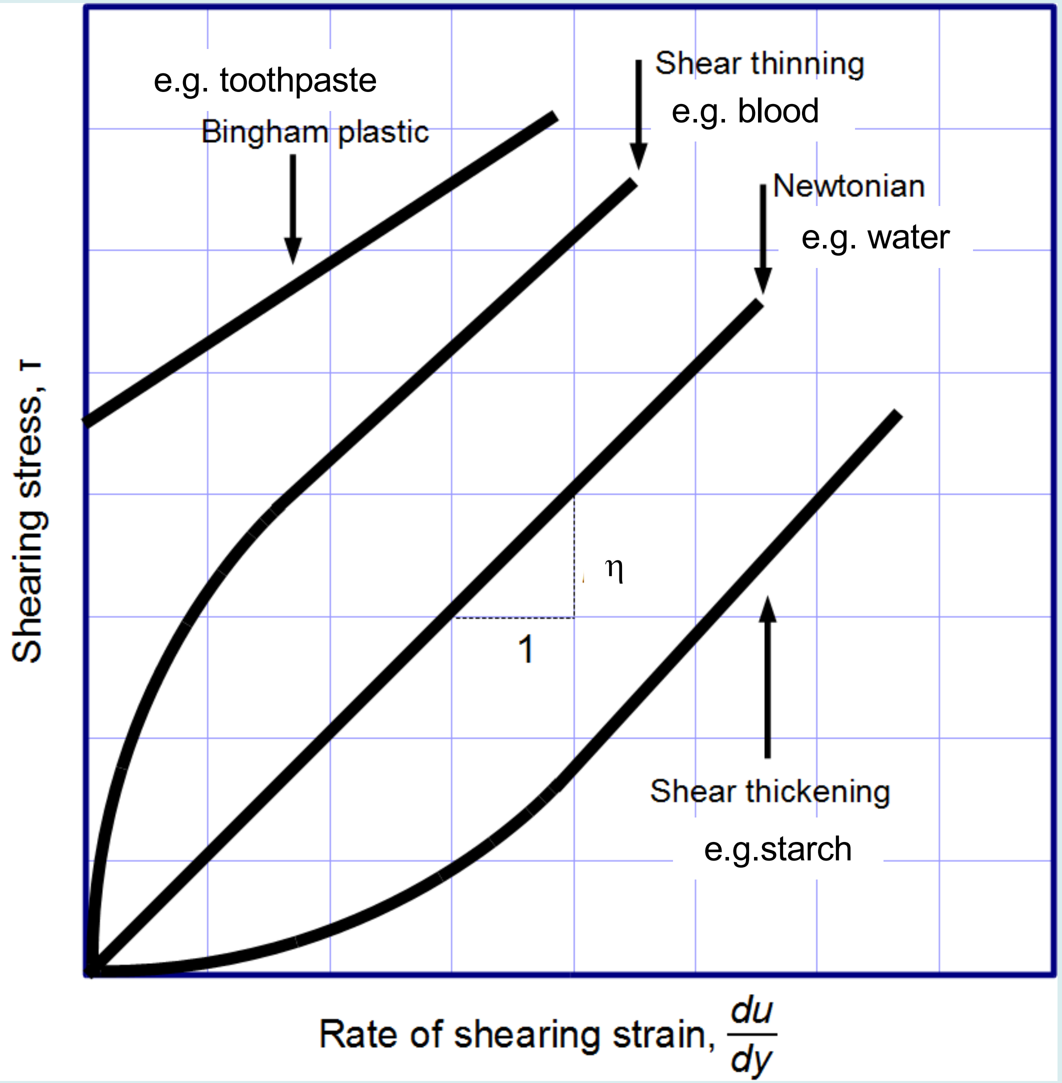
2. Non-ideal (real)

a. Newtonian (viscous)

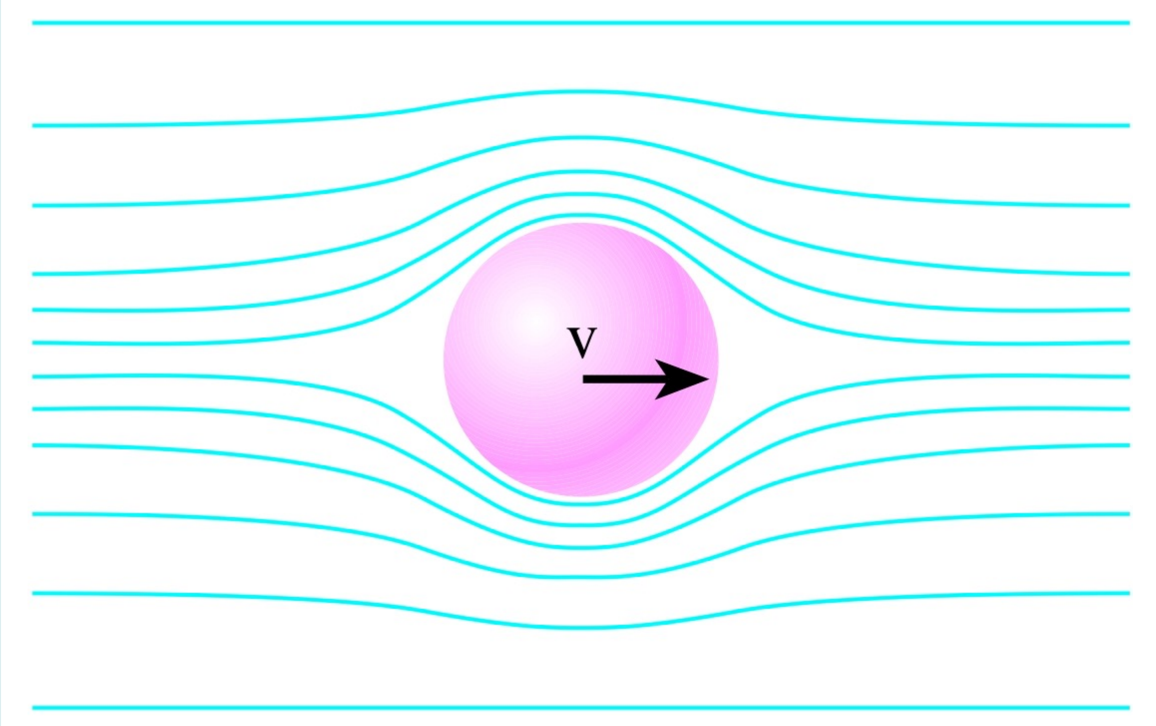
η independent of shear stress

b. Non-newtonian (anomalous)

η changes with shear stress



Friction on spherical particles – Stokes law



Georg Gabriel Stokes
(1819-1903)

Frictional force is proportional with the velocity:

$$u = \frac{\upsilon}{F} \quad , \quad u = \frac{1}{6\pi\eta r}$$

$$F_s = 6\pi\eta r\upsilon$$

u is the mobility of the sphere, which is the value of the velocity resulting from a unity of force.

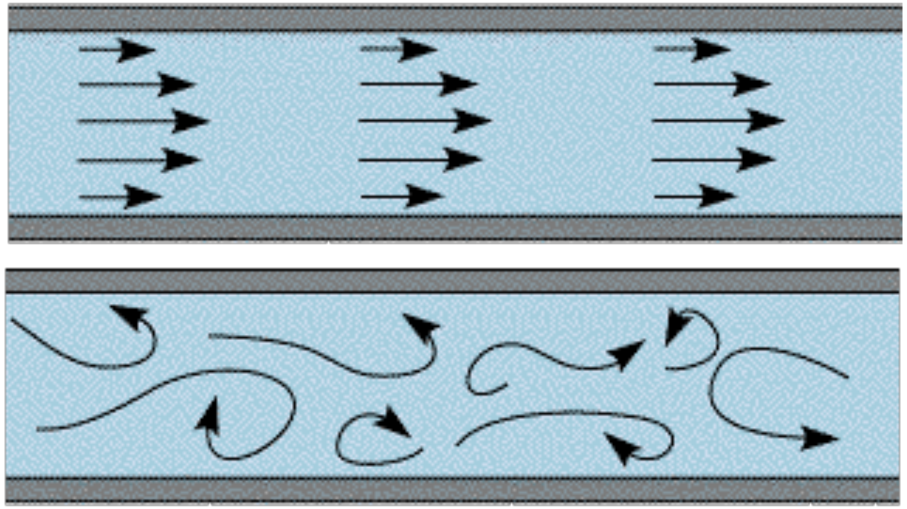
Types of flow:

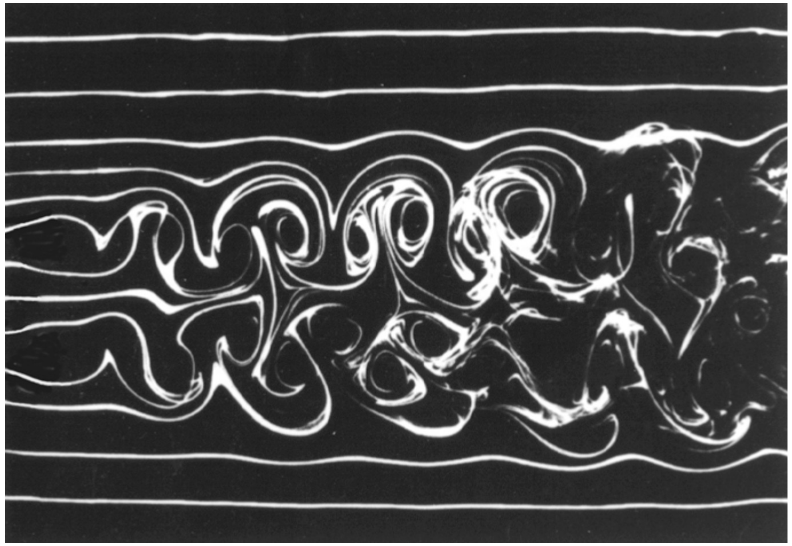
laminar flow – flow happens in layers, which do not mix

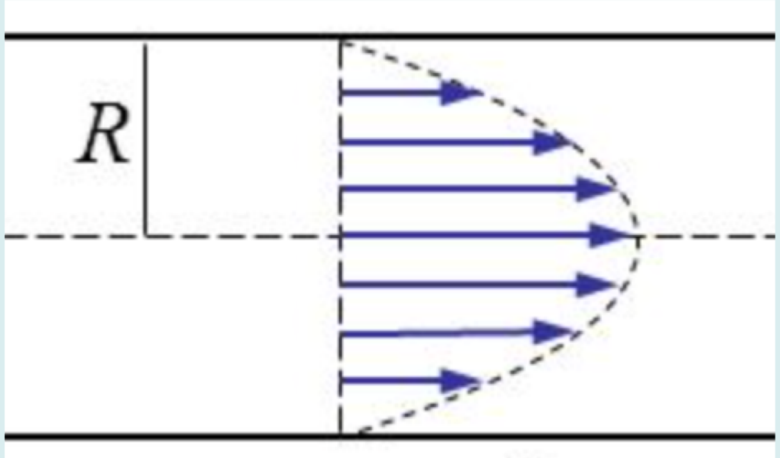
turbulent flow – layers are mixing (chaotic streamlines)

laminar flow

turbulent flow

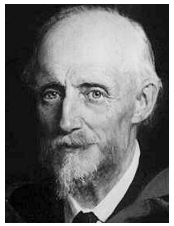






parabolic velocity profile

Reynolds number (Re):



Osborne Reynolds (1842-1912)

$$Re = \frac{v r \rho}{\eta}$$

v = flow rate (m/s)
r = tube radius (m)
ρ = density of fluid (kg/m^3)
η = viscosity (Ns/m^2)

$Re \approx 2000$

Re > 1000 – turbulency appears

Bernoulli's law

conservation of energy in ideal fluids

Daniel Bernoulli
(1700-1782)

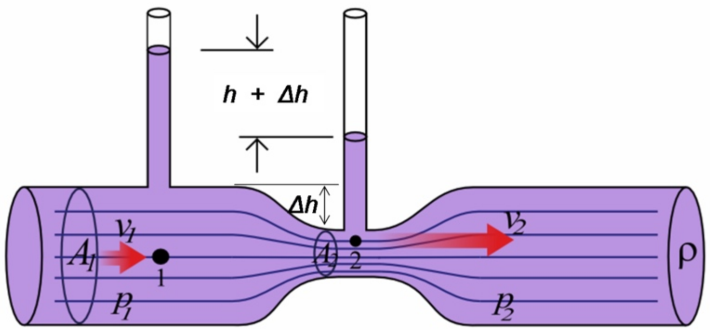
$$p + \frac{1}{2}\rho v^2 + \rho g h = \; const$$

p = static pressure

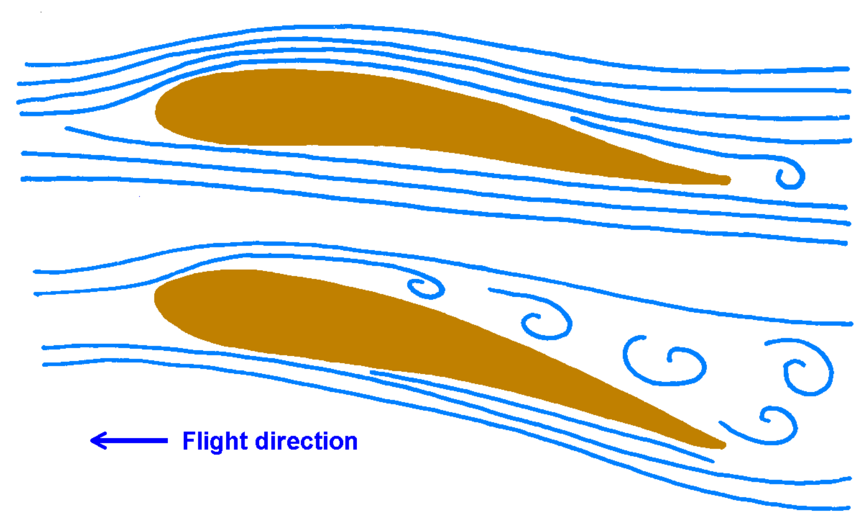
$\frac{1}{2}\rho v^2$ = dynamic pressure

$\rho g h$ = hydrostatic pressure

Giovanni Battista
Venturi
(1746-1822)



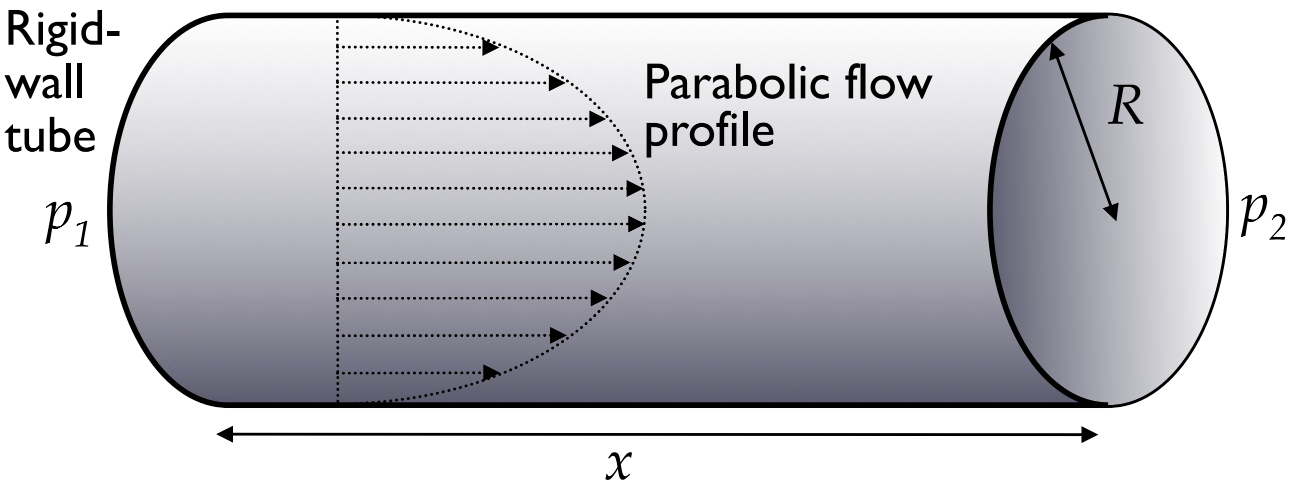
Static pressure drops at the
tube narrowing



Applications: sprayer, atomizer, aspirator (vacuum pump), inspirator (Bunsen burner), diffuser, jet engine, wing profile, vacuum cleaner, biphasic pulse in aorta insufficiency

Hagen-Poiseuille's law

Stacionary flow of newtonian fluids and gases in rigid-wall tubes




G.H.L. Hagen (1797-1884)


J.-L.-M. Poiseuille (1799-1869)

V = volume
t = time
R = tube radius
η = viscosity
p = pressure
x = tube length

$V/t = I_V$ = volumetric vlow rate
$\Delta p/\Delta x$ = pressure gradient, maintained by p_2-p_1 *(negative!)*
A = cross-sectional area of tube
I_V = volumetric flow rate

$$I_V = \frac{V}{t} = -\frac{R^4\pi}{8\eta}\frac{\Delta p}{\Delta x}$$

$$I_V = -\frac{R^4\pi}{8\eta\Delta x}\Delta p \Rightarrow -\Delta p = R_{tube}\cdot I_V \Rightarrow U = R\cdot I$$

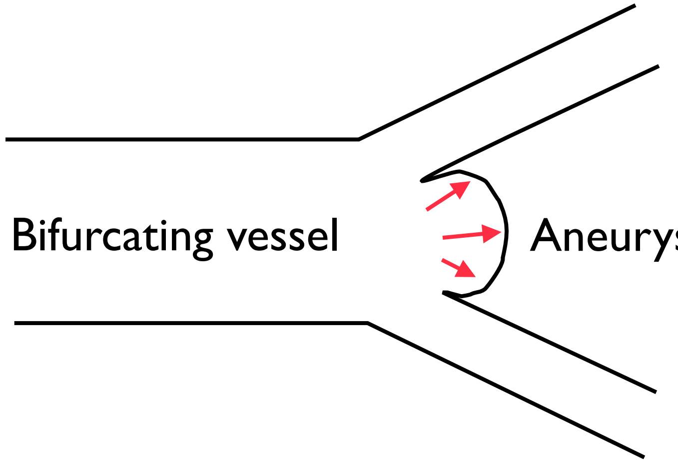
$1/R_{tube}$ — Ohm's law!

$$\frac{\Delta v}{\Delta r} \sim r \Rightarrow \left(\frac{\Delta v}{\Delta r}\right)_{max} = R$$

Shear stress is maximal at the tube wall

Medical significance of fluid flow

Bernoulli's law:



Formation of aneurysm (pathological expansion of blood vessel):

- Expansion of vessel: diameter increases
- Flow rate decreases, accoring to continuity equation
- Static pressure increases due to Bernulli's law
- Aneurysm pregrediates - positive fedback mechanism leading to catastrophy

Hagen-Poiseuille's law:

$$\frac{V}{t} = \frac{R^4 \pi}{8\eta} \frac{\Delta p}{\Delta x}$$

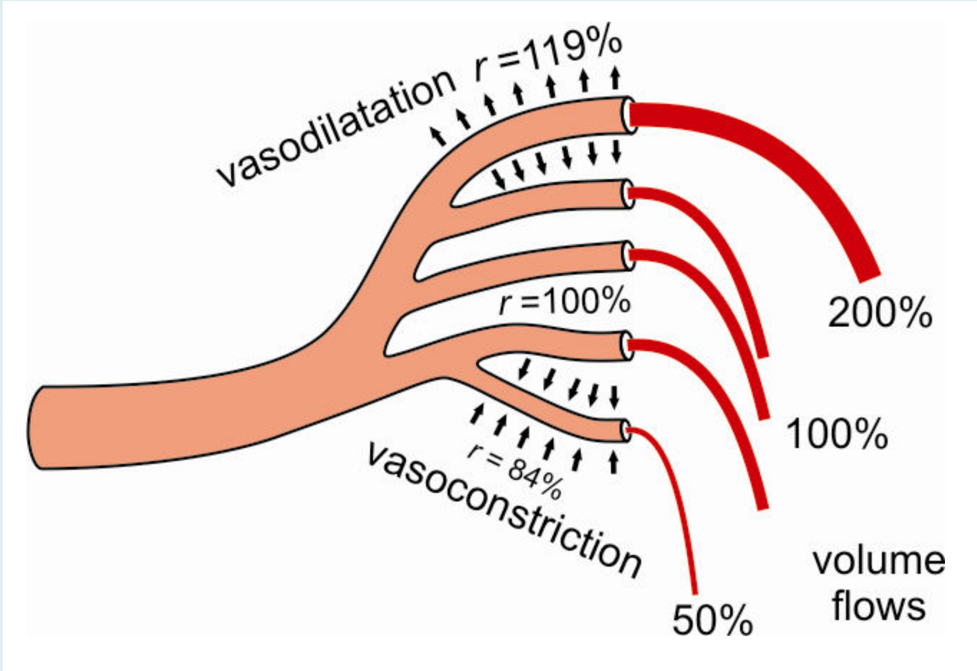
Flow intensity, hence the delivered oxygen quantity, may be **drastically reduced** in certain pathological conditions:

- constriction of blood vessels (e.g., diabetes, Bürger's disease)
- change in blood viscosity (e.g., fever, anaemia)
- Reduction of vessel diameter by half leads to a reduction of volumetric flow by 1/16!

Medical significance of fluid flow

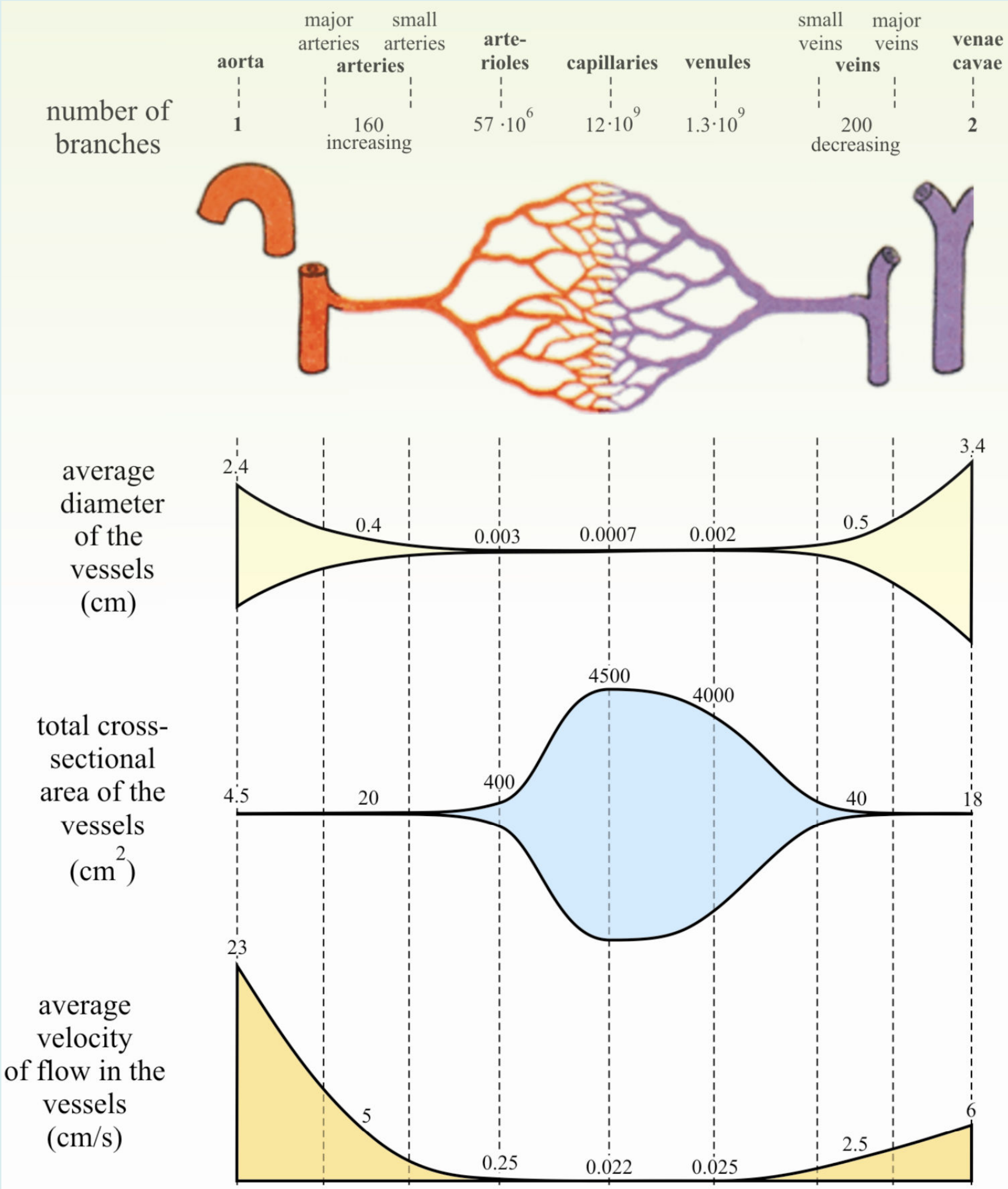
regulation of the intensity of blood flow
(Hagen-Poiseuille's law)

$$I_V = \frac{V}{t} = \frac{R^4 \pi}{8\eta} \frac{\Delta p}{\Delta x}$$

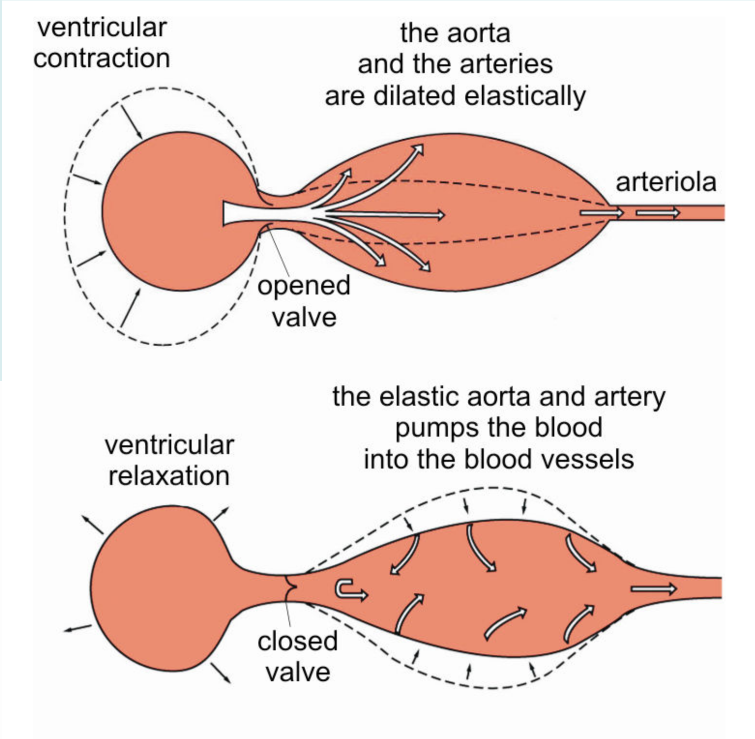


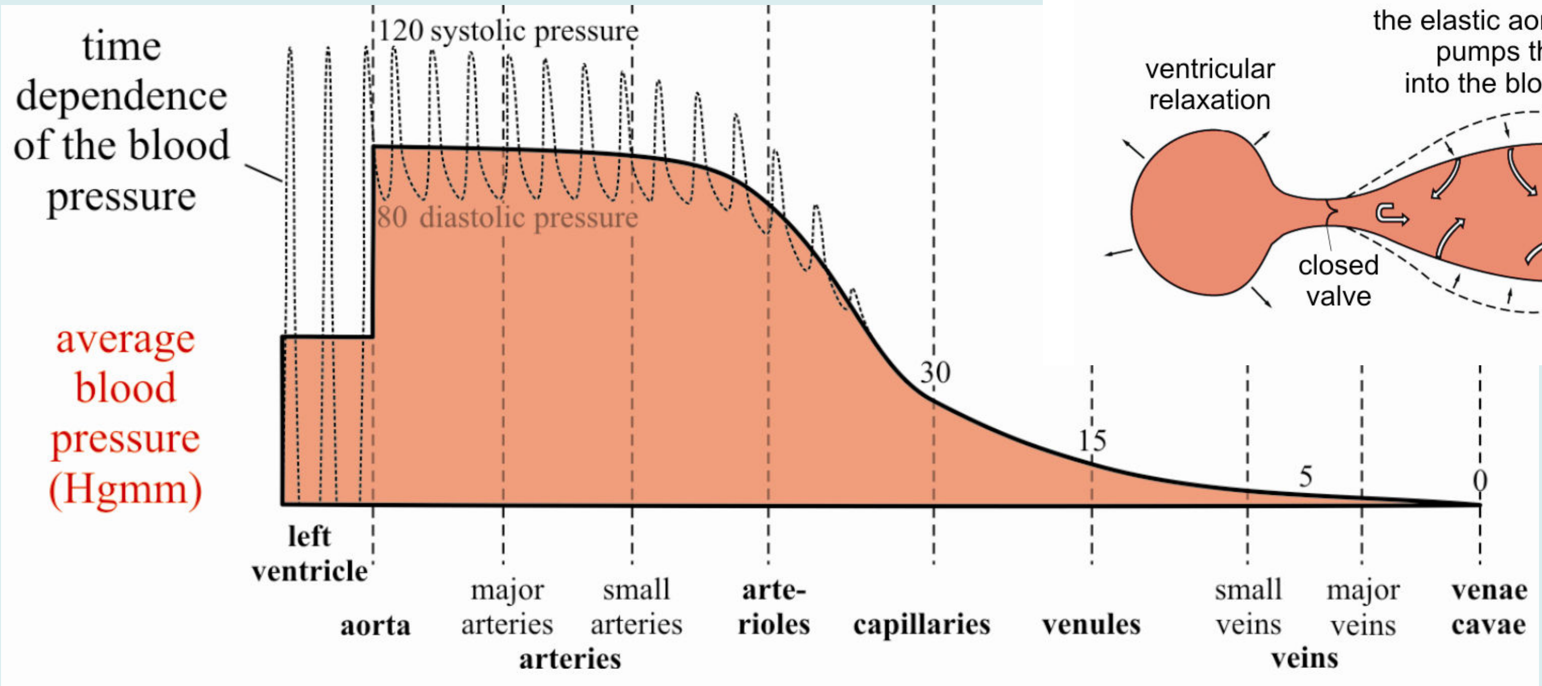
Physiological regulation of the blood flow depending on the needs of tissues/organs

Flow parameters in the circulation



Flow conditions in the arterial system





Viscosity of blood

<table>
<tr><td colspan="3">55-60% of body mass is water
42 kg (70 kg body mass)</td></tr>
<tr><td>2/3 intracellular
28 kg</td><td colspan="2">1/3 extracellular
14 kg</td></tr>
<tr><td></td><td>1/3 plasma
4-5 kg</td><td>2/3 intersticium
9-10 kg</td></tr>
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Blood:

Average volume: 5 l
Average viscosity: 5 mPas
Average density: 1.05 g/cm^3
Composition: 40-45 % corpuscular, 55-60 % plasma

Viscosity of blood

1. Hematocrit (htc, ϕ):

$$htc = \frac{V_{cells}}{V_{total}}$$

Normal range: 0.4-0.5.

Viscosity of blood as suspension
(in the physiologically relevant htc range):

$$\lg \eta_s = A + B\phi$$

η_s=suspension viscosity
A, B=empirical constants

Viscosity of blood

2. Plasma viscosity

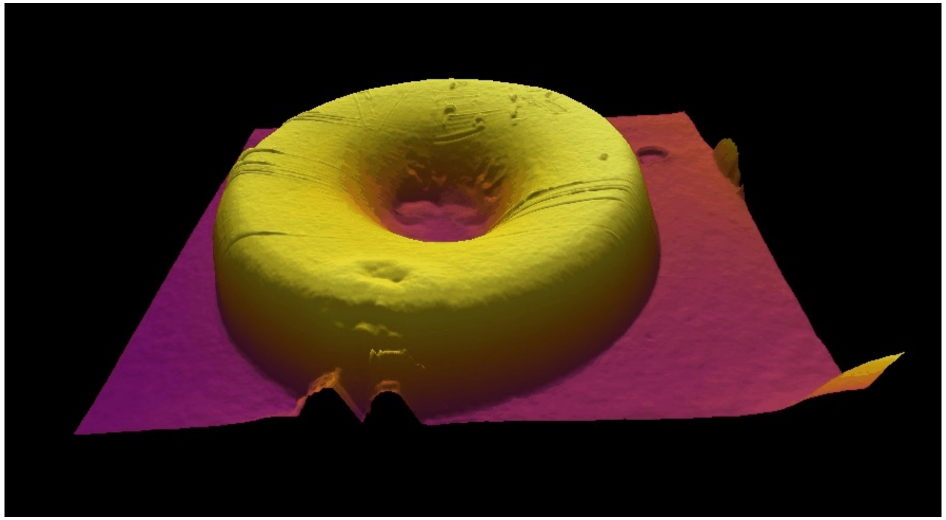
- Depends on plasma proteins.
- In *paraproteinaemias* (e.g. myeloma multiplex or plasmocytoma) the concentration of immunoglobulins is high, leading to increased viscosity.

Plasma protein	**Normal concentration**	**% ratio**	**Function**
Albumin	35-50 g/l	55%	maintenance of colloind osmotic pressure, transport
Globulins	20-25 g/l	38%	Part of the immune system
Fibrinogen	2-4.5 g/l	7%	Blood coagulation

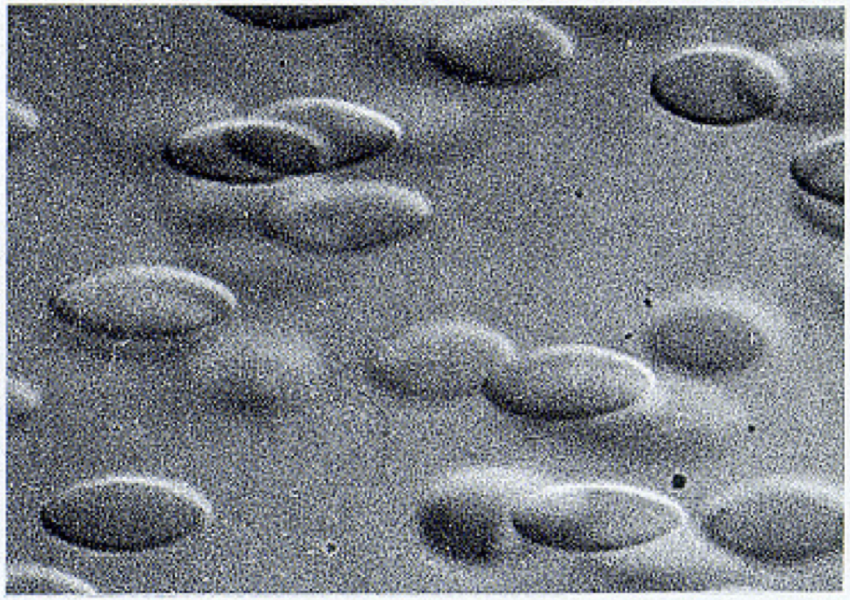
Viscosity of blood

3. Plasticity of red blood cells

- 65% suspension of blood-cell-size particles is rock hard.
- By contrast, a 95% blood suspension if fluid, with viscosity of ~20 mPas!
- Deformation of red blood cells: droplet, parachute, arrowhead shapes.



Disc-shaped cell with 7-11 um diameter



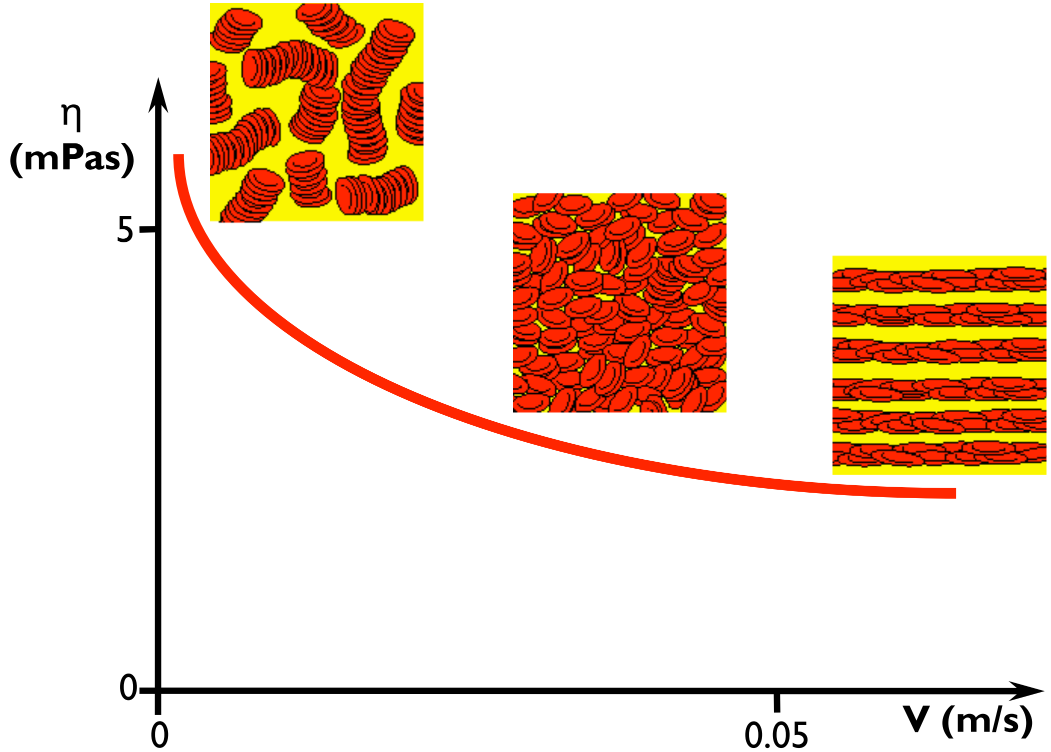
Viscosity of blood

4. Aggregation of red blood cells

- Stack or roleaux formation.
- More pronounced at low flow rates

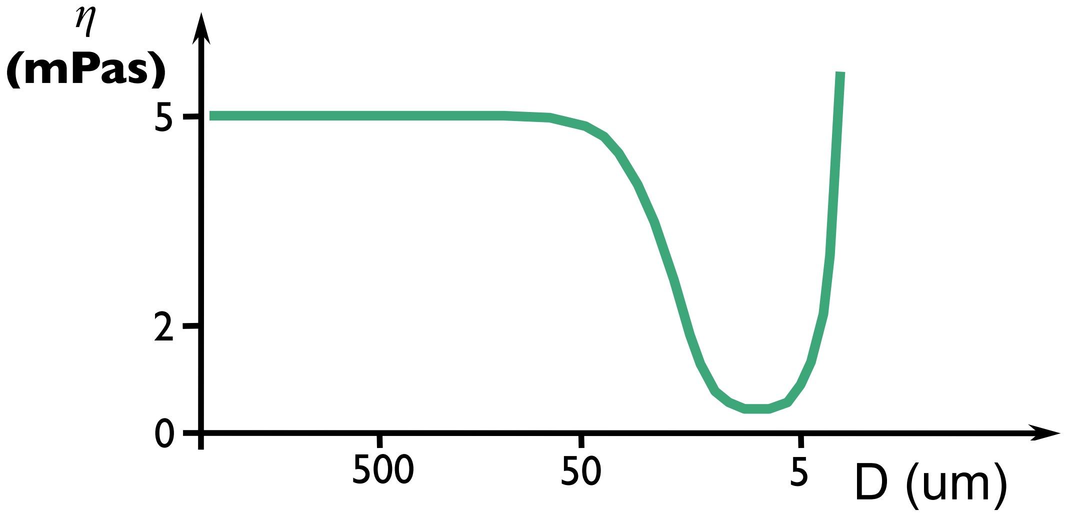
Roleaux (stack)

5. Flow rate, velocity gradient

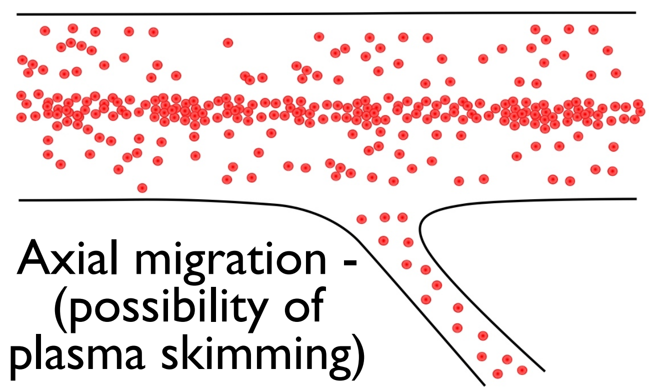


Viscosity of blood

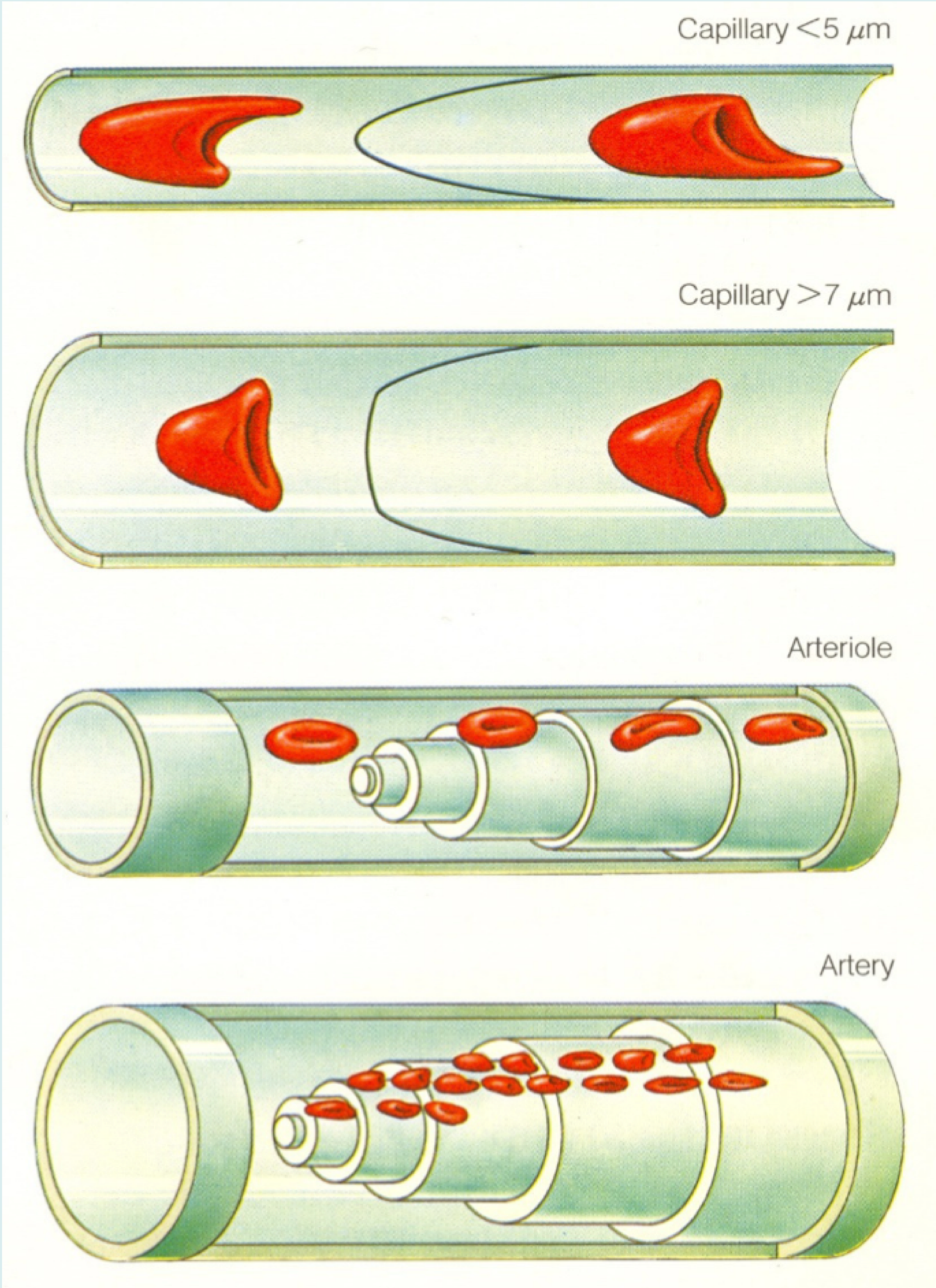
6. Blood vessel diameter



- With a decrease of vessel diameter, the anomalous (non-newtonian) behavior of blood becomes more pronounced.
- *Axial migration*: the red blood cells line up in the axis of the vessel (Bernoulli's law). In the axis the velocity gradient decreases, and near the vessel wall it increases. Increase in velocity gradient decreases apparent viscosity (Fåhraeus-Lindquist effect).



Adaptation of RBC due to deformability



Related chapters:

Damjanovich, Fidy, Szöllősi: Medical biophysics

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1.1
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1.5

Practicals: Flow