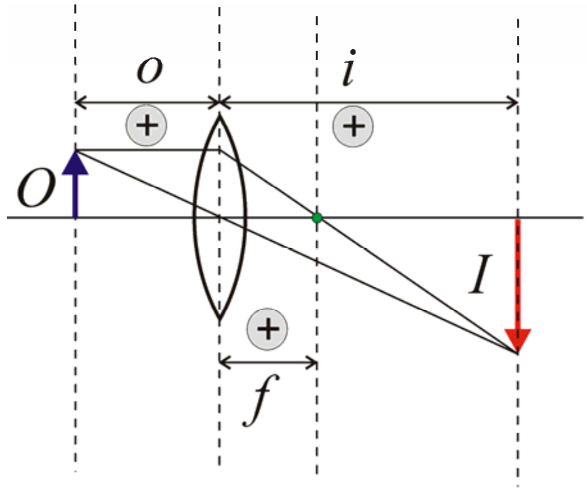
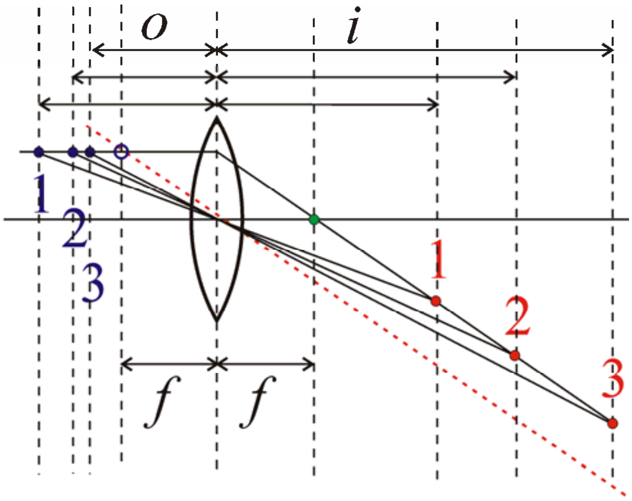
Image formation by lenses (thin lens approximation)





Lens equation and **lens-makers' equation**:

$$\frac{1}{o}+\frac{1}{i}=\frac{1}{f}=(n-1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$$

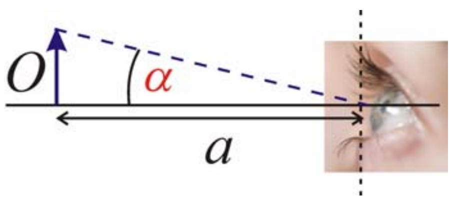
r_1, r_2: radii of curvature of the lens surface,

n: refractive index of the medium of the lens.

Simple magnifier

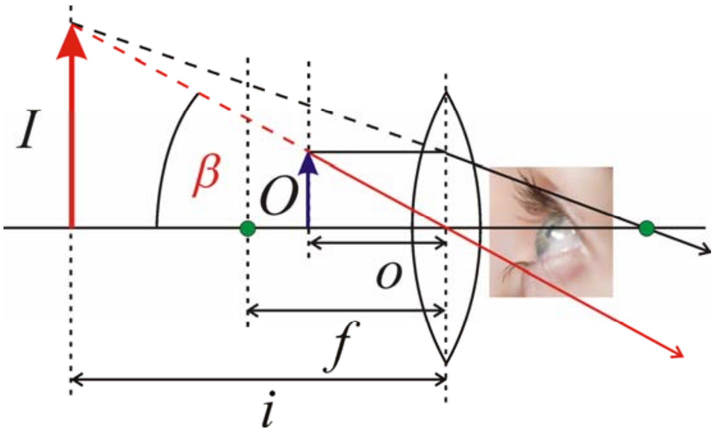
We have to compare two cases: eye looks at the O object

1. without lens from the conventional near point ($a \approx 25$ cm), under the angle of α



2. with lens from the distance o, under the angle of β

I virtual image



Angular magnification (definition):

$$N = \frac{tg\beta}{tg\alpha} \qquad \text{and we use} \qquad \frac{1}{o} = \frac{1}{f} - \frac{1}{i}$$

In our case (simple magnifier):

$$N = \frac{tg\beta}{tg\alpha} = \frac{\frac{I}{i}}{\frac{O}{a}} = \frac{\frac{O}{o}}{\frac{O}{a}} = \frac{a}{o} = a\left(\frac{1}{f} - \frac{1}{i}\right).$$

Two possible answers:

I. if $i = -a$ than $N = \frac{a}{f} + 1,$

II. if $i = -\infty$ than $N = \frac{a}{f}$

In the I. case eye looks at the virtual image with accommodation, in the II. case without accommodation, eye is focused at infinity, thus $o = f$.

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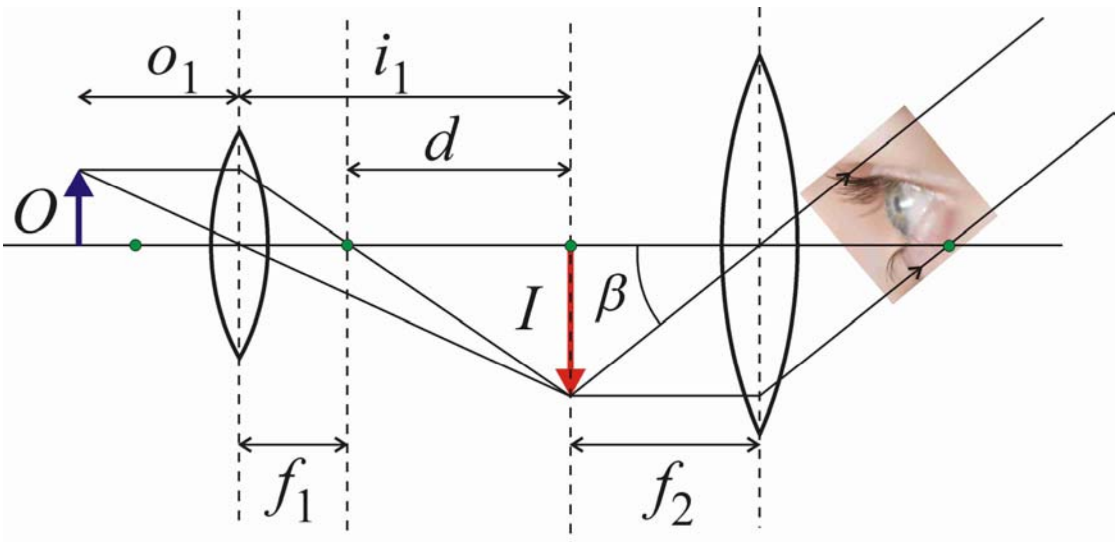
Lens systems (1) microscope



Without accommodation, eye is focused at infinity.

Angular magnification of microscope:

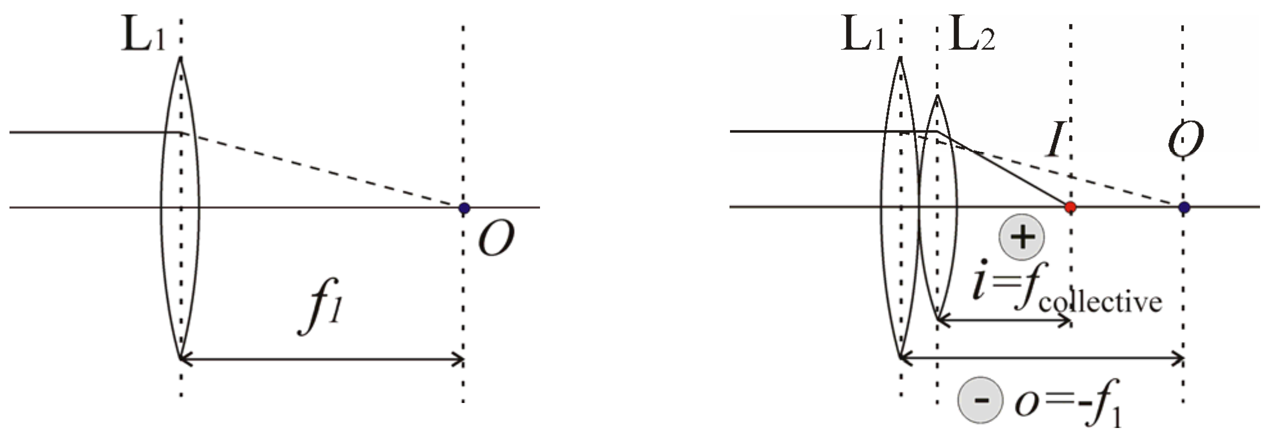
$$N=\frac{tg\beta}{tg\alpha}=\frac{\dfrac{I}{f_2}}{\dfrac{O}{a}}=\frac{I}{f_2}\frac{a}{O}=\frac{I}{O}\frac{a}{f_2}=\frac{i_1}{o_1}\frac{a}{f_2};$$

$$\frac{1}{o_1}=\frac{1}{f_1}-\frac{1}{i_1}=\frac{i_1-f_1}{f_1 i_1}=\frac{d}{f_1 i_1}$$

$$N=\frac{d}{f_1 i_1}\frac{i_1 a}{f_2}=\frac{da}{f_1 f_2}$$

Lens systems (2) **power** (refractive strength)

How high the collective focal length of two close juxtaposed lenses is $\{L_1(f_1), L_2(f_2)\}$?



Let's apply the lens equation for O as a virtual object.

$$-\frac{1}{f_1}+\frac{1}{f_{collective}}=\frac{1}{f_2} \qquad \frac{1}{f_{coll.}}=\frac{1}{f_1}+\frac{1}{f_2}=D_{coll.}=D_1+D_2$$

In such cases powers are added. Units [1/m], dioptre, [dpt].
Application e.g.: glasses, contact lenses.

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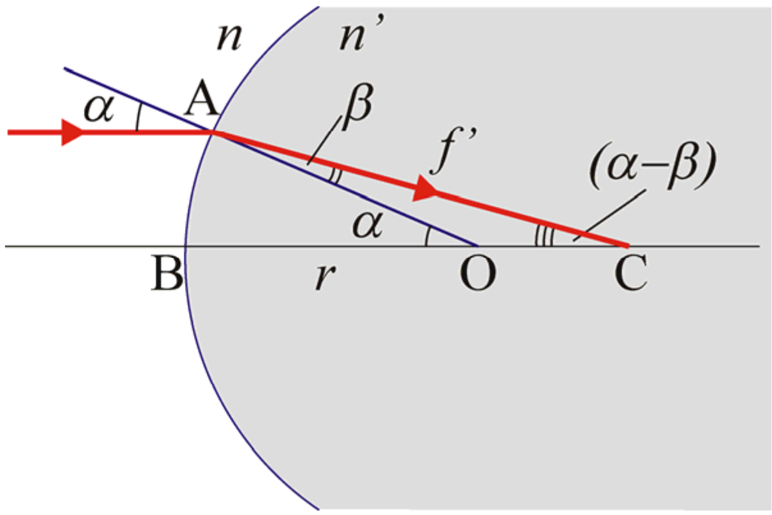
Image formation by simple curved surface (sphere with radius r):



For small angles:

1. $$\frac{\sin\beta}{\sin\alpha} = \frac{n}{n'} \approx \frac{\beta}{\alpha}$$

For the arc AB:

2. $$f'(\alpha - \beta) \approx r\,\alpha$$

$$\frac{\alpha-\beta}{\alpha} = \frac{r}{f'} \qquad\qquad 1 - \frac{\beta}{\alpha} = \frac{r}{f'}$$

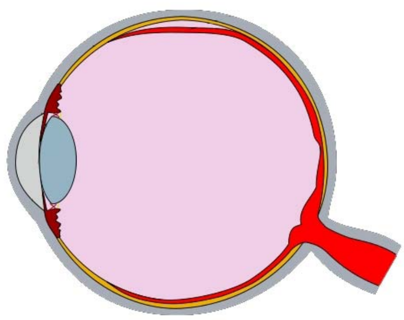
Substitution according to equation 1.:

$$1 - \frac{n}{n'} = \frac{r}{f'}, \qquad\qquad \frac{n'-n}{n'} = \frac{r}{f'}$$

The power in this case:

$$D = \frac{n'}{f'} = \frac{n'-n}{r}$$

Application: for the human eye
e.g. the power of cornea



medium	r [mm]	n	$n'-n$	D [dpt]
air		1		
			0,37	48
cornea	7,7	1,37		

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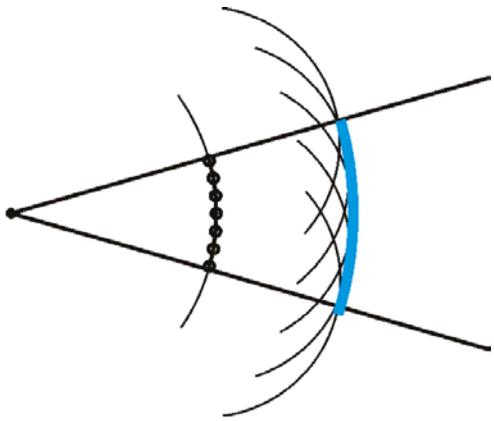
There are phenomena that cannot be explained by this model.

Physical or wave optics

(other model)

Its bases: **Huygens–Fresnel-principle**



According to the Huygens principle, elementary waves originate from every point of a wavefront, and the new wavefront is the common envelope of these elementary waves.

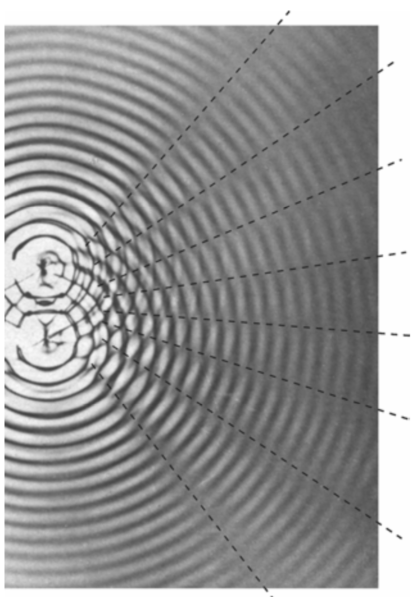
The laws of rectilinear propagation, the reflection and refraction can be described by this model as well.

Fresnel supplemented this by observing that the **superposition** principle is also in effect during the formation of the new wave front, which is nothing else than the quantitative formulation of the empirical fact that waves will propagate through each other without disturbance. Interference.

Waves (we learned about them earlier; dynamics, „repetition")

E.g. „water wave": it can be observed directly.
Because it changes slowly enough (low frequency, f) and the typical (wave) size is large enough (long wavelength, λ).

„Light waves" are different.



At certain conditions patterns can be formed, which don't or slowly change in time, and their size is much larger than the wavelength, λ.

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Interference (two or more waves meet)

the most important phenomenon in connection with waves

Incoherent and coherent waves

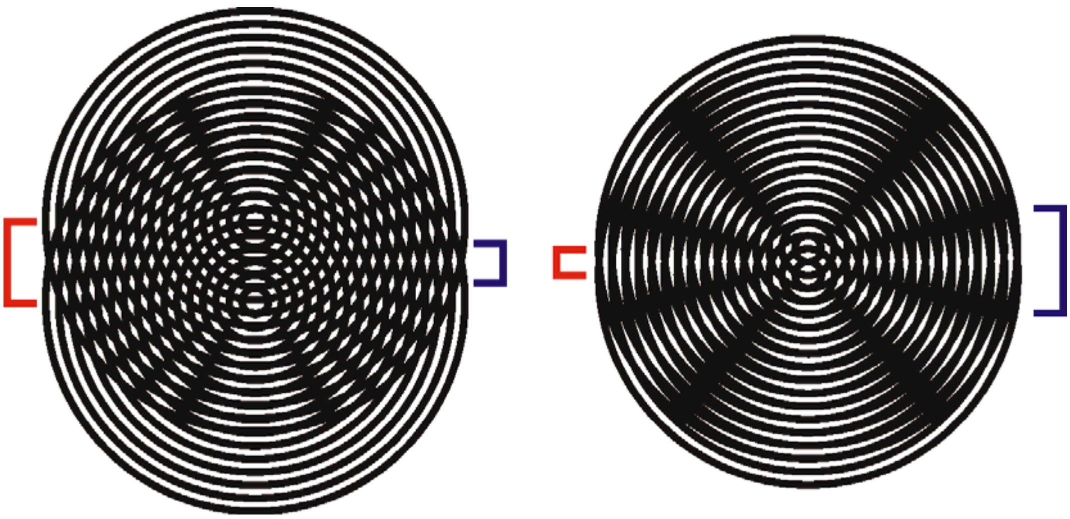
 

Rise of coherent waves is controlled in space and time, they are synchronized somehow.

Light interference

Nothing but the produced patterns can be observed.
Conditions for existent of observable patterns in the case of point like sources:

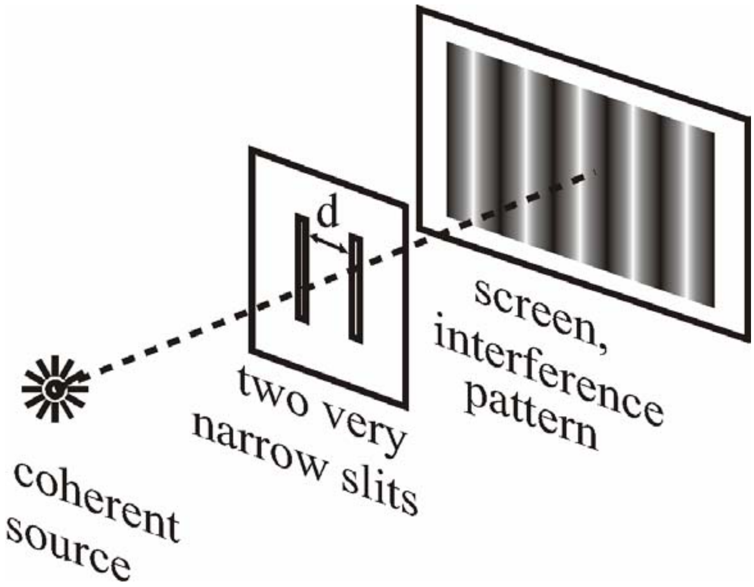
1. coherent waves (e.g. difference of phases ($\Delta\varphi$) is constant)
2. distance of sources is commensurable with the wavelength (λ).



The smaller the distance of sources (red mark),
the bigger the typical size of the pattern (blue mark).

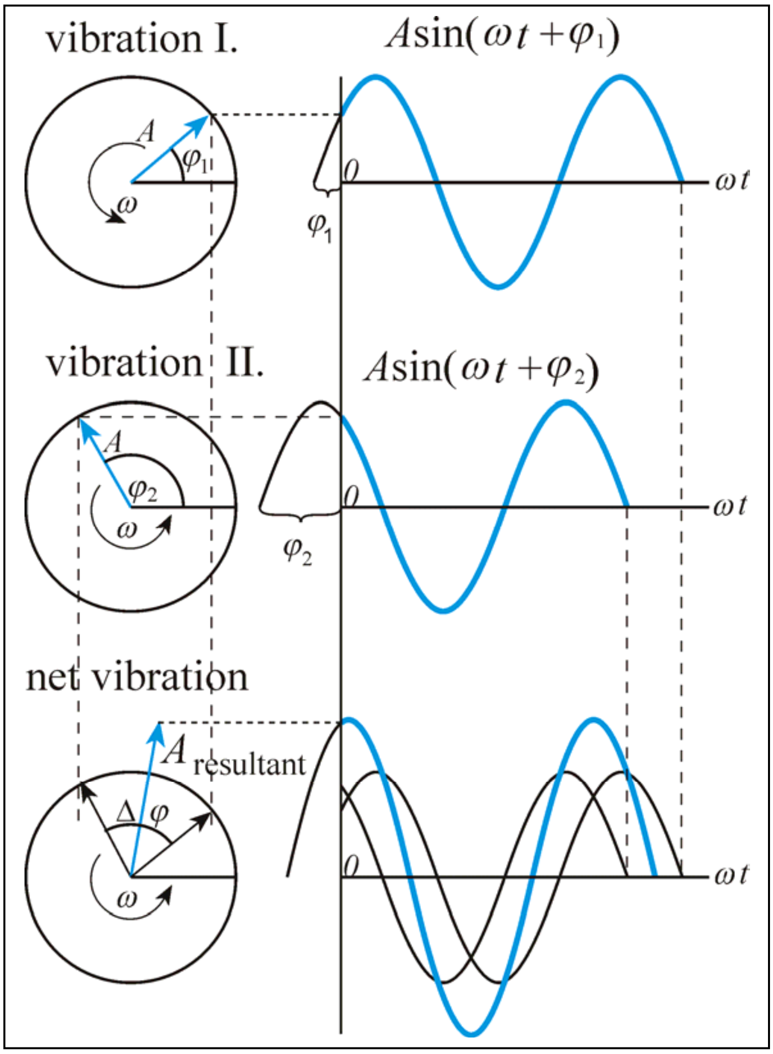
Typical experiment and pattern of light interference

Young's
double slit experiment
(diffraction)



The places of constructive and destructive interference are determined by the difference in phase ($\Delta\varphi$).

At a certain place the vibrational states are demonstrated by rotating vectors:



The amplitude of the net vibration ($A_{resultant}$) is given by the vector sum of the components (A).