

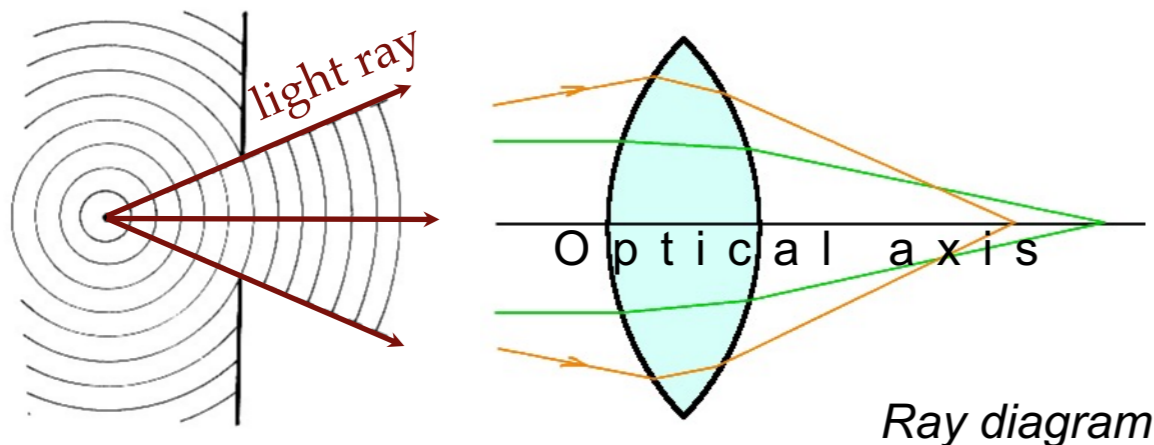
WAVE OPTICS

MIKLÓS KELLERMAYER

Geometric optics and wave optics

Geometric optics

If light propagates through a slit much larger than its wavelength, then the spreading of the wavefront (phase) may be simplified into a line (“light ray”).



Ray diagram

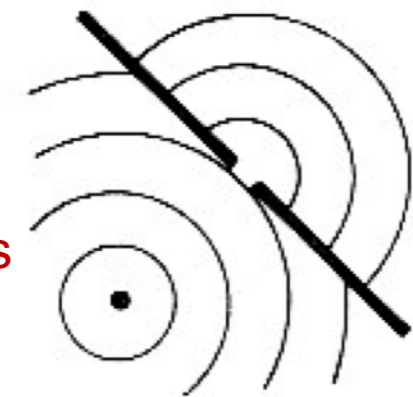
- Optical (light) ray (“light beam”): abstraction, mathematical line.
- Arrows represent the direction of energy propagation.
- Optical axis: line connecting the midpoint of optical components (e.g., lenses).
- Principle of reversibility: the direction of energy propagation (arrows) may be reversed.

Speed of propagation of light in **vacuum**: $c = 2.99792458 \times 10^8 \text{ ms}^{-1}$
In optically denser media the speed of propagation is reduced (c_1).
This may be expressed with the **absolute refractive index** (n_1):

Wave optics

If light propagates through a slit comparable or smaller than its wavelength, then its wave properties must be taken into account.

Some phenomena cannot be explained with geometric optics



Important parameters of the propagating wave:

- **Period** (T)
- **Frequency** ($f = 1/T$)
- **Velocity** (v, c)
- **Wavelength** (λ): distance covered in a period:

$$\lambda = cT = \frac{c}{f}$$

$$n_1 = \frac{c}{c_1}$$

Wave: propagating oscillation

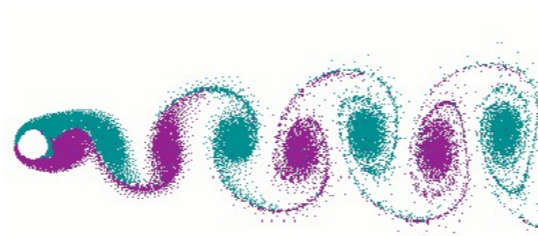
What is an oscillation?

Example:
Mechanical
oscillation



- Tacoma Narrows Bridge (“Gallopín’ Gertie”)
- (“*Gertie the Dinosaur*” (1914), cartoon, Winsor McCay)
 - Opening: July 1, 1940.
 - During wind (50-70 km/h): oscillation for hours
 - Oscillation amplitude initially 0.5 m, then, after snapping of a suspension cable, up to 9 m!
 - Collapse: November 7, 1940.

Explanation:
Kármán vortices “shake”
the object



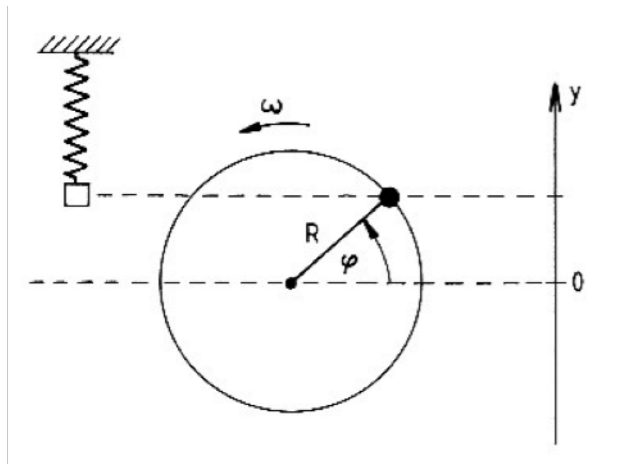
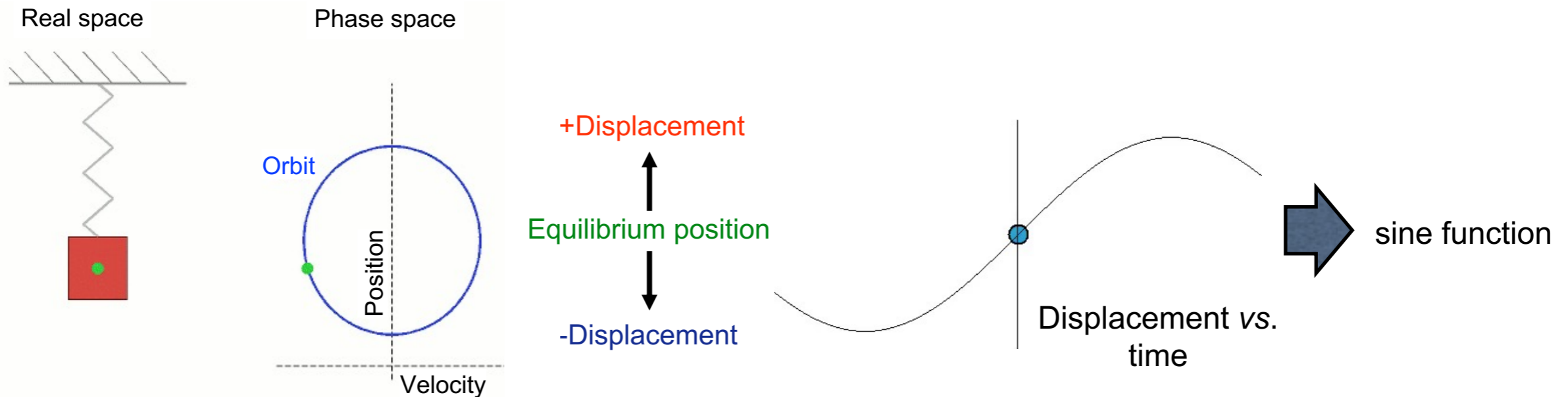
Kármán vortex street



Theodore
von Kármán
1881-1963

Harmonic oscillation

Restoring force acts on a system displaced out of equilibrium (e.g., mass on a spring).



ϕ = phase angle at time t
 y = displacement at time t
 ω = angular velocity (ϕ/t)
 R = length of rotating unit vector = maximal displacement (amplitude)

$$y = R \sin \phi$$

Because $\phi = \omega t$: $y = R \sin(\omega t)$

If the initial phase angle (ϕ_0) differs from 0: $y = R \sin(\omega t + \phi_0)$

Because angular velocity (ω) is the full circular orbit (2π) per period (T):

$$y = R \sin\left(\frac{2\pi}{T}t + \phi_0\right)$$

Important parameters of the propagating wave:

- Period (T)
- Frequency ($f=1/T$)
- Velocity (v, c)
- Wavelength (λ): distance covered in a period:

$$\lambda = cT = \frac{c}{f}$$

Types of waves

- According to **source**:

1. Mechanical: elastic deformation propagating through elastic medium
2. Electromagnetic: electric disturbance propagating through space (vacuum)

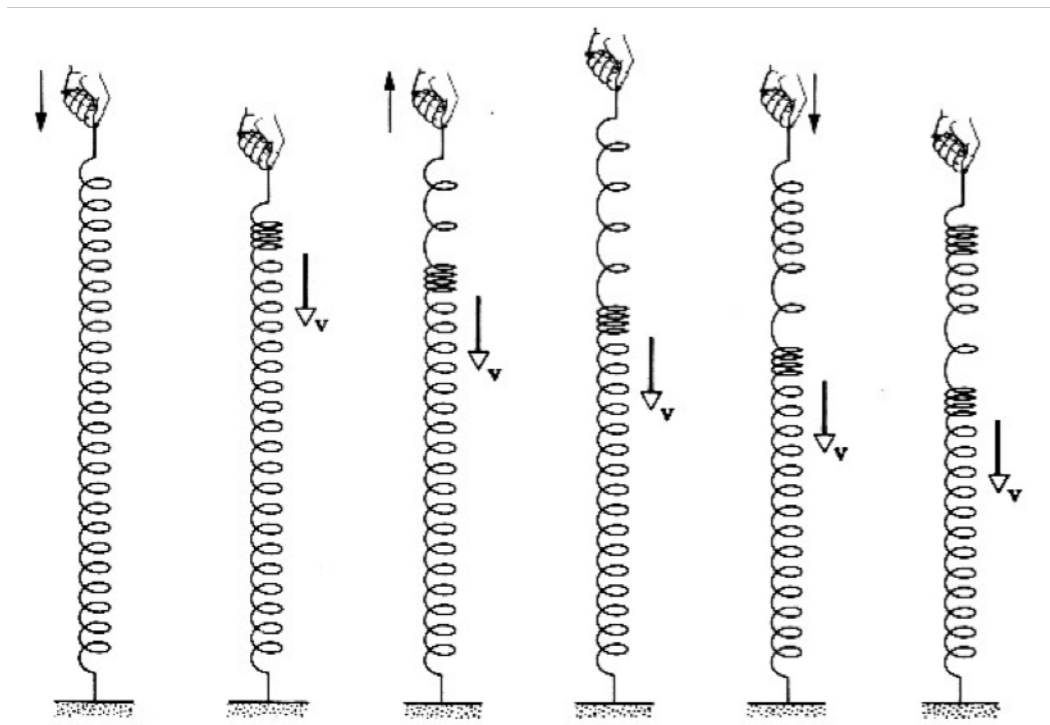
- According to **propagation dimension**:

1. One-dimensional (rope)
2. Surface waves (pond)
3. Spatial waves (sound)

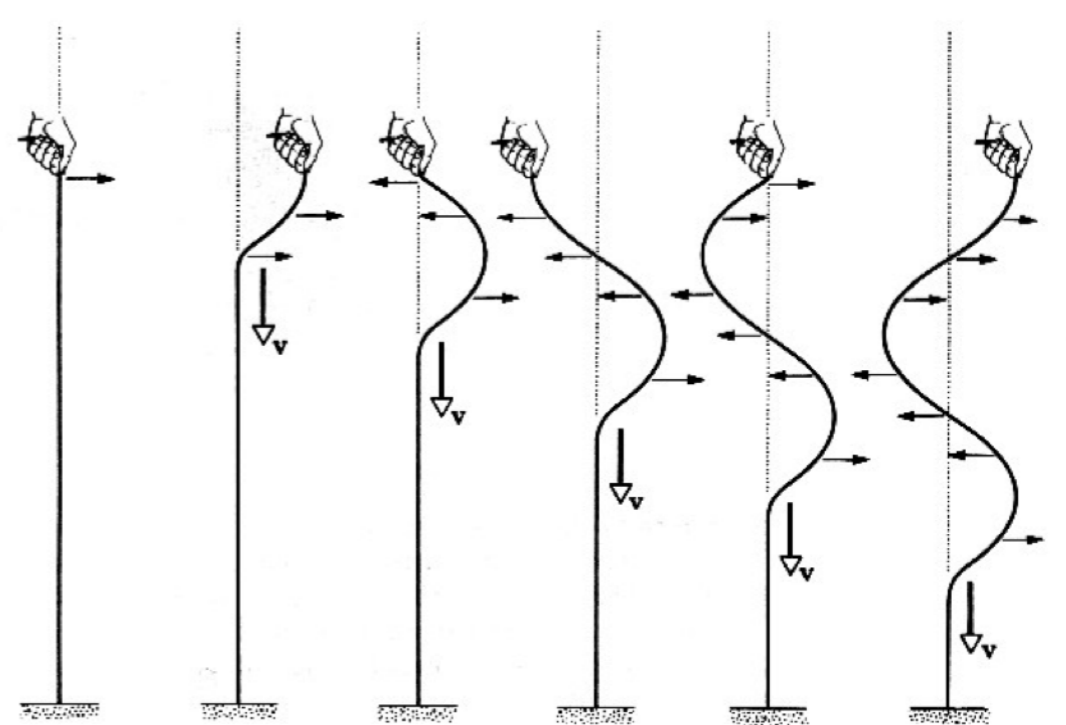


- According to **relative direction of oscillation and propagation**:

1. Longitudinal



2. Transverse



Wave phenomena I.

Diffraction

Huygens-Fresnel principle:
every point of a wavefront is the source of further waves

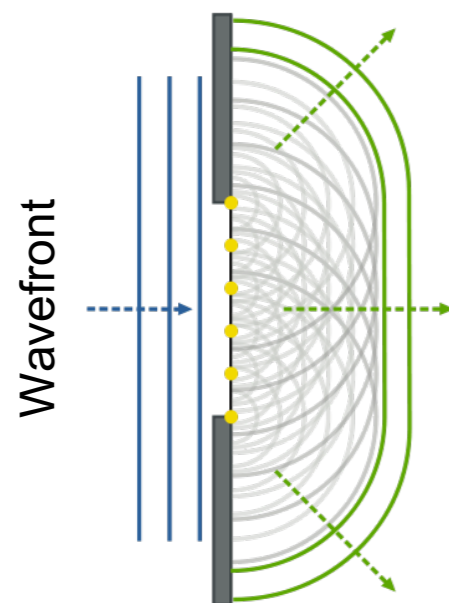
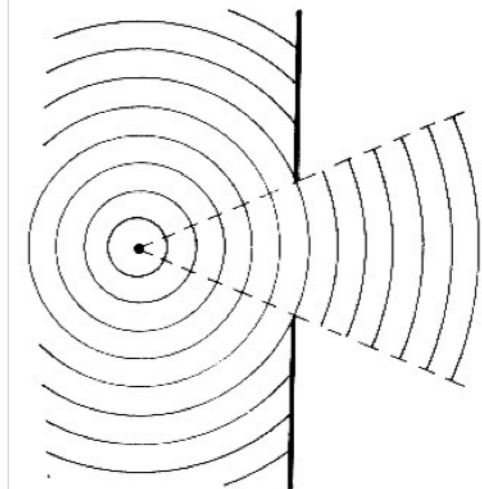


Christiaan Huygens
(1629-1695)



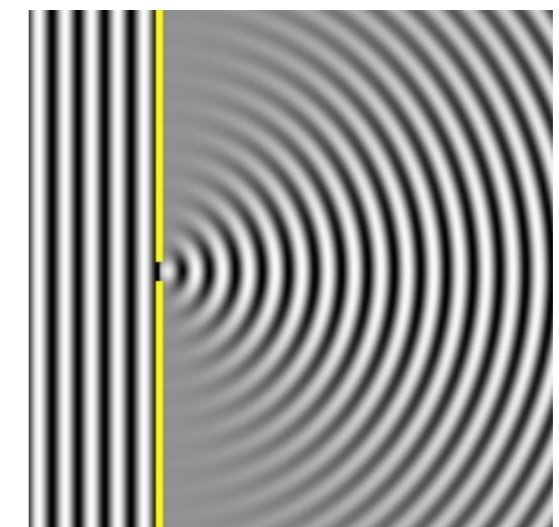
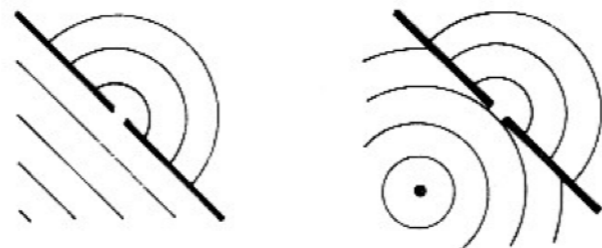
Augustin-Jean Fresnel
(1788-1827)

Slit much greater than
the wavelength (λ)



The wave appears in the
“shaded” areas, too.

Slit much smaller than
wavelength (λ)



Wave phenomena II.

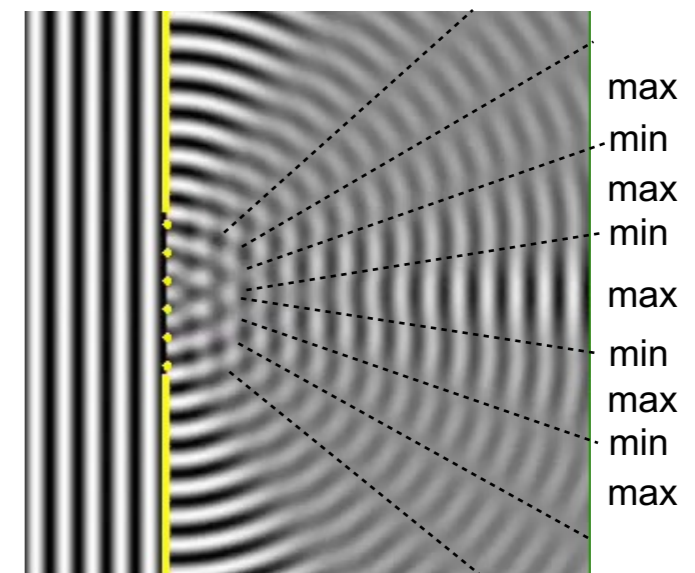
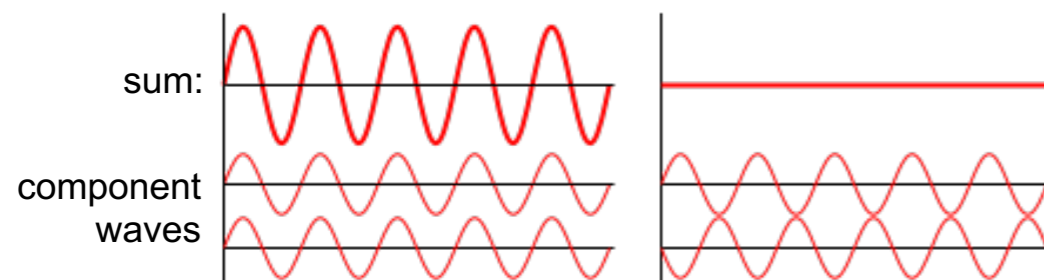
interference

Principle of superposition

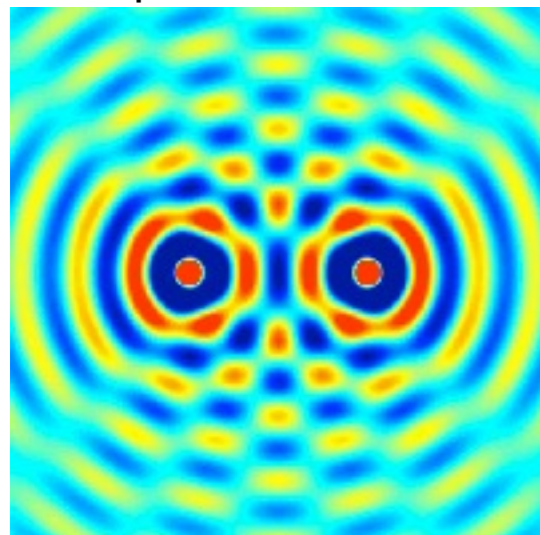
Slit comparable to wavelength
(=pointlike slits separated by distance d , where $d \sim \lambda$)

Waves in phase
($\Delta\varphi=0$): amplification

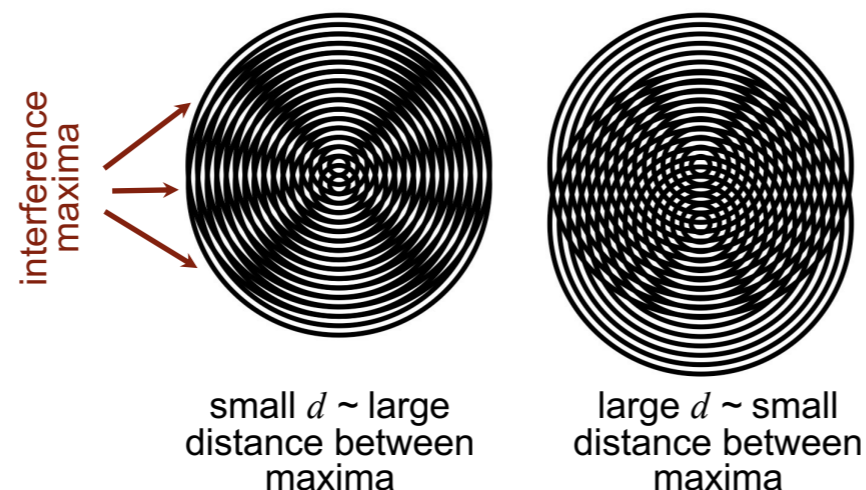
If $\Delta\varphi=\pi$:
cancellation



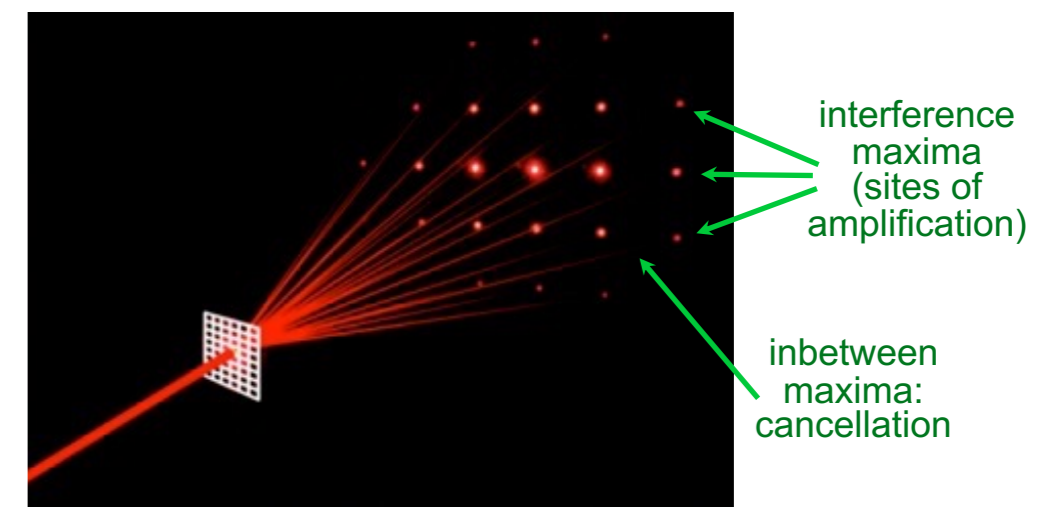
Interference of waves
emerging from two
point sources.



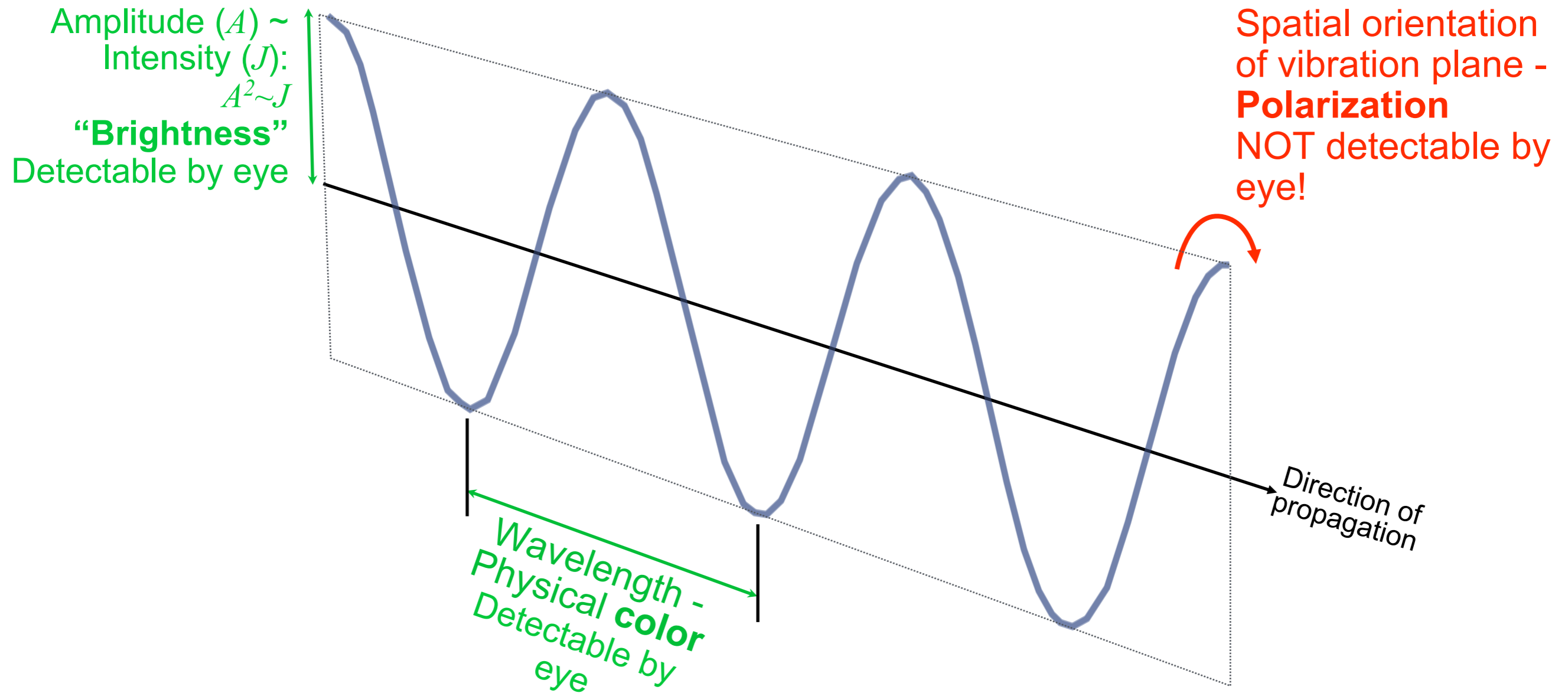
Interference pattern depends
on distance (d) separating the
pointlike slits



Diffraction pattern of a 2D optical grating



Detectable parameters of the light wave



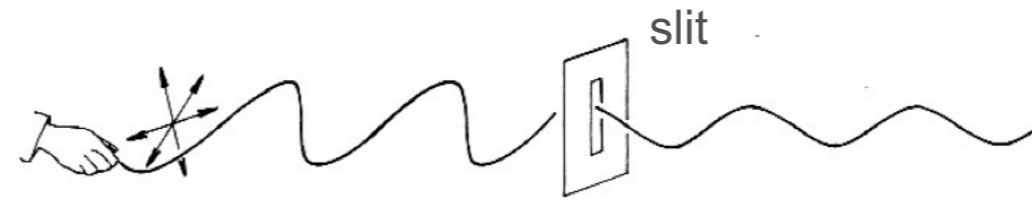
Wave phenomena III.

Polarization

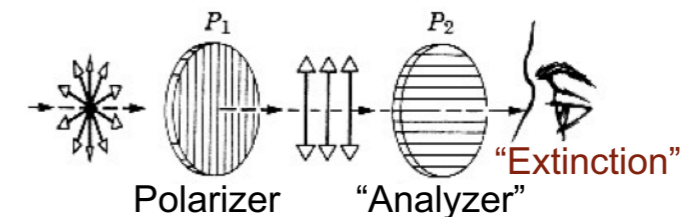
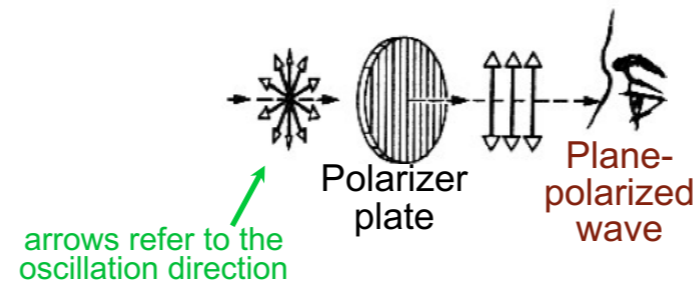
- Polarization: oscillation is oriented in some *preferred* direction
- Birefringence is related to polarization: anisotropic propagation velocity
- Only *transverse* waves can be polarized.



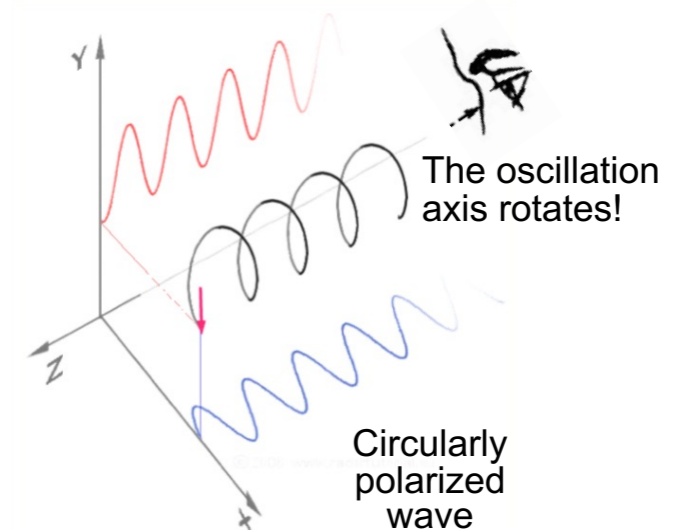
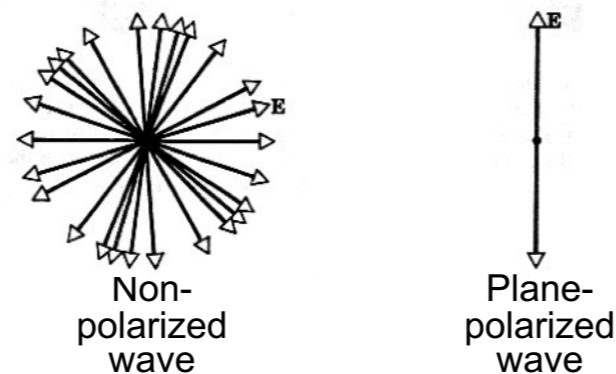
Polarization of
Mechanical waves



Polarization of
Electromagnetic waves



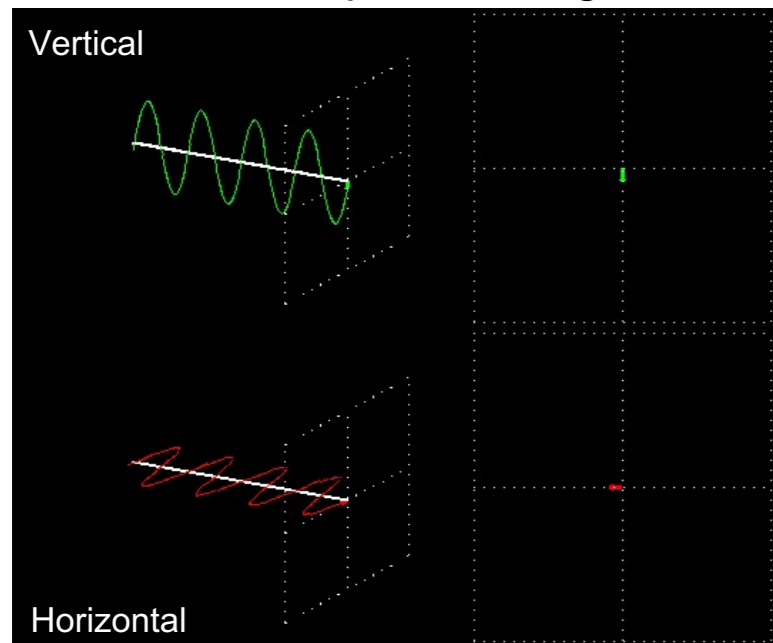
Polarization can be understood by observing the **head-on** view of the wave:



Polarized light and its interactions

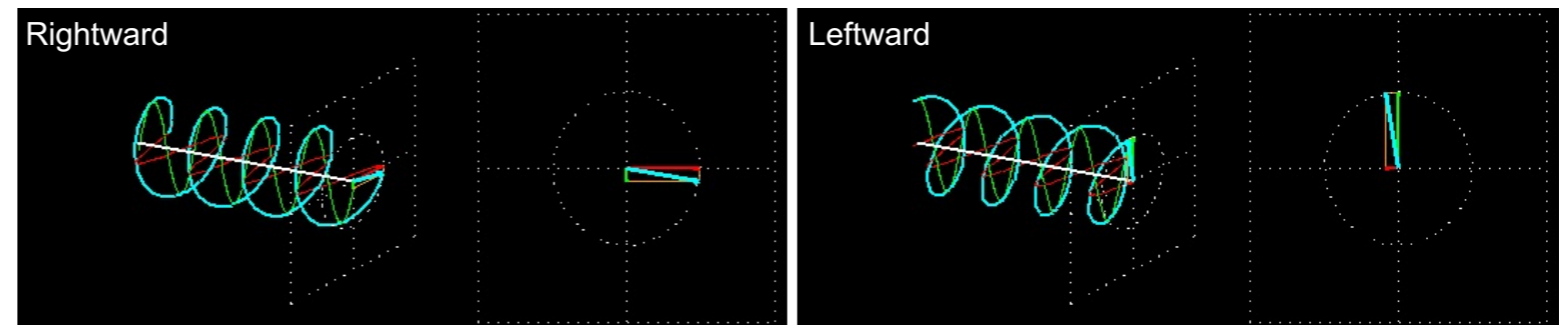
Direction of the vibration (electric or magnetic field) has preferred orientation

1. Plane-polarized light

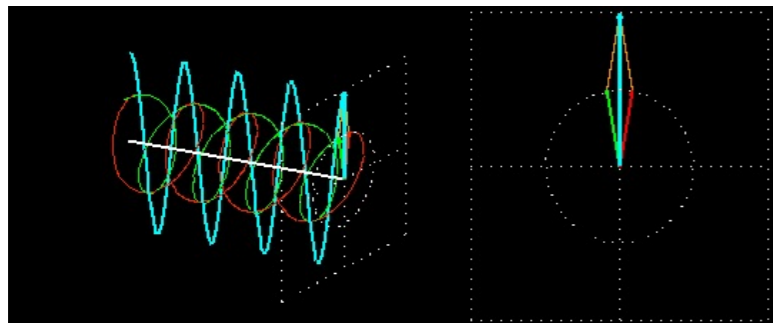


2. Circularly polarized light:

Superposition of vertically and horizontally polarized waves with identical λ and $\lambda/4$ phase shift

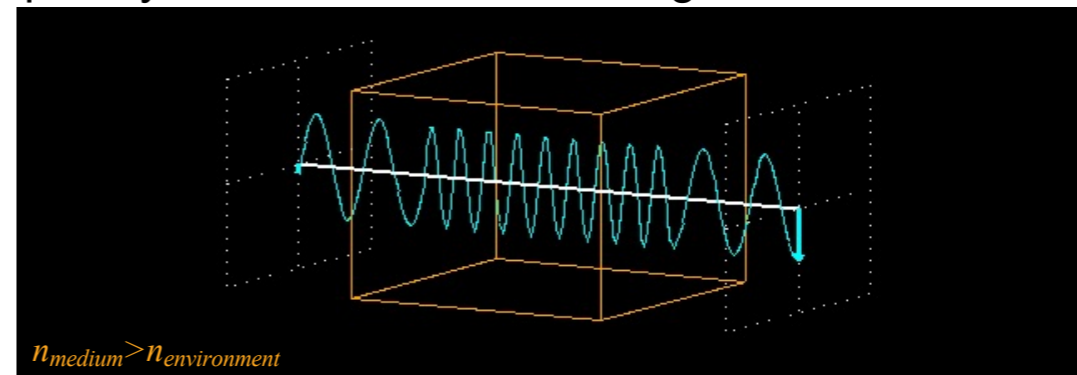


3. Superposition of a right- and leftward circularly polarized wave results in plane-polarized light.

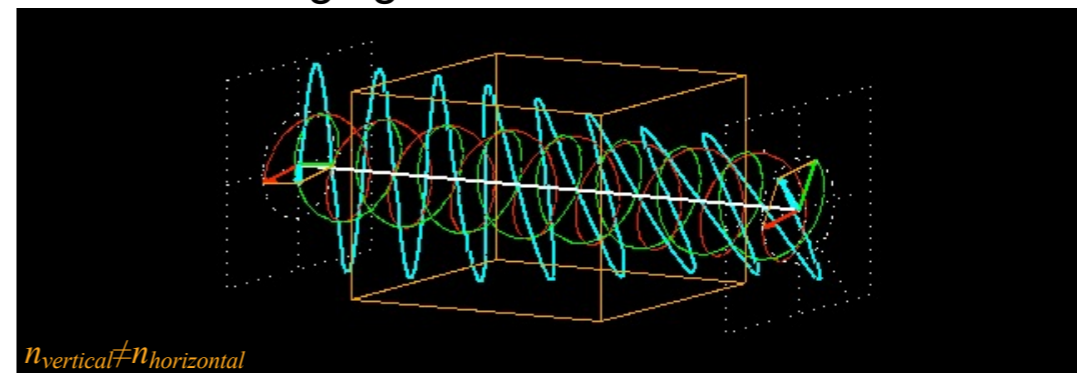


Orientation of polarization plane depends on the relative phase of the two circularly polarized waves

4. Light decelerates in optically dense medium; because its frequency is constant, its wavelength becomes reduced.



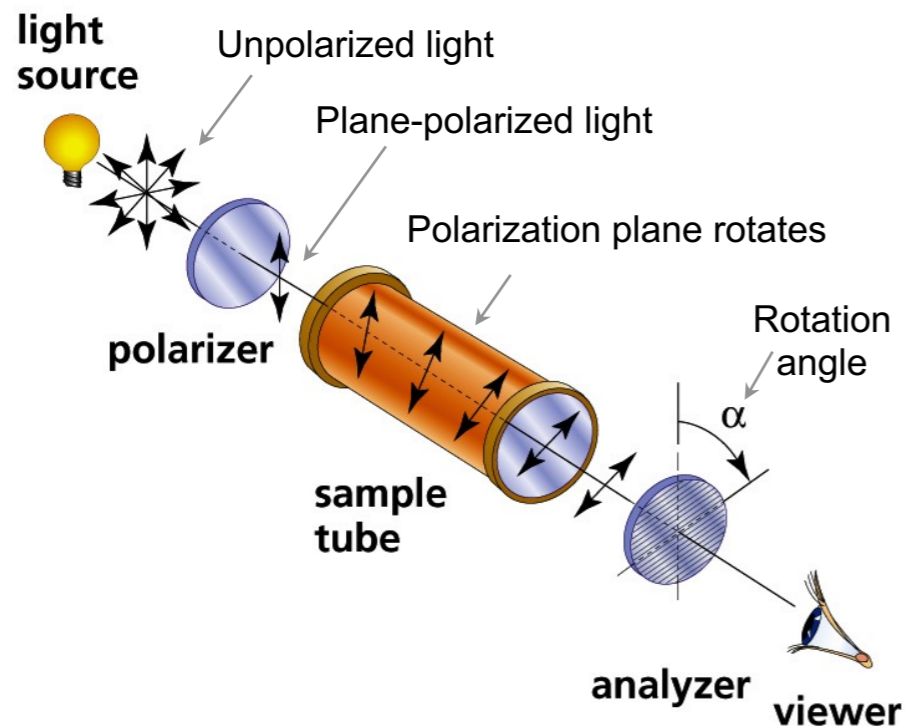
5. In an anisotropic* medium a phase shift occurs between the circularly polarized components: the polarization plane of the emerging wave becomes **rotated**.



*Anisotropy (birefringence): refractive index (\sim light speed) is orientation dependent (i.e., in different directions within the sample, light propagates with different speeds).

Applications of polarization

Polarimetry



Rotation angle depends on the concentration (c) of the optically active* material:

$$\alpha = [\alpha]_D^{20} \cdot c \cdot l$$

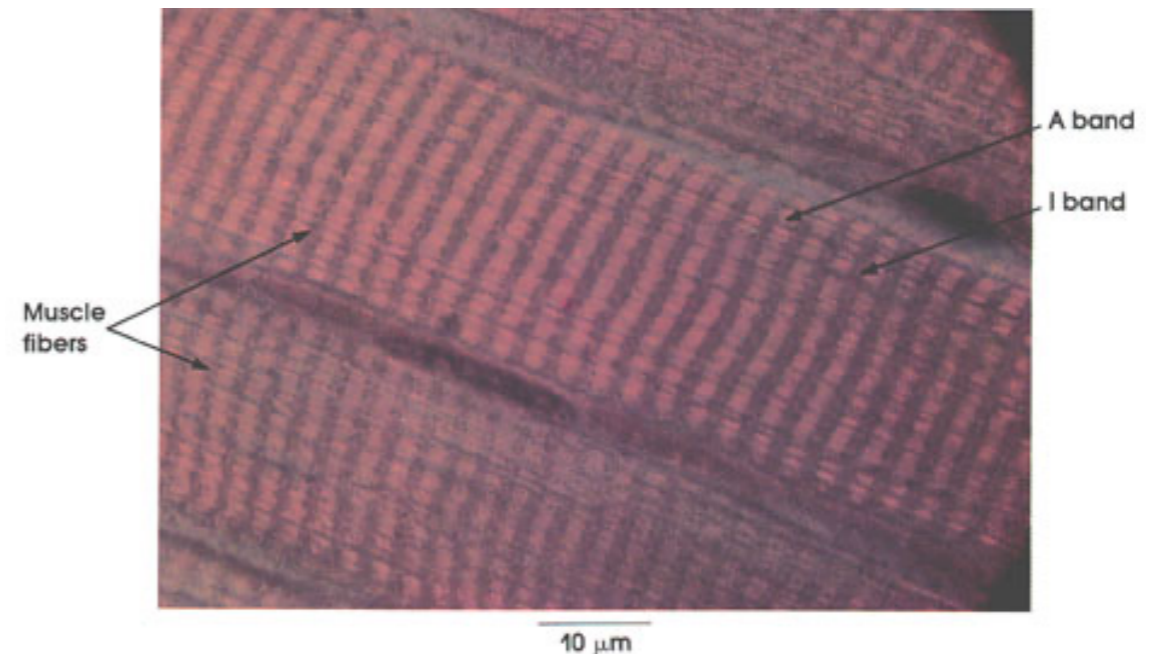
$[\alpha]$ = specific angle of rotation (“20”: room temperature; “D”: emission spectral line of Na $\lambda=589$ nm)

l = length of sample tube

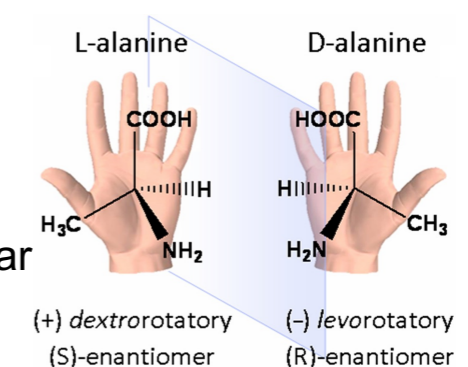
*Optically active material: contains **chiral** (mirror-symmetric) molecules that rotate the plane of polar

Polarization microscopy

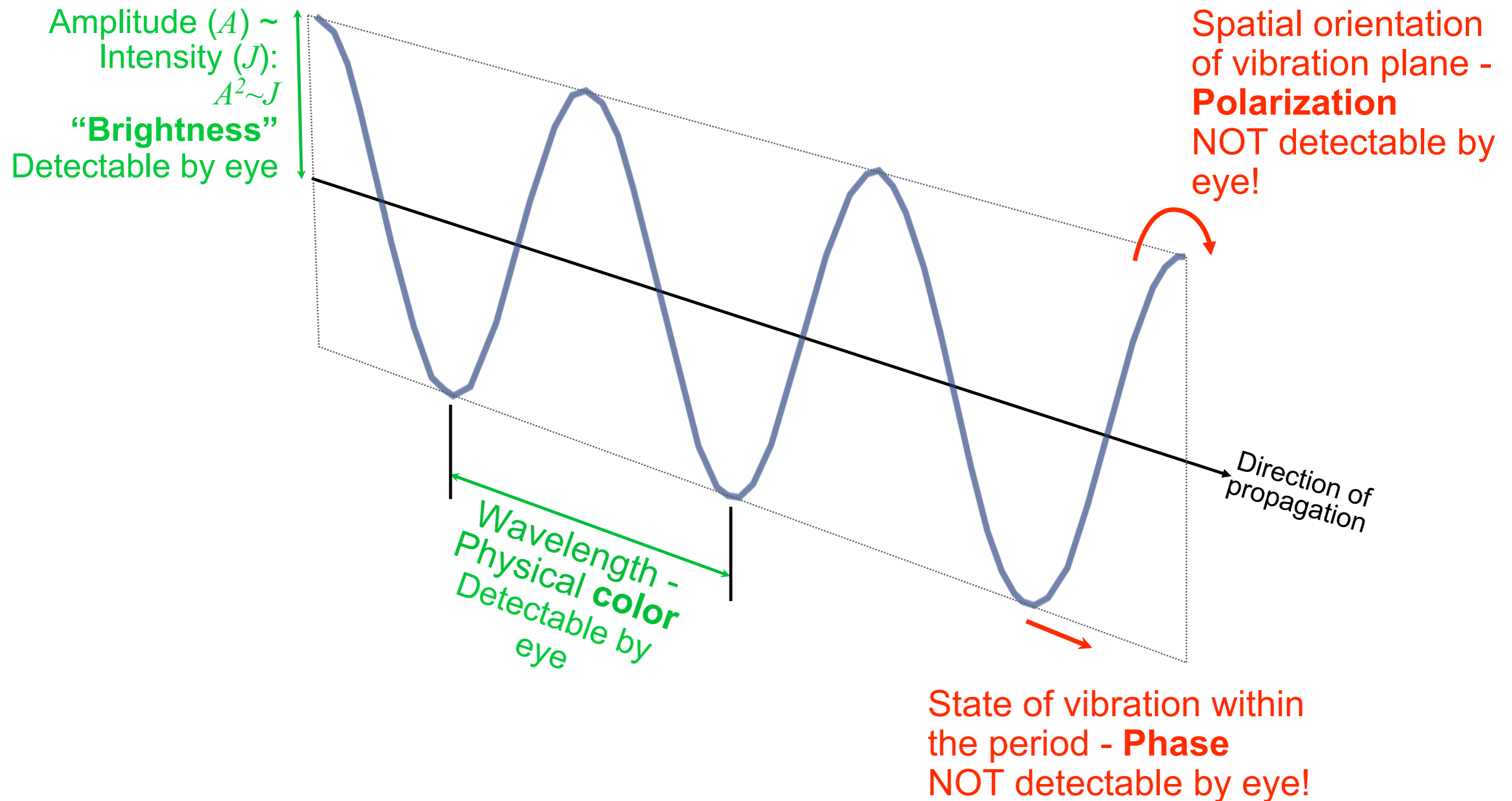
Cross-striated skeletal muscle in the polarization microscope



- A-band: anisotropic (birefringent) region (contains myosin helically ordered myosin molecules)
- I-band: isotropic region



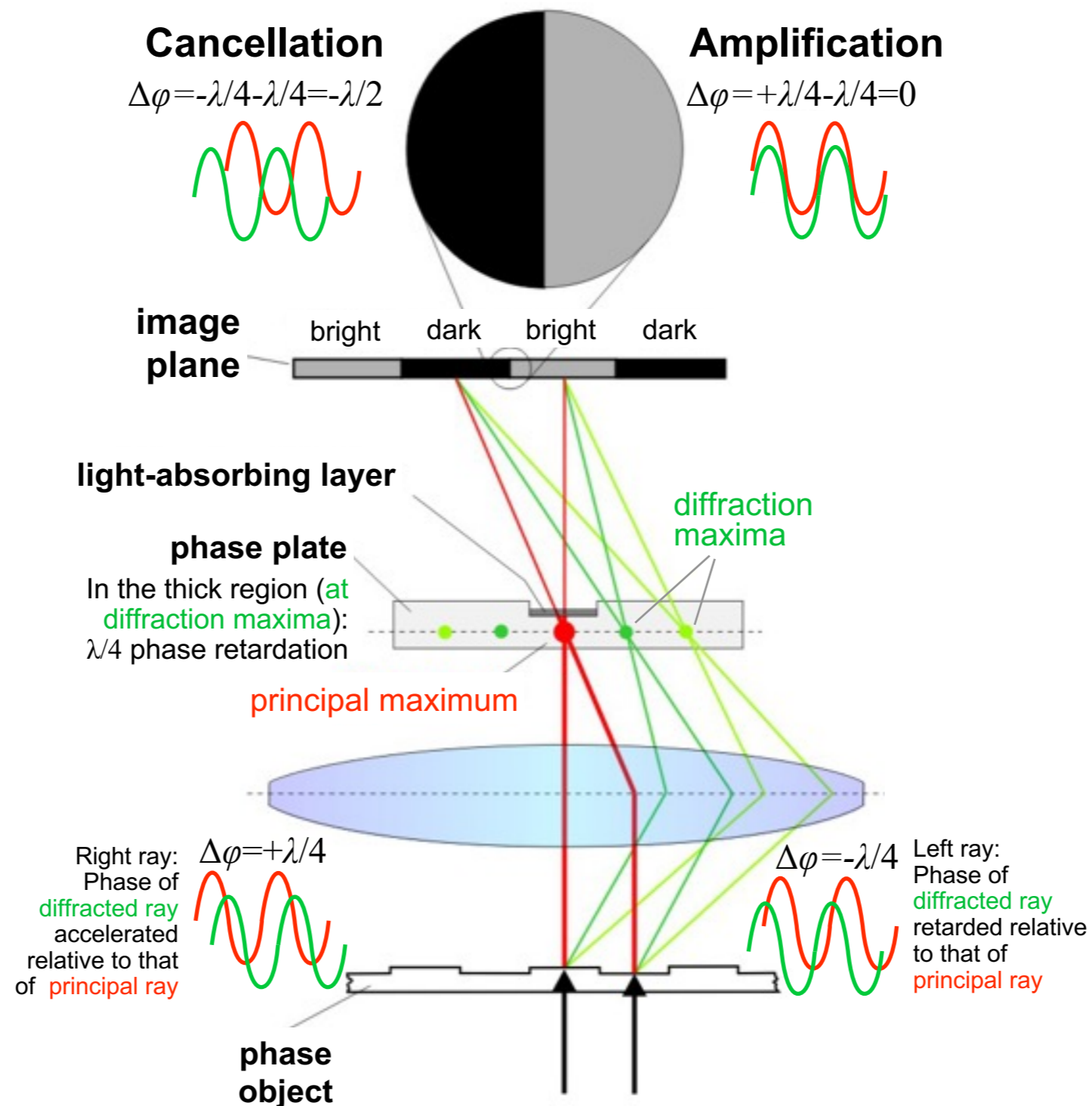
Detectable parameters of the light wave



Phase contrast microscopy

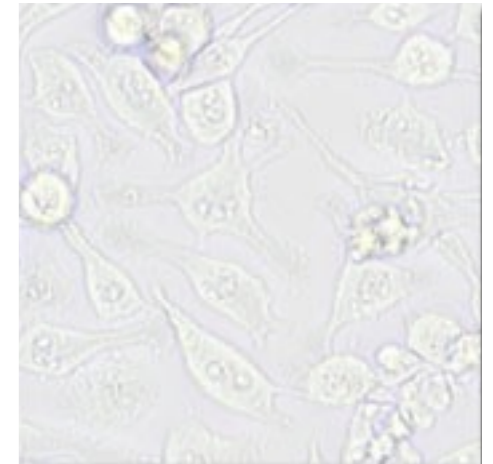


Frits Zernike (1888-1966)
Nobel-prize

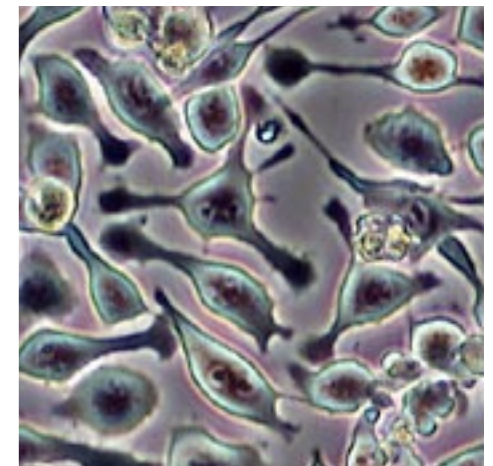


- Phase: shows the state of vibration within the entire period (2π).
- Expressed with the phase angle (ϕ).
- Phase difference between waves ($\Delta\phi$): phase shift (retardation or acceleration)

Live (unstained) cells



Bright-field microscopic image

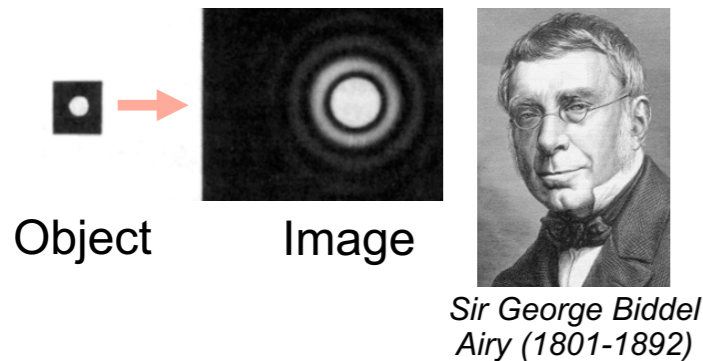


Phase-contrast microscopic image

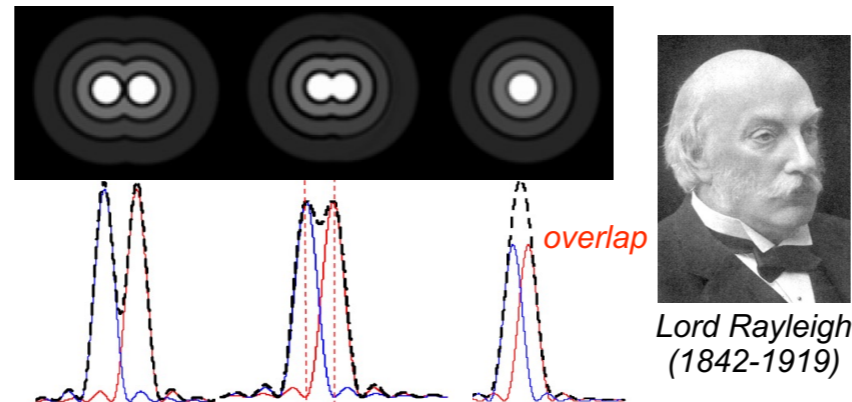
Resolution of the human eye I.

Diffraction limit

Because of diffraction: image of a point object is an Airy disk



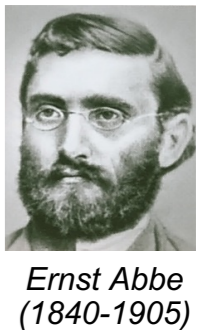
Rayleigh criterion: objects may be resolved if their corresponding Airy disks do not overlap



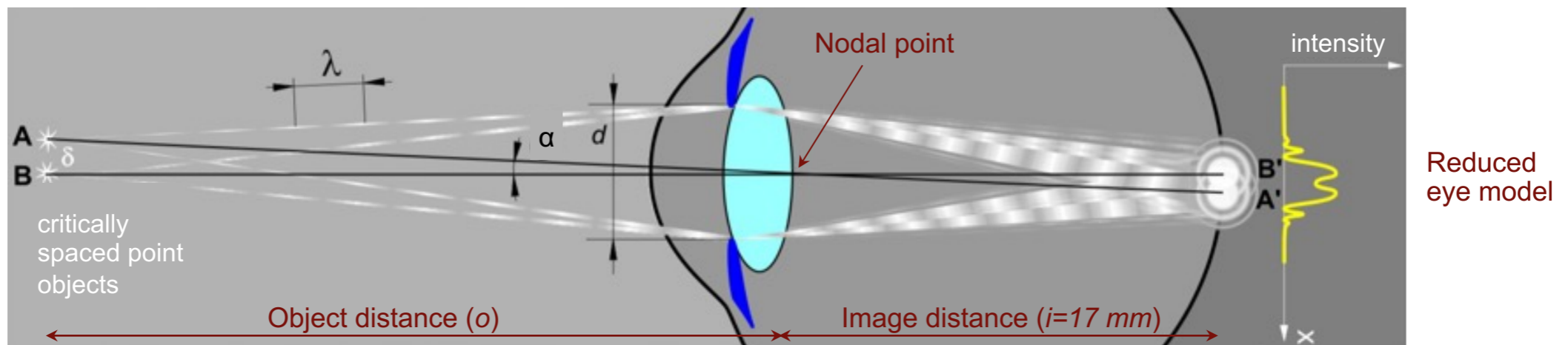
Smallest resolved distance has a limit (Abbe equation):

$$d = \frac{0.61\lambda}{n \sin \alpha}$$

λ = wavelength
 n = refractive index of medium
 α = angle between axis and outermost ray




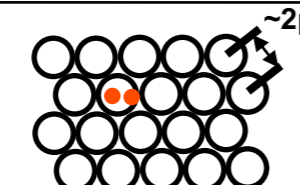


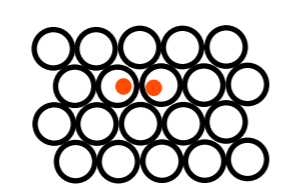


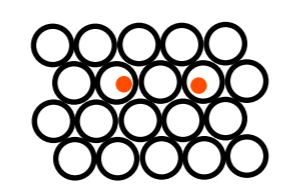

Diffraction limit of the human eye



Limiting visual angle: $\alpha_H = 1.22 \frac{\lambda}{d}$ Smallest angle of view at which two closely spaced objects may be resolved.
 At average wavelength (550 nm) and pupil diameter (4 mm): **0.6'** (angular minutes)

Resolution of the human eye II.

Biological limit: receptor cell density

Object	Image on receptors	Sensed image
		
		
		

- Condition of resolution: at least one inactivated receptor cell falls in between two activated ones. The limiting angle of view under this condition is $(\alpha_B) \approx 0.8'$
- The diffraction and biological limits of the human eye are **comparable!**

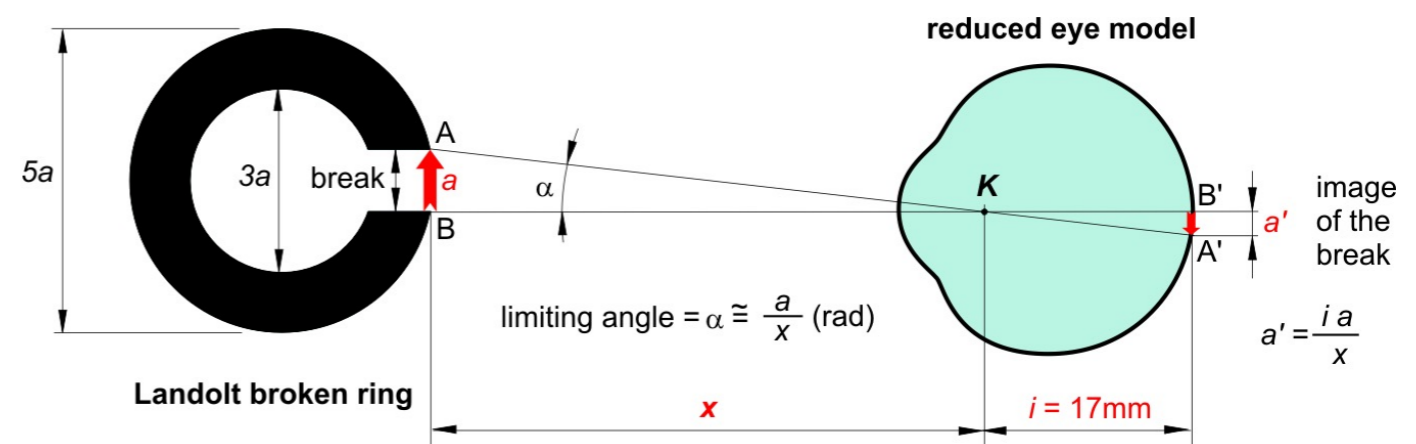
Visual Acuity (“visus”, vision):

$$\text{visual_acuity} = \frac{1'}{\alpha} 100\%$$

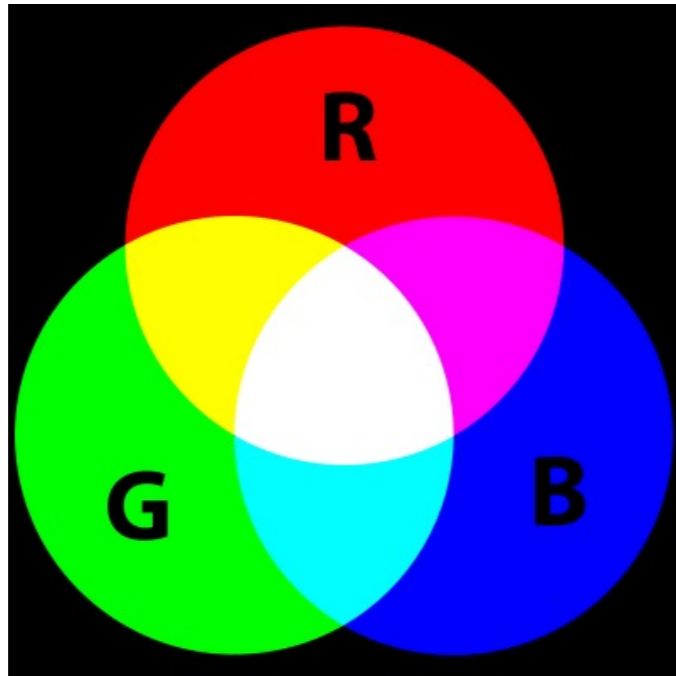
α = experimental (measured) visual angle

Average visual angle in healthy humans: $1'$ (= 100% vision)

Measurement of visual acuity



Color coding, color vision



Additive color coding

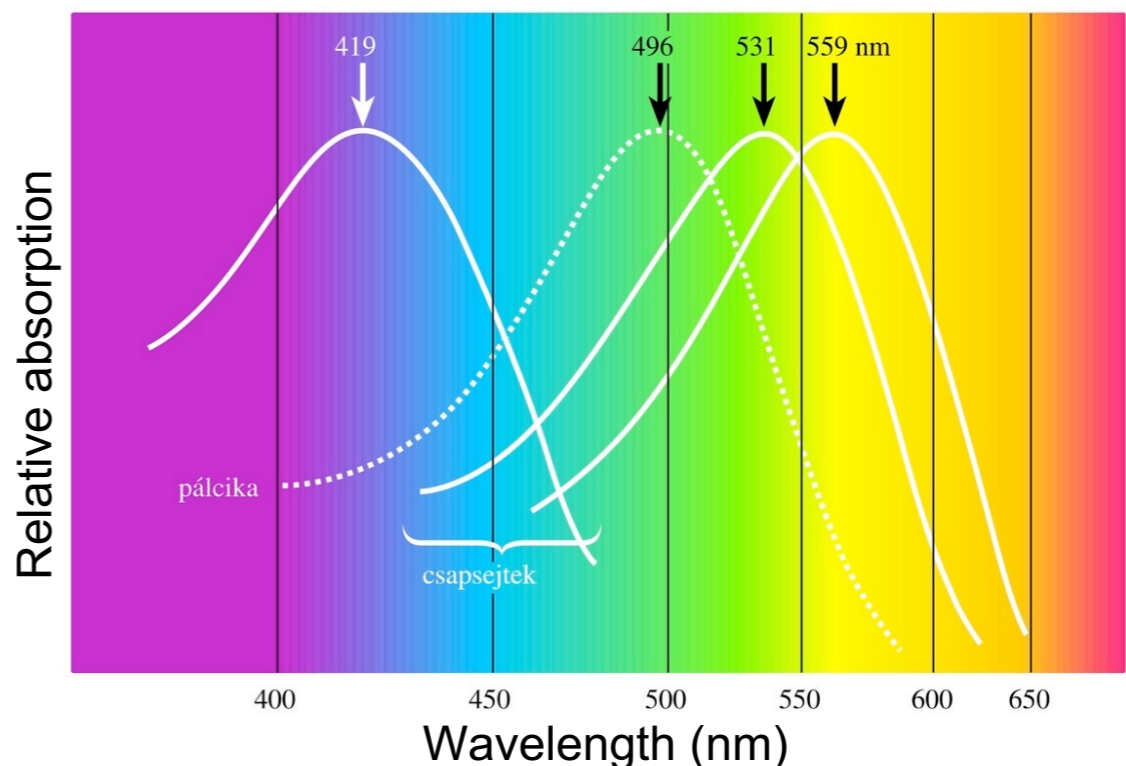
Any color may be generated by mixing three basic colors (R =red, G =green, B =blue) with varying weighing factors.

$$X = rR + gG + bB$$

In the human eye:

- 3 different color-sensitive receptors.
- Each receptor absorbs in different regions of the visible spectrum ($R=64\%$, $G=32\%$, $B=2\%$).

Absorption spectra of the human color-sensitive receptors (cones)



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