Mathematical and Physical Basis of Medical Biophysics

Lecture 1

Mathematics Necessary for Understanding Biophysical Laws.

6th September 2021

Gergely AGÓCS

How to Get Prepared?

- university = **autonomous learning**
- sources:
  - **your** notes made in the lectures; ***only in the first four weeks***


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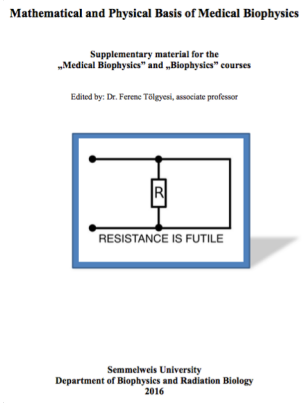
- sources:
  - **your** notes made in the lectures; ***only in the first four weeks***
  - Tölgyesi: *Mathematical and Physical Basis of Medical Biophysics* (2016)
  - on-line material: https://itc.semmelweis.hu/moodle/course/view.php?id=313
    - subject requirements
    - lecture schedule and slides
    - textbook


Mathematical and Physical Basis of Medical Biophysics

Supplementary material for the „Medical Biophysics" and „Biophysics" courses

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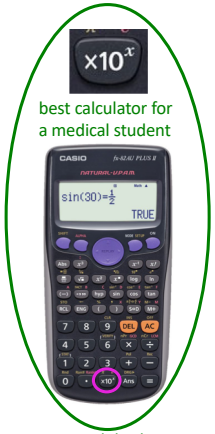
2016

How to Use Scientific Notation?



best calculator for a medical student

natural display

still okay (but less convenient)

linear input

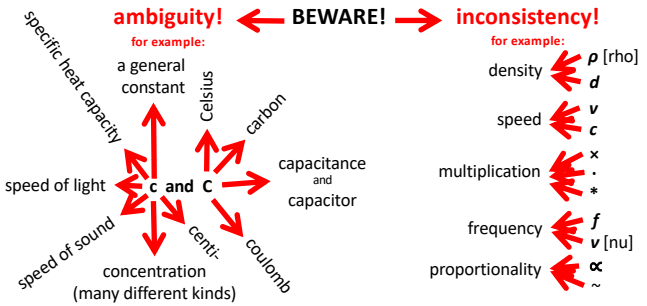
not allowed

programmable graphical display

Use of Symbols in Science

In science we use a large number of **Latin** and **Greek** letters (and their combinations) as symbols, so it is inevitable to learn the Greek alphabet.

However, the number of quantities and units is much greater than the number of available letters, and this can lead to confusion. Your help: CONTEXT



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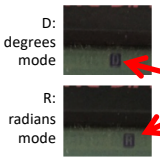
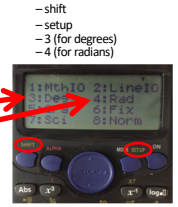
Angles

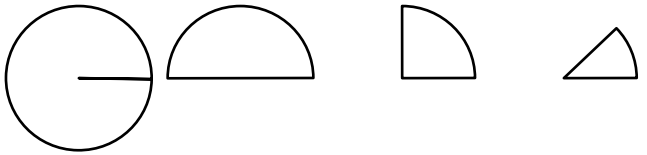


revolution: one turn
degree: practical, traditional unit
radian: scientific unit, arc/radius

1 revolution = 360° = 2π rad

1° = 60′ = 3600″





one **revolution** 360° **degrees** 2π **radians**

half **revolution** 180° **degrees** π **radian**

quarter **revolution** 90° **degrees** π/2 **radian**

1/8 **revolution** 45° **degrees** π/4 **radian**

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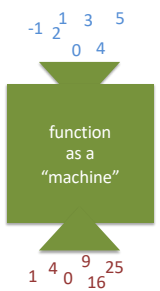
What is a Function?

Unambiguous assignment of one set of values to an other set of values

INPUT (ARGUMENT, INDEPENDENT VARIABLE)
x



OUTPUT (VALUE, DEPENDENT VARIABLE)
f*(*x*) or *y

DOMAIN

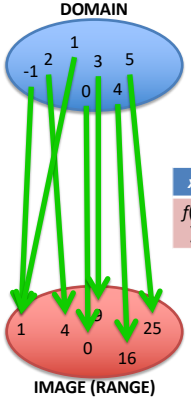


IMAGE (RANGE)

$x \mapsto f(x)$ or $y = f(x)$

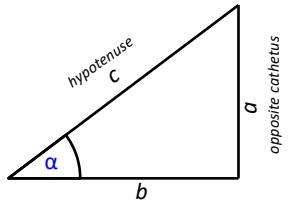
x	–1	0	1	2	3	4	5
f(*x*)	1	0	1	4	9	16	25

$x \mapsto f(x)$ or $y = f(x)$

f is the function defining the relationship between *x* and *f*(*x*)

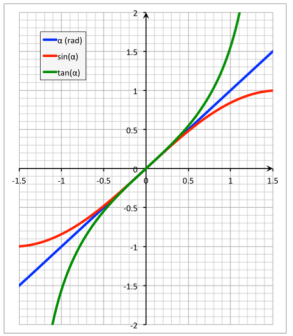
Trigonometric Functions

degree: practical, traditional unit
radian: scientific unit, arc/radius
1 revolution = 360° = 2π rad



sine: $\sin(\alpha) = a/c$
cosine: $\cos(\alpha) = b/c$
tangent: $\tan(\alpha) = \mathrm{tg}(\alpha) = a/b$

for small angles (<10° ≈ 0.2 rad):
$\sin(\alpha) \approx \alpha\ [\mathrm{rad}] \approx \tan(\alpha)$



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Linear Function

INTEGRAL FORM

VARIABLES: dependent variable, independent variable

$y = a \cdot x + b$

PARAMETERS: slope (gradient, increment), y-axis intercept

"DIFFERENTIAL" FORM

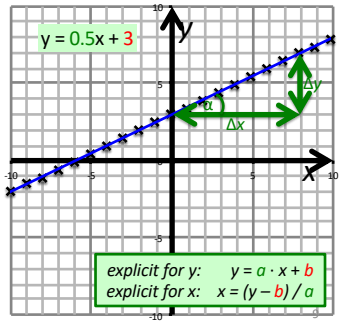
$\Delta y \propto \Delta x$

The **change** of the dependent variable is proportional to the **change** of the independent variable

if $x = 0$ then $y = b$

if $\Delta x = 1$ then $\Delta y = a$

$a = \Delta y / \Delta x = \tan\alpha$



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Linear Function: Some Examples

from the Biophysics Formula Collection

#1: The ideal gas law (I.35)

$pV = nRT$ (if n & V are constant)

$p = nR/V \cdot T + 0$

$y = a \cdot x + b$

#2: Photoelectric effect (II.37)

$E_{kin} = hf - W_{em}$

$E_{kin} = h \cdot f + (-W_{em})$

$y = a \cdot x + b$

#3: Attenuation coefficient (II.85)

$\mu = \mu_m \cdot \rho$

$\mu = \mu_m \cdot \rho + 0$

$y = a \cdot x + b$

#4: Ohm's law

$R = U/I$

$I = 1/R \cdot U + 0$

$y = a \cdot x + b$

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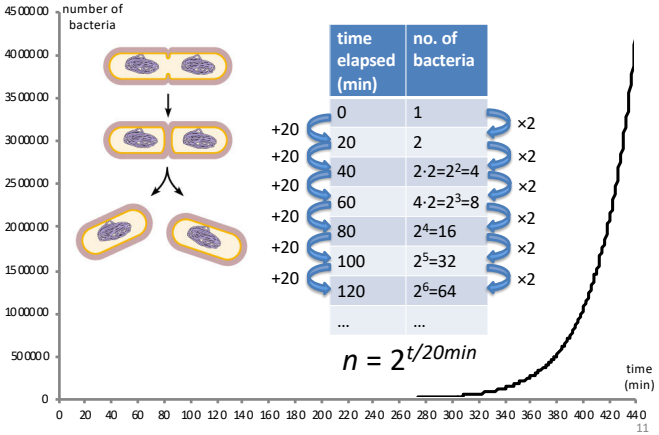
Exponential Function: Example #1



time elapsed (min)	no. of bacteria	
0	1	
+20	20	2	×2
+20	40	$2 \cdot 2 = 2^2 = 4$	×2
+20	60	$4 \cdot 2 = 2^3 = 8$	×2
+20	80	$2^4 = 16$	×2
+20	100	$2^5 = 32$	×2
+20	120	$2^6 = 64$	×2
...	...		

$n = 2^{t/20min}$

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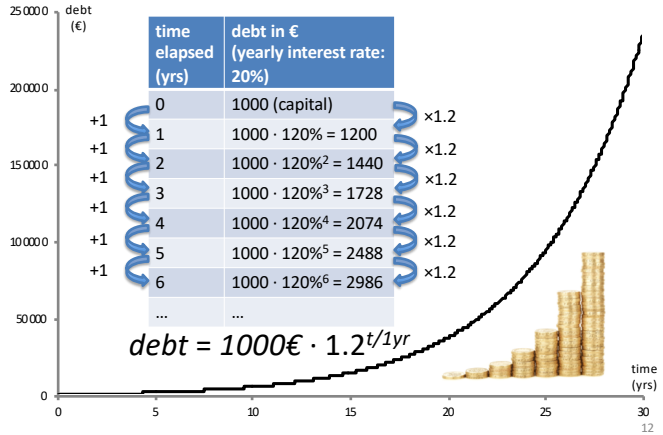
Exponential Function: Example #2



time elapsed (yrs)	debt in € (yearly interest rate: 20%)	
0	1000 (capital)	
+1	1	1000 · 120% = 1200	×1.2
+1	2	$1000 \cdot 120\%^2 = 1440$	×1.2
+1	3	$1000 \cdot 120\%^3 = 1728$	×1.2
+1	4	$1000 \cdot 120\%^4 = 2074$	×1.2
+1	5	$1000 \cdot 120\%^5 = 2488$	×1.2
+1	6	$1000 \cdot 120\%^6 = 2986$	×1.2
...	...	

$debt = 1000€ \cdot 1.2^{t/1yr}$

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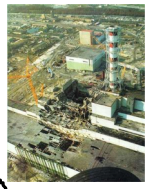
Exponential Function: Example #3



time elapsed (yrs)	Cs-137 (in PBq) from Chernobyl (half life: 30 yrs)	
0	85 (total fallout)	
+30	30	85 / 2 = 85 · 2^{-1} = 42.5 ×1/2
+30	60	85 / 2^2 = 85 · 2^{-2} = 21.3 ×1/2
+30	90	85 / 2^3 = 85 · 2^{-3} = 10.6 ×1/2
+30	120	85 / 2^4 = 85 · 2^{-4} = 5.3 ×1/2
+30	150	85 / 2^5 = 85 · 2^{-5} = 2.7 ×1/2
+30	180	85 / 2^6 = 85 · 2^{-6} = 1.3 ×1/2
…	…	

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Exponential Function

INTEGRAL FORM

$y = b \cdot a^x$

PRACTICAL MODIFICATIONS:
- the base number is preferred to be *e*
- a new factor parameter *p* (or 1/*k*) is necessary in the exponent
- use a negative sign in the exponent
- *b* is rather denoted by y_0

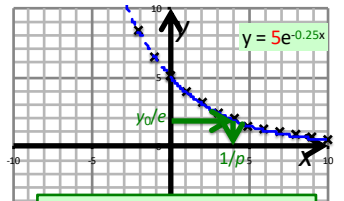
VARIABLES: dependent variable, independent variable

$y = y_0 \cdot e^{-px} = y_0 \cdot e^{x/k}$

PARAMETERS: pre-exponential coefficient, exponential coefficient

if $x = 0$ then $y = y_0$

if $y = y_0/e$ then $x = 1/p = k$



explicit for y: $y = y_0 \cdot e^{-px}$

explicit for x: $x = \ln(y / y_0) / (-p)$

"DIFFERENTIAL" FORM

$\Delta y/y \propto \Delta x$

The **relative change** of the dependent variable is proportional to the **change** of the independent variable

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Exponential Function: Linearization

graphical linearization
plot *y* on a log scale as a function of x:
*the relationship **looks** linear but it **is** still exponential*

INTEGRAL FORM

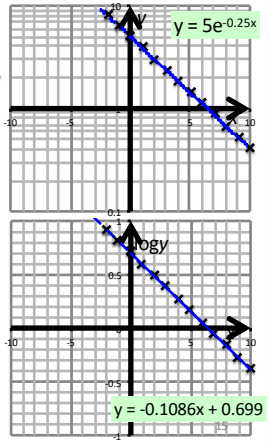
$y = y_0 \cdot e^{-px}$

$\log y = \log\left(y_0 \cdot e^{-p \cdot x}\right)$

$\log y = \log y_0 + \log\left(e^{-p \cdot x}\right)$

$\log y = \log y_0 - p \cdot x \cdot \log e$

$\underbrace{\log y}_{y} = \underbrace{-p \cdot \log e}_{a} \cdot x + \underbrace{\log y_0}_{b}$



intercept = log(y_0)
log(5) = 0.699
slope = $-p \cdot \log(e)$
$-0.25 \cdot \log(e) = -0.1086$

arithmetical linearization
plot log(*y*) as a function of *x*:
*the relationship **is** linear*

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Exponential Function: Some Examples

from the Biophysics Formula Collection

#1: Law of radiation attenuation (II.11)

$J = J_0 \cdot e^{-\mu x}$

$y = y_0 \cdot e^{-px}$

#2: Boltzmann's distribution (I.25)

$n_i = n_0 \cdot e^{-\Delta\varepsilon/(kT)}$

$y = y_0 \cdot e^{-x/k}$

#3: Decay law (II.96)

$N = N_0 \cdot e^{-\lambda t}$

$y = y_0 \cdot e^{-px}$

#4: Discharging an RC circuit (VII.2)

$U = U_0 \cdot e^{-t/(RC)}$

$y = y_0 \cdot e^{-x/k}$

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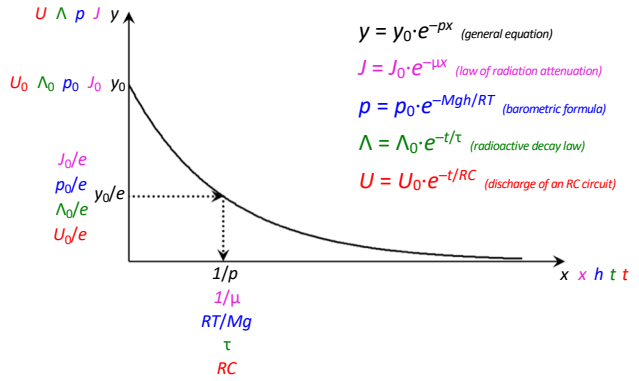
Graph of Exponential Functions

from the Biophysics Formula Collection



$y = y_0 \cdot e^{-px}$ *(general equation)*

$J = J_0 \cdot e^{-\mu x}$ *(law of radiation attenuation)*

$p = p_0 \cdot e^{-Mgh/RT}$ *(barometric formula)*

$\Lambda = \Lambda_0 \cdot e^{-t/\tau}$ *(radioactive decay law)*

$U = U_0 \cdot e^{-t/RC}$ *(discharge of an RC circuit)*