**Slide 1**



# Physical Foundations of Dental Materials Science
## 7.
**Mechanical properties of materials 1. Elasticity**

Keywords:
- ❖ 1 Stress-strain
- ❖ 2 Elasticity
- ❖ 3 Deformations

**E-book chapter 14, 15.**

**Problems:** 4.1, 4.3, 4.4, 4.5, 4.6, 4.9, 4.11, 4.16, 4.17, 4.18, 4.21, 4.22, 4.23, 4.24.

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## Mechanical propeties of materials

Stiffness, elasticity, toughness, hardness are the most importamt parameters when considering mastication, implantology or orthodontics.

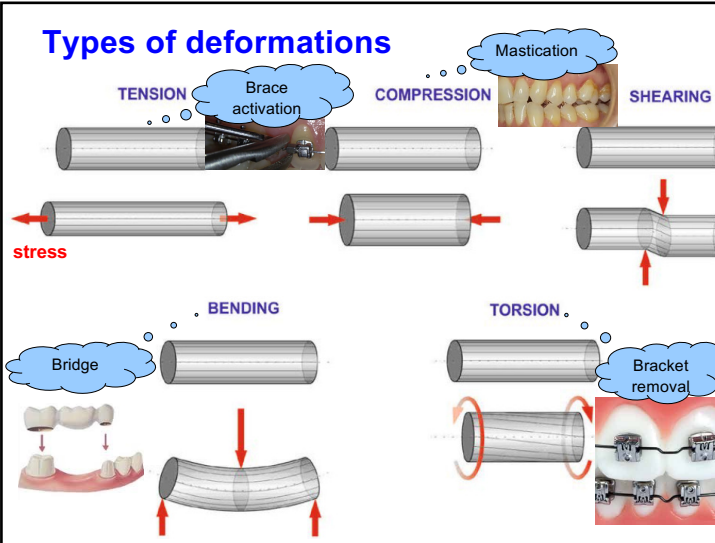Mechanical stress (force) is applied on material.

↓

The consequence is usuallay some kind of deformation.
(change in shape depends on the direction and point of application)

↓

Depending on the materials mechanical properties, deformation may be reversible or irreversible.

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## Types of deformations

Mastication

Brace activation

TENSION     COMPRESSION     SHEARING

**stress**

Bridge

BENDING     TORSION

Bracket removal



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**elastic** *adjective* able to resume its normal shape spontaneously after being stretched or compressed

**rigid** *adjective* unable to bend or be forced out of shape; not flexible

**plastic** *adjective* relating to the permanent deformation of a solid without fracture by the temporary application of force

**strong** *adjective* able to withstand force, pressure, or wear

**weak** *adjective* liable to break or give way under pressure; easily damaged

**solid** *adjective* firm and stable in shape; not liquid or fluid, strongly built or made of strong materials

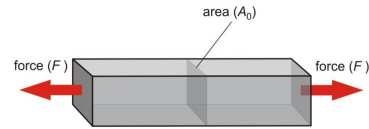**tough** *adjective* resistant to fractue or breaking
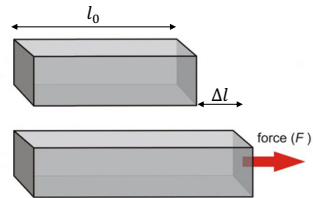
**brittle** *adjective* liable to break easily

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## Basic consepts

**Stress**

$$\sigma = \frac{F}{A_0}$$

Dimenson: $\left[\frac{N}{m^2} = Pa\right]$

area ($A_0$)

force ($F$)        force ($F$)

**Strain (deformation)**

$$\varepsilon = \frac{\Delta l}{l_0}$$

Dimensionless: $\left[\frac{m}{m}\right]$

$l_0$

$\Delta l$

force ($F$)

**Strain is proprtional to stress!**

$$\sigma \sim \varepsilon$$

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## Stress-strain diagram

Stress ($\sigma$)

tangent

plastic range

elastic range

proportional range

permanent deformation

fracture

strain ($\varepsilon$)
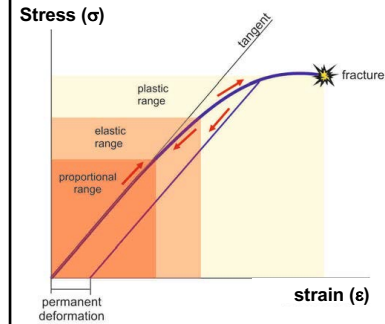
**Elastic range**
Range of reversible deformation. Unloaded length ($l_0$) recovers when released. Hysteresis may occour.

**Proportional range**
*(part of elastic range)*
Deformation is linearly proportional to the load. No hysteresis. *(see Hooke's law: Biophysics, Rasonance lab)*

**Plastic range**
After a critical stress value, object undergoes irreversible change of its structure. Unloaded length ($l_0$) does not recover. Permanent deformation of object.
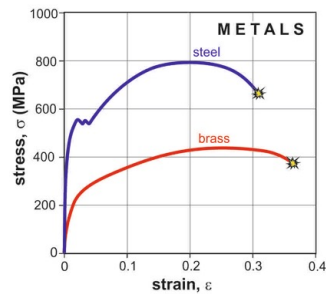
**Fracture**
Desintegration of object.

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## Typical stress-strain curves
### Metals

METALS

steel

brass

stress, $\sigma$ (MPa)

strain, $\varepsilon$

Short elastic range

Serious plastic range

Metals are highly deformable $\varepsilon_{max}$~0.3 (~30%)

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## Typical stress-strain curves
### Ceramics

$Al_2O_3$

CERAMICS

glass

stress, $\sigma$ (MPa)
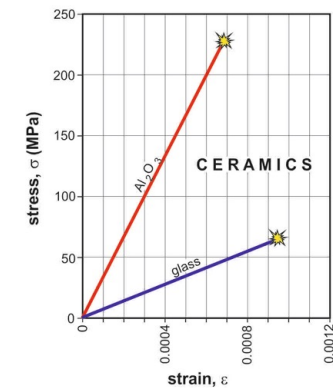
strain, $\varepsilon$

Extremly short elastic range $\varepsilon_{max}$<0.001 (less than ~0.1%)

No plastic range!

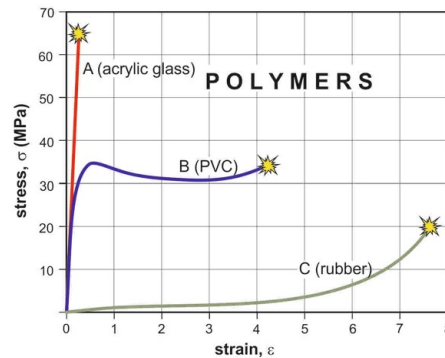Ceramics are brittle, they rupture after a short extension/compression....

## Typical stress-strain curves
### Polymers



Polymers show variouis behaviours

Typically they are deformable $\varepsilon_{max}$ from 0.1 to 8

Acrylic glass
No plastic range - brittle

PVC
Large plastic range
Plastic polymer

Rubber
Elastic polymer
$\varepsilon_{max}$ up to 7-8 (~700-800%)

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## Additional factors determining the stress-strain diagram
*The main decisive is the material's structure (e.g. amorphous or crystalline)*

1. Type of stress. (For example the behavior of ceramics is different in the case of tension or elongation.)

2. The shape, size commonly the geometry of the object. (It is more difficult to bend a thicker rod than a thin.)

3. The run-time of the stress: *stress rate*. (For example the glass is broken into pieces using a hammer but a bullet launched by a gun produces a small hole only.)
Mechanical properties – introduction

4. Temperature. (Some metals are very plastic and tough at room temperature but become more brittle cooling them.)
These conditions must be presented in the stress-strain diagram.

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## Hooke's law of elaticity
**Strain is proprtional to stress!**

$$\sigma \backsim \varepsilon$$

$$\sigma = E\varepsilon$$
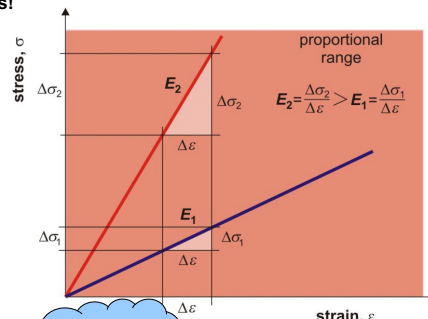
$$\boxed{\frac{F}{A_0} = E\frac{\Delta l}{l_0}}$$

$$F = \frac{EA_0}{l_0}\Delta l$$

$$F = k\Delta l$$

Proportionality coeffitients:



$E_2 = \frac{\Delta\sigma_2}{\Delta\varepsilon} > E_1 = \frac{\Delta\sigma_1}{\Delta\varepsilon}$

The symbol *D* is also used for spring constant!

**Young's modulus**
*(material stiffness)*

$$E = \frac{\sigma}{\varepsilon} = \frac{F}{A_0}\frac{l_0}{\Delta l} \qquad E = \left[\frac{N}{m^2} = Pa\right]$$
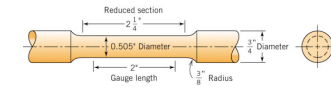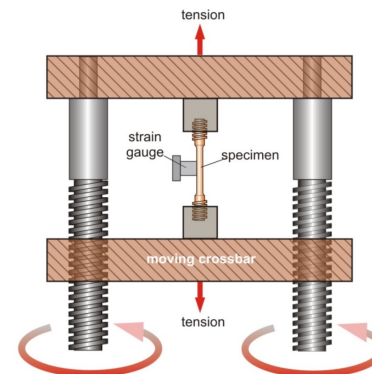
**Body stiffness**

$$k = \frac{F}{\Delta l} = \frac{EA_0}{l_0} \qquad k = \left[\frac{N}{m}\right]$$
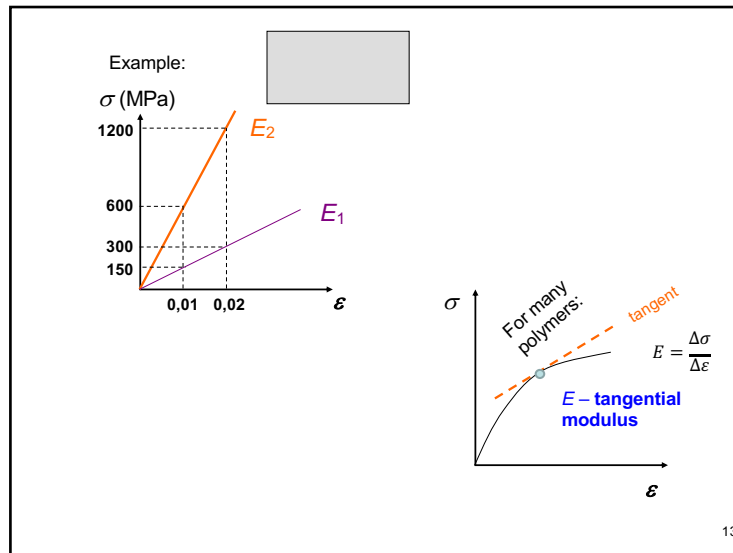
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## Testing



**The result of shuch measurement is determined by:**

- Type of stress (tension, ...)

- sample geometry



- Timecourse of stress
  - constant
  - changing

- temperature

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Example:

$\sigma$ (MPa)

$E_2$

1200

600

$E_1$

300
150

0,01  0,02   $\varepsilon$

$\sigma$

For many polymers:

tangent

$E = \dfrac{\Delta\sigma}{\Delta\varepsilon}$

$E$ – **tangential modulus**

$\varepsilon$

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## Slide 14

### Quantification of stiffness

**Young's modulus of various materials**

GPa = $10^9$ N/m$^2$

**Spring constant**
Easy to measure
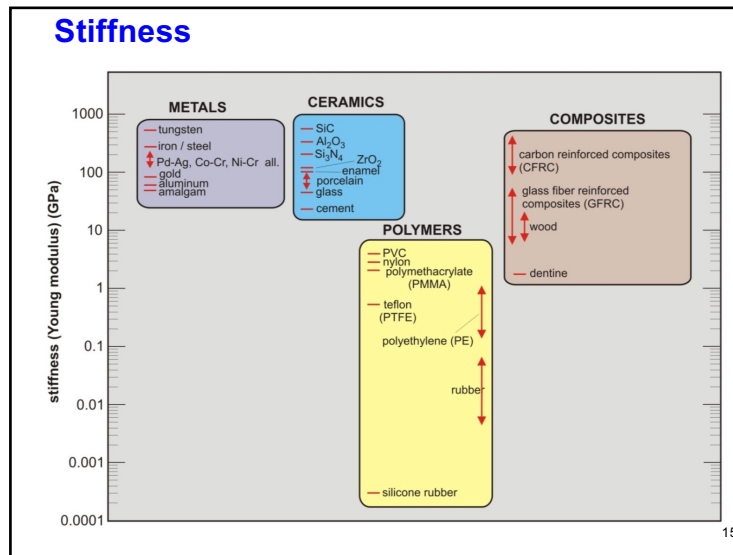Depends on size and shape of object

**Young's modulus**
Hard to measure (we need to know the object's dimensions)
Shape independent

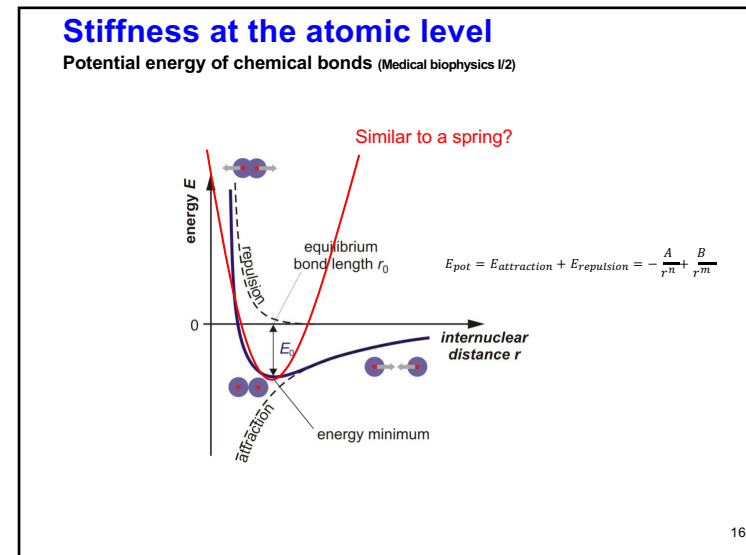1/E – reciprocal of E is used to express compliance!

| material | E (GPa) |
|---|---|
| dentine | ≈ 15 |
| enamel | ≈ 100 |
| silicon rubber | ≈ 0.0003 |
| steel | 200-230 |
| amalgam | 50-60 |
| glass | 60-90 |
| porcelain | 60-110 |
| gold | 79 |
| gold alloys | 75-110 |
| Pd-Ag alloys | 100-120 |
| titanium | 110 |
| titanium alloys | 105-120 |
| Co-Cr alloys | 120-220 |
| Ni-Cr alloys | 140-190 |

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## Slide 15

### Stiffness

stiffness (Young modulus) (GPa)

1000
100
10
1
0.1
0.01
0.001
0.0001

**METALS**
— tungsten
— iron / steel
↕ Pd-Ag, Co-Cr, Ni-Cr all.
— gold
— aluminum
— amalgam

**CERAMICS**
— SiC
— Al$_2$O$_3$
— Si$_3$N$_4$  — ZrO$_2$
— enamel
— porcelain
— glass
— cement

**COMPOSITES**
carbon reinforced composites (CFRC)
glass fiber reinforced composites (GFRC)
wood
— dentine

**POLYMERS**
— PVC
— nylon
— polymethacrylate (PMMA)
— teflon (PTFE)
polyethylene (PE)
rubber
— silicone rubber

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## Slide 16

### Stiffness at the atomic level

**Potential energy of chemical bonds** (Medical biophysics I/2)

Similar to a spring?

energy E

repulsion

equilibrium bond length $r_0$

0

$E_0$

internuclear distance r

attraction

energy minimum

$E_{pot} = E_{attraction} + E_{repulsion} = -\dfrac{A}{r^n} + \dfrac{B}{r^m}$

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# Young's modulus

**Young's modulus decreases with temprature increasing**



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# Changes in the cross-sectional dimension

**Stretched objects become narrower or… ?**

$\Delta d$ is negative!

*Engineering stress vs. Real stress*

$$\frac{\Delta d}{d} = -\mu \frac{\Delta l}{l}$$

$\mu$: Poisson's ratio (no dimension)
$\mu_{max} = 0.5$ (*usually between 0.3-0.4*)

**Auxetic materials** have negative Poisson's number: They become thicker when stretched. (bullet proof vests, compression bandages)

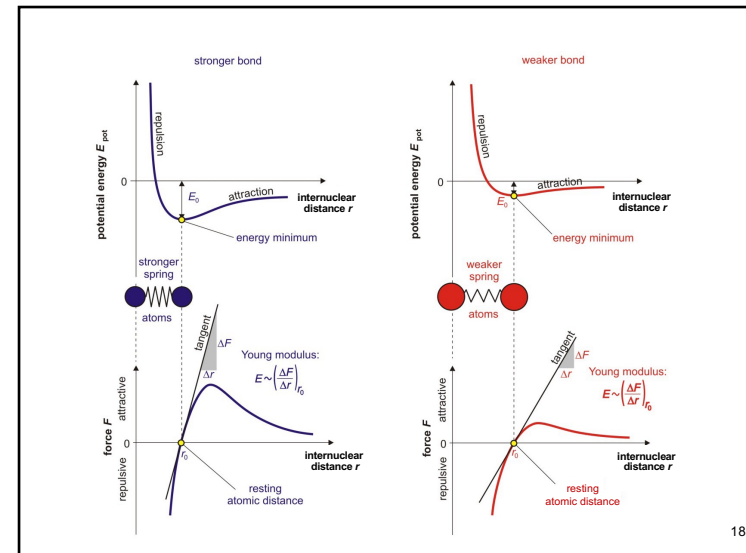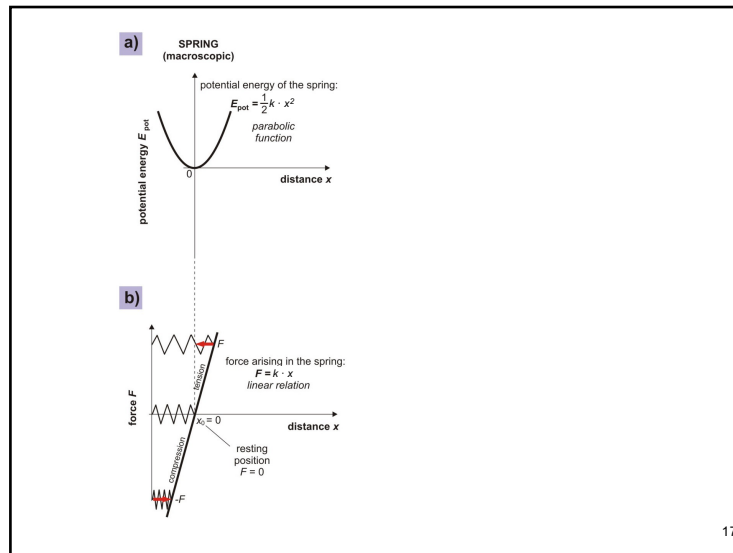http://www.youtube.com/watch?v=PLDbSWSm5i8

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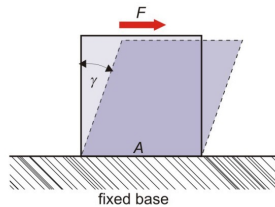
## Deformations in other directions

**Compression**
Same as stretch, but signs of stress and strain are opposite, Hooke's law is valid.

**Shear**
Force is parallel to the surface,
Shear stress: $\sigma_{shear} = \frac{F_{shear}}{A}$
Strain is characterized by the $\gamma$ angle



$\sigma_{shear} = G\gamma$    $G$: Shear modulus
                     $\gamma$: in radians!

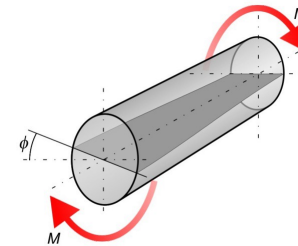$G = \frac{E}{2(1+\mu)}$    *Depends on Young's modulus and Poisson's ratio!*

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## Deformations in other directions

**Torsion**
Can be derived from shear. Torque (M) is applied on the object.
Angle of torsion ($\phi$) is measured in radians!
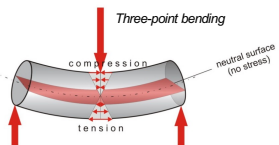


$$M = G\frac{r^4\pi}{2l}\phi$$

$G$: Shear modulus
(contains E and $\mu$)

$r$: radius
$\phi$: angle of torsion
$l$: length

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## Deformations in other directions

*Three-point bending*



**Bending**
Compression on the side of force applied, tension on the opposite side of object. Neutral surface at the middle – does not deform

CROSS-SECTIONS    SECONDARY MOMENT OF AREA

circle    $\Theta = \frac{\pi}{4}\cdot r^4$

ring    $\Theta = \frac{\pi}{4}\cdot (r_2^4 - r_1^4)$

rectangle    $\Theta = \frac{1}{12}ab^3$

Cantilever deflection

$$F = 3E\frac{\Theta}{l^3}s$$

*Body stiffness in case of deflection*

$G$: Force
$E$: Young's modulus
$\Theta$: secondary moment of area
$s$: deflection
$l$: length

*In case of bending the force required depends stronger on the shape!*
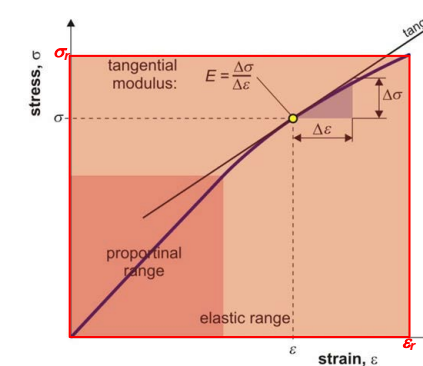
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## Elasticity beyond the proportional range

**Elastic limit**
End of the elastic range on the y-axis ($\sigma_r$) – maximum reversible strain

**Elastic strain recovery**
End of the elastic range on the y-axis ($\varepsilon_r$) – maximum reversible stress



tangent
tangential modulus: $E = \frac{\Delta\sigma}{\Delta\varepsilon}$
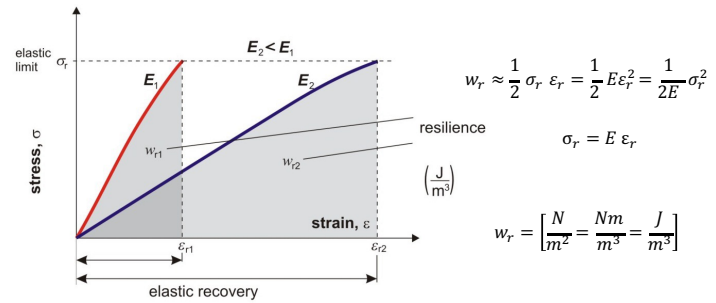
Tangential Young's modulus

$$E = \frac{\Delta\sigma}{\Delta\varepsilon}$$

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## Resilience
*Work done on the material per unit volume till the elastic limit.*



$$w_r \approx \frac{1}{2}\, \sigma_r \, \varepsilon_r = \frac{1}{2}\, E \varepsilon_r^2 = \frac{1}{2E}\, \sigma_r^2$$

$$\sigma_r = E\, \varepsilon_r$$

$$w_r = \left[ \frac{N}{m^2} = \frac{Nm}{m^3} = \frac{J}{m^3} \right]$$

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