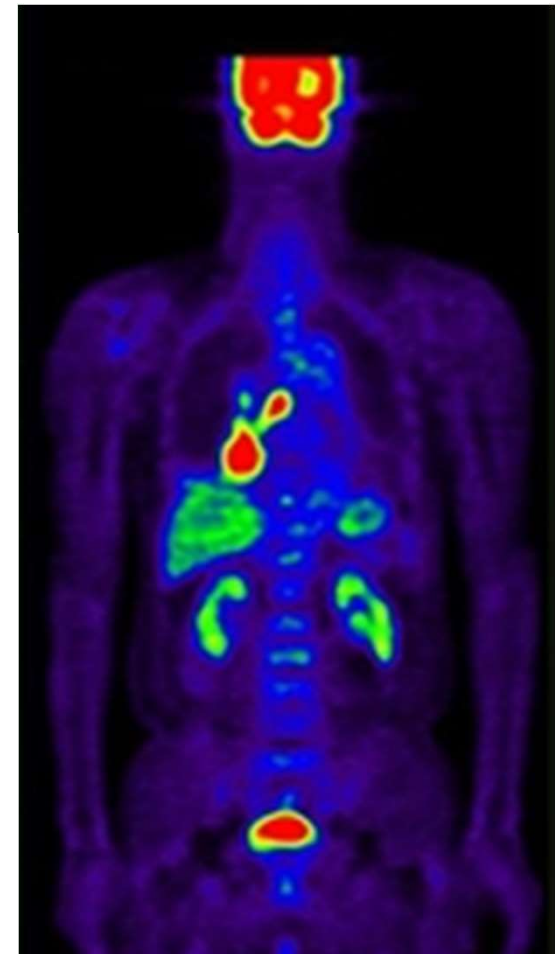
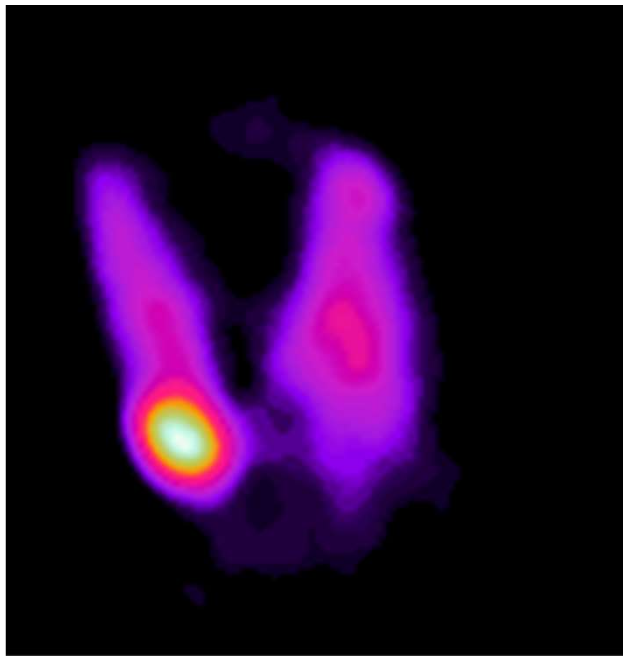


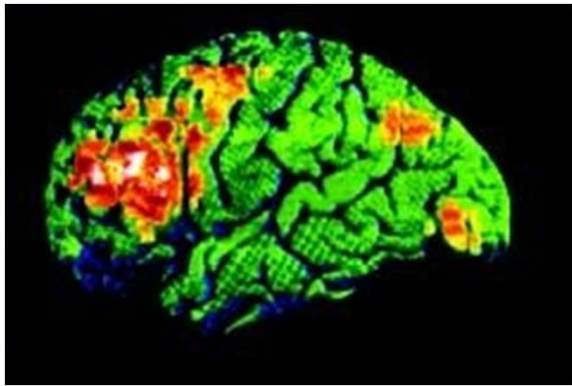
# The atomic nucleus. Radioactivity. Nuclear radiations

László Smeller

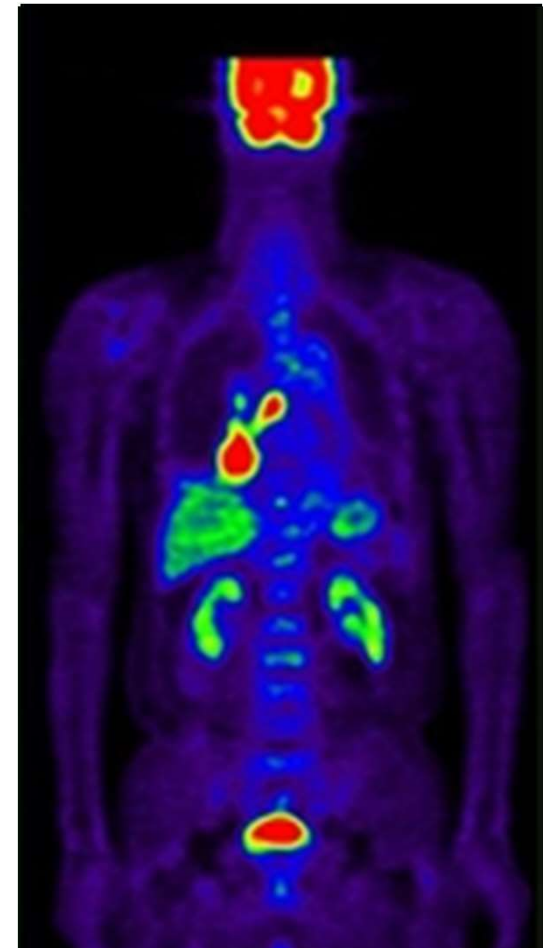
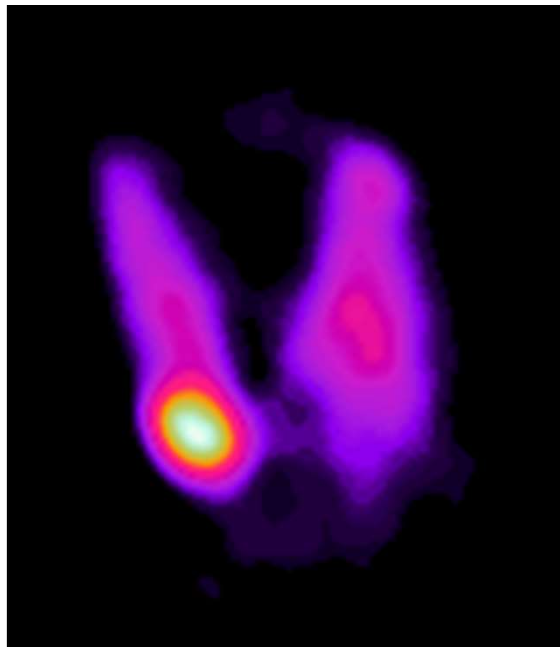


# Why?




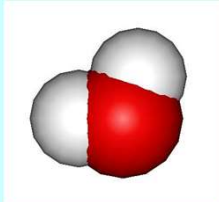
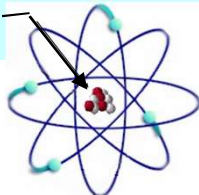
Medical/pharmaceutical applications of nuclear radiation:



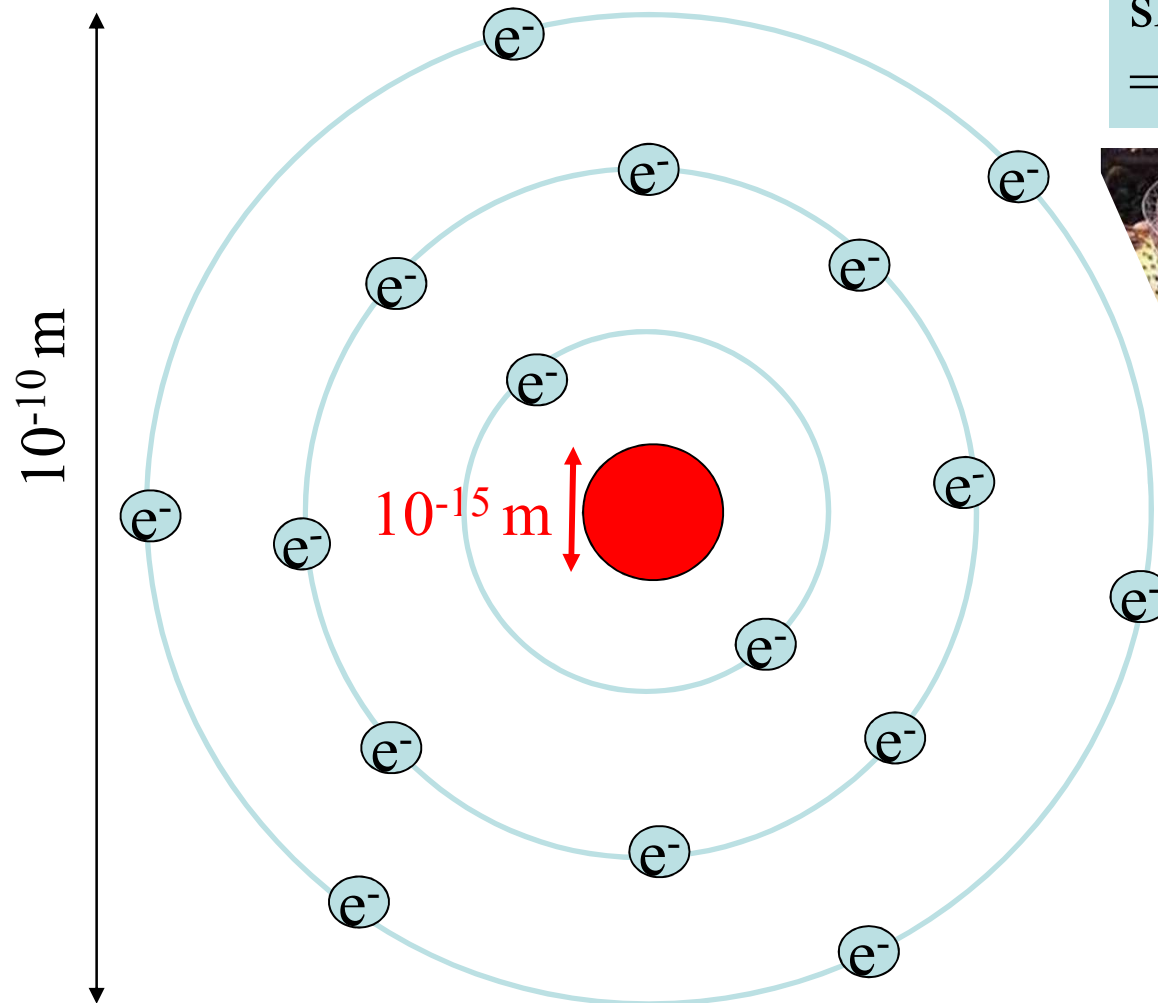
Nuclear imaging  
Radiotherapy  
Pharmacokinetics



# Length scale of the nature

m		
$10^0$	meter	men 
$10^{-3}$	millimeter	letters you can read
$10^{-6}$	micrometer	size of a cell (e.g. erythrocyte) 
		$\varnothing 7\mu\text{m}$
$10^{-9}$	nanometer	protein 
$10^{-10}$	– angstrom	diameter of an atom, bond length
		H atom $\varnothing \approx 1$ angstrom (Å)
$10^{-12}$	picometer	wavelength of the X-ray 
$10^{-15}$	femtometer	size of the nucleus 

# The electrons and the nucleus

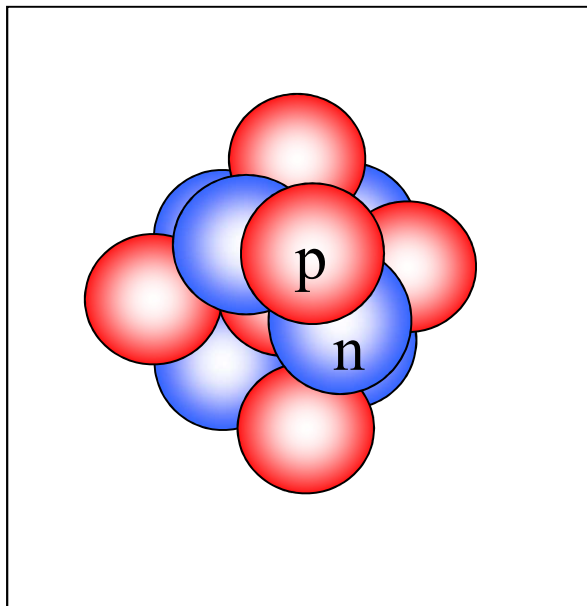


Changes in the electron  
shell:  
=> chemical processes



**Changes of the  
nucleus:  
=> radioactivity**

# Structure of the nucleus

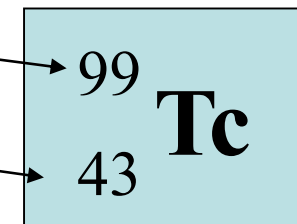


Elementary charge =  $1,6 \cdot 10^{-19} \text{ C}$

	charge	mass
<b>proton</b>	<b>+1 e</b>	<b>1 atomic mass unit</b>
<b>neutron</b>	<b>0</b>	<b>1 atomic mass unit</b>

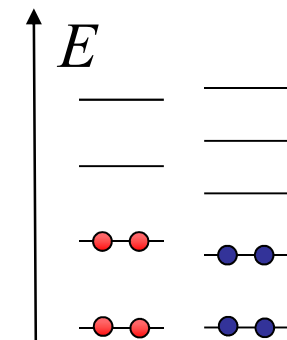
$A$  (mass number) = number of protons  
 + number of neutrons  
 $Z$  (atomic number) = number of protons

99 nucleon, 43 proton and 56 neutron



# Stability of the nucleus

- Coulomb force: destabilization  
(electrostatic repulsion between the protons)
- Nuclear force: very strong  
attractive force  
acts only on short range ( $\sim \text{fm}$ )  
independent on the charge
- Quantized energy levels for the nucleus.
- Typical binding energy is in the MeV range  
 $eV = 1,6 \cdot 10^{-19} \text{ J}$



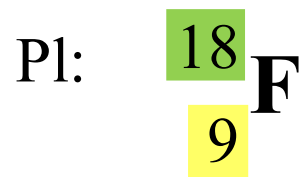
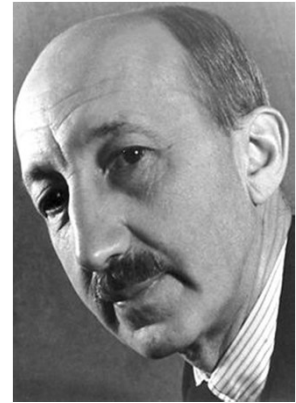
# Isotopes

Number of protons is the same

Number of neutrons is different

Variants of the same element

⇒ the chemical properties are identical.



unstable  
(radioactive)



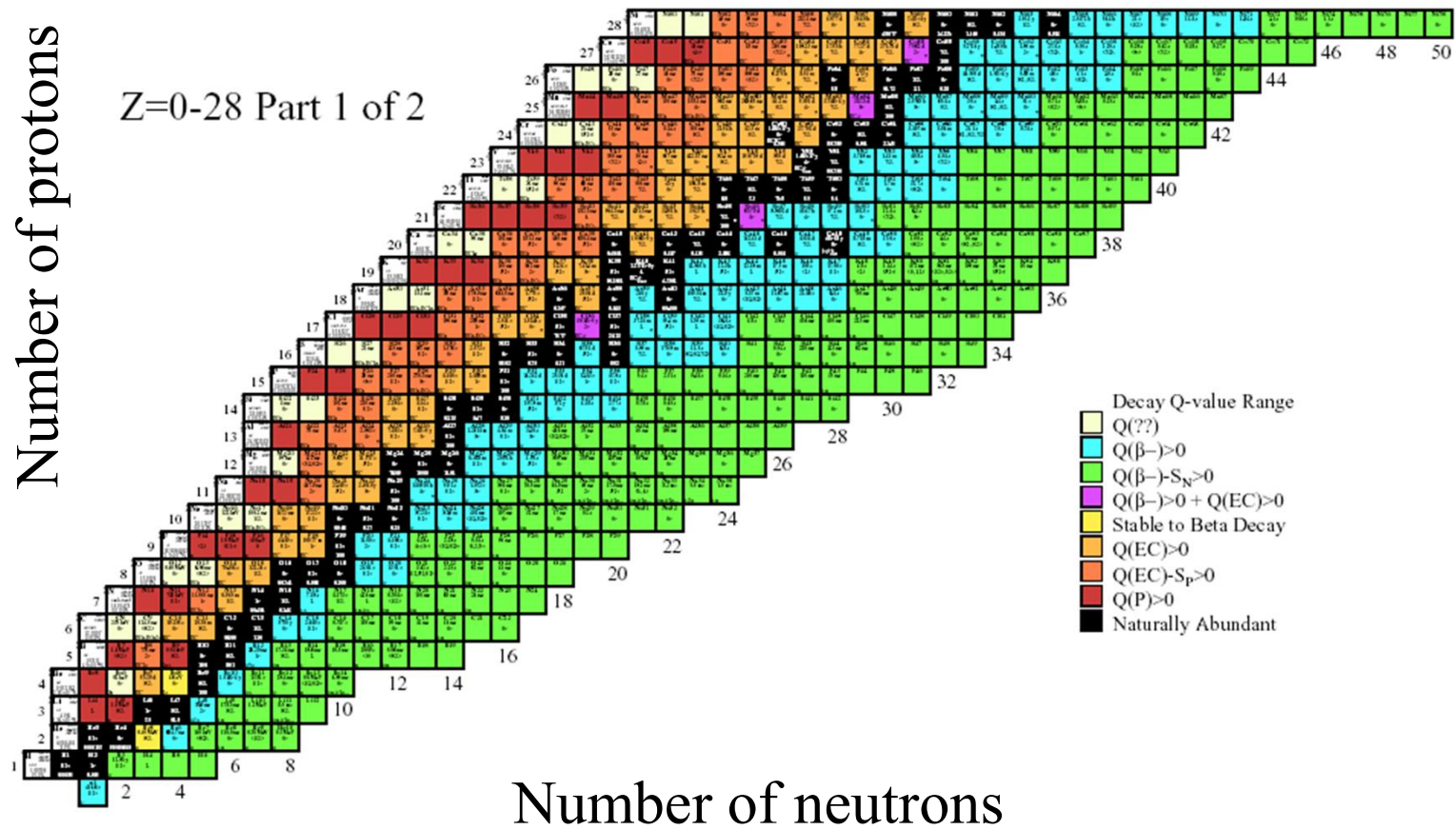
stable

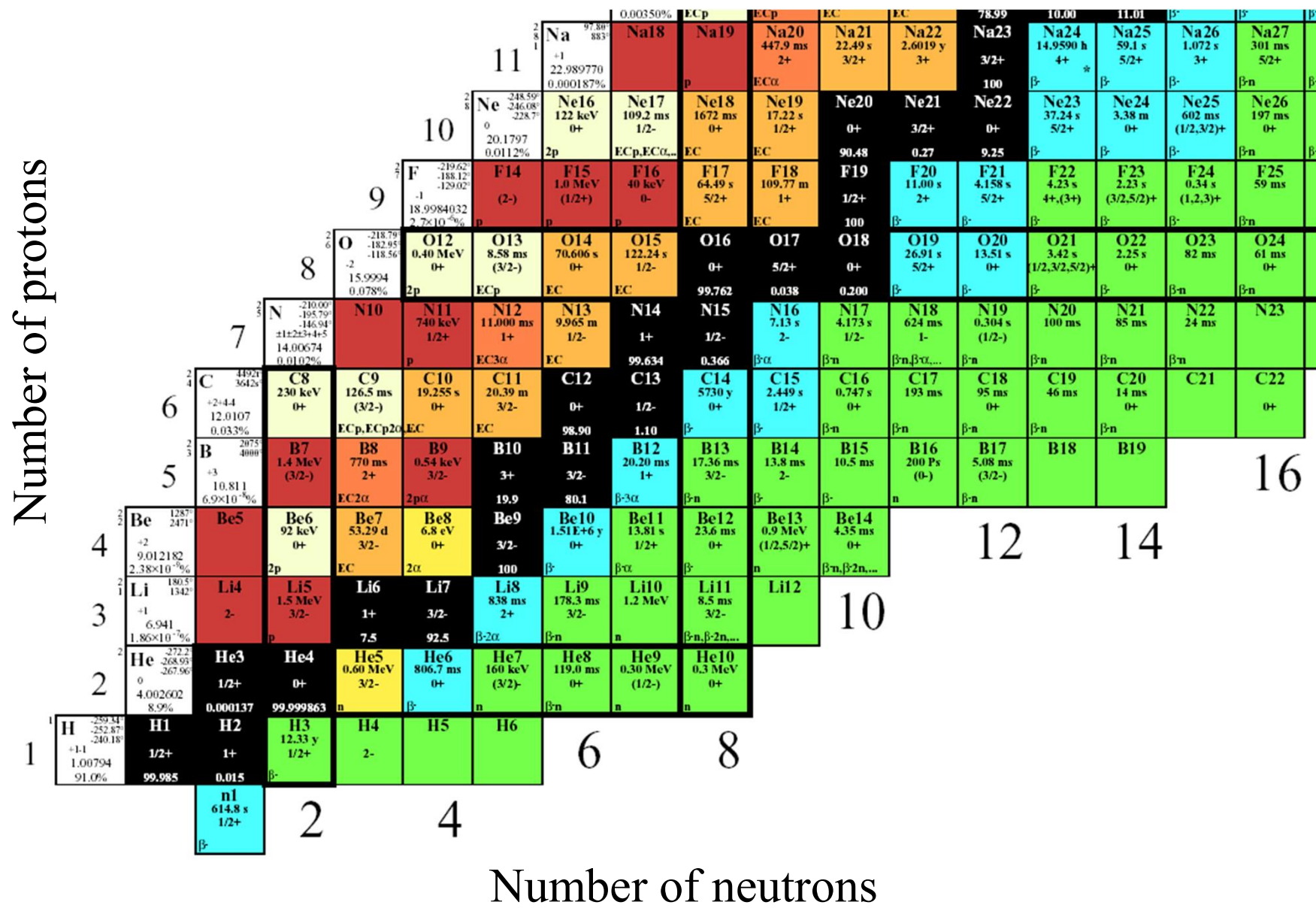


unstable  
(radioactive)

isotope  $\leftrightarrow$  radioactive isotope

# Table of isotopes





Number of protons

1	2	3	4	5	6	0.01012%										99.989879%										0.3508																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																					
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Number of neutrons

# Radioactive decays and particles

$\alpha$  - decay

$\alpha$  - particle =  ${}^4_2\text{He}$  nucleus

$\beta$  - decay :  $\beta^-$   
 $\beta^+$

$\beta^-$  particle = electron

$\beta^+$  particle = positron

Isomeric transition

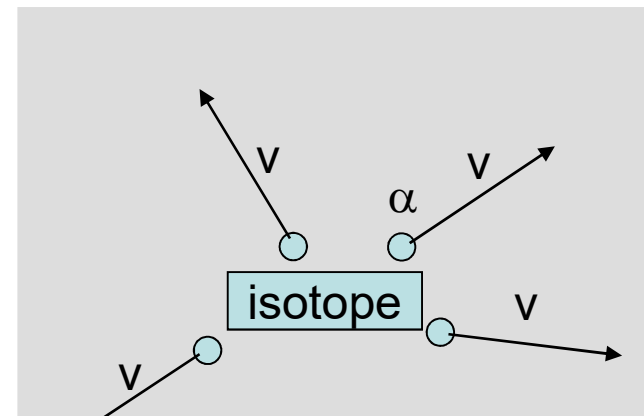
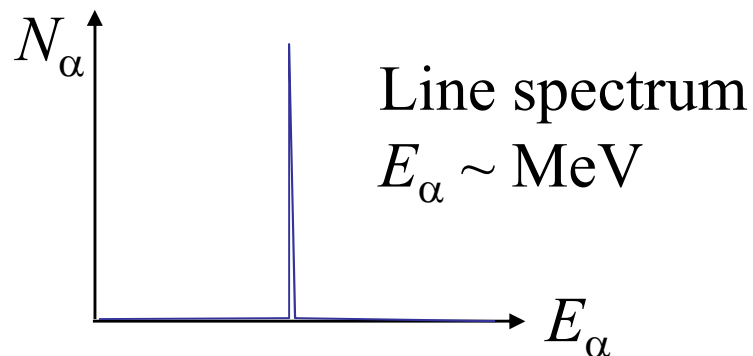
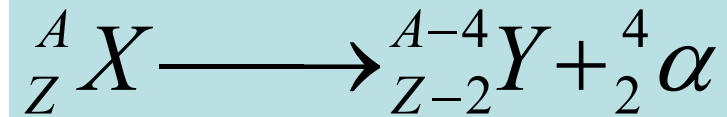
$\gamma$ -ray

K-electron capture

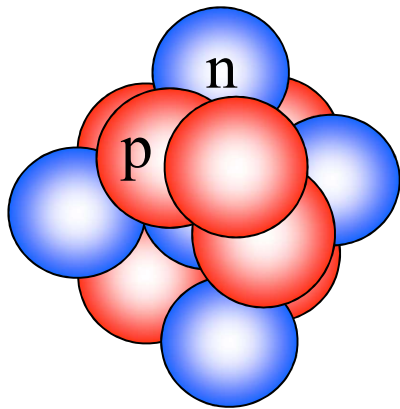
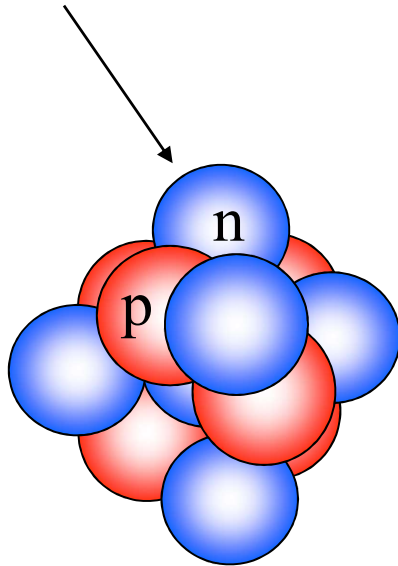
characteristic x-ray photon

# $\alpha$ - decay

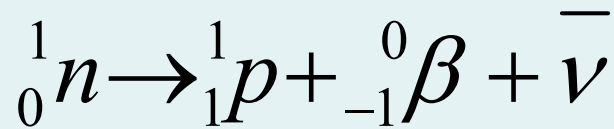
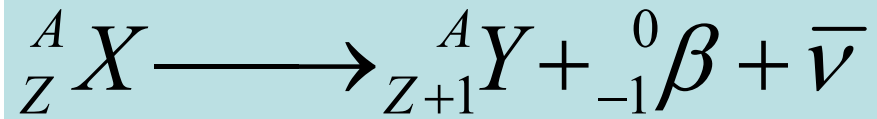
$\alpha$  - decay: an  $\alpha$  particle ( ${}^4\text{He}$  nucleus) will be emitted  
typical for the heavy atoms



neutron surplus

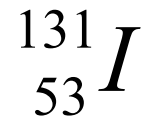
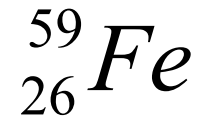
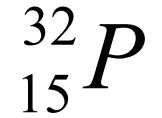
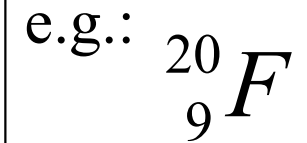


# $\beta^-$ - decay



remains in  
the nucleus

leave the  
nucleus

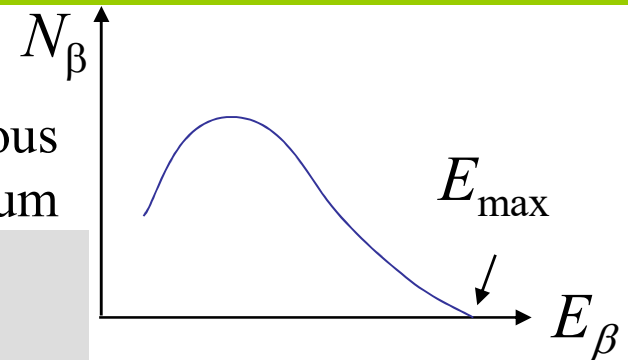
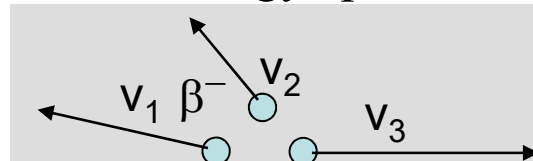


$e^-$   $\beta^-$ -ray

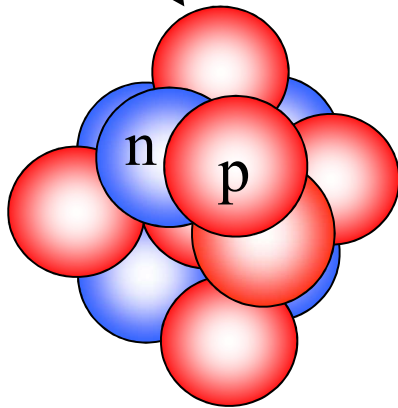
$\bar{\nu}$

continuous  
energy spectrum

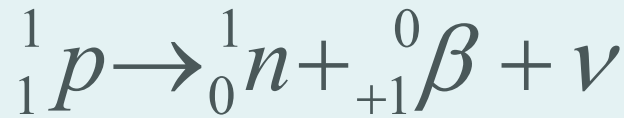
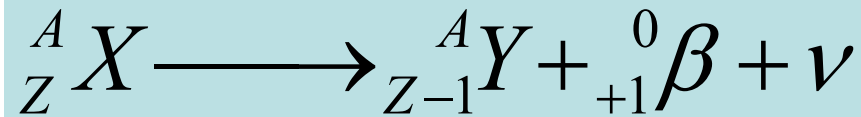
$$\beta^- = {}^0_{-1} \beta = e^-$$



proton surplus

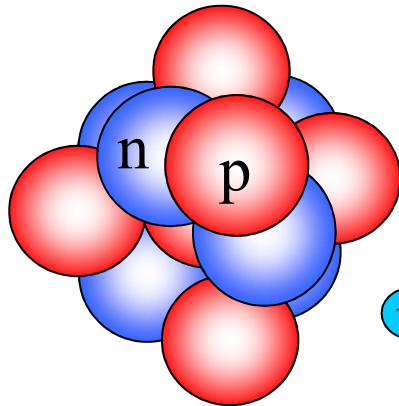


# $\beta^+$ - decay



remains in  
the nucleus

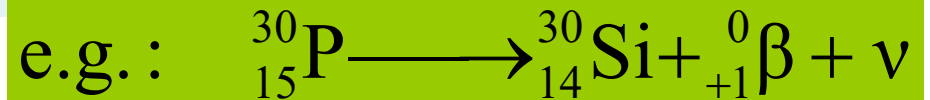
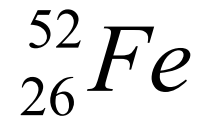
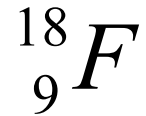
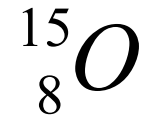
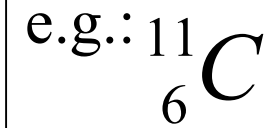
leave the  
nucleus

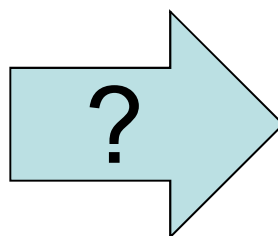


$\beta^+$ -ray

continuous energy spectrum

These isotopes must be produced  
artificially (e.g. in cyclotron)

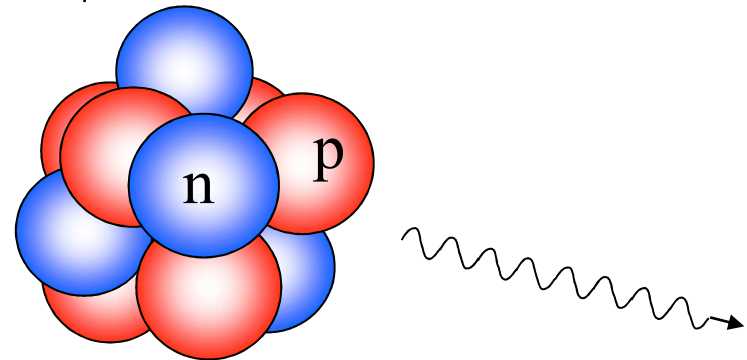
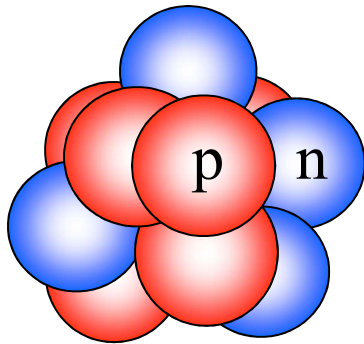
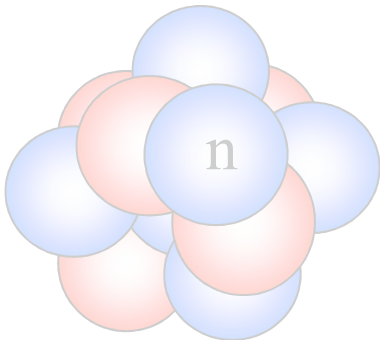
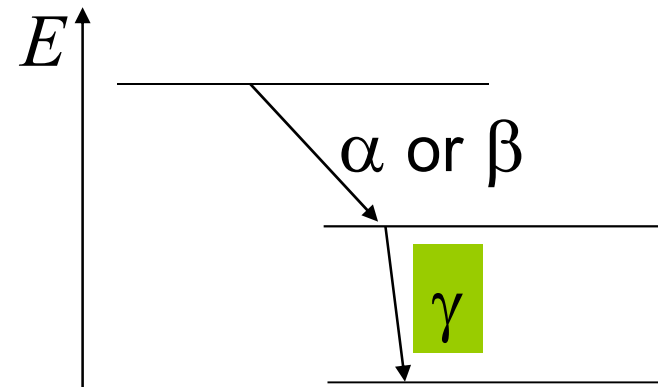




# Prompt $\gamma$ -radiation

The daughter nucleus might have an  
**energetically unfavoured** arrangement of nucleons.  
(excited state)

The surplus energy will normally be emitted immediately ( $< \text{ps}$ ) in form of the  $\gamma$  radiation



Atomic number, mass number are unchanged.

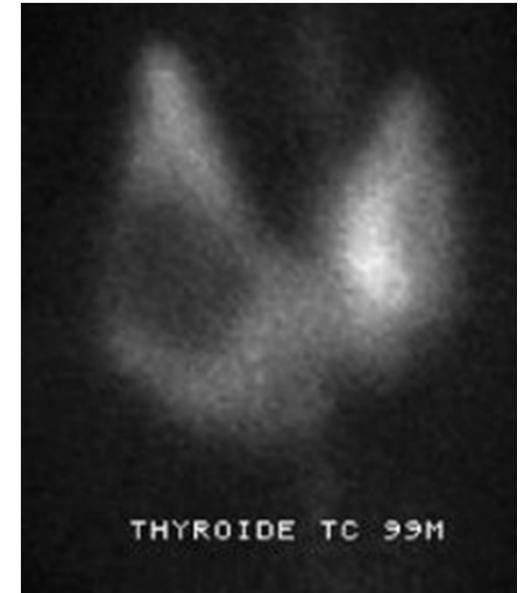
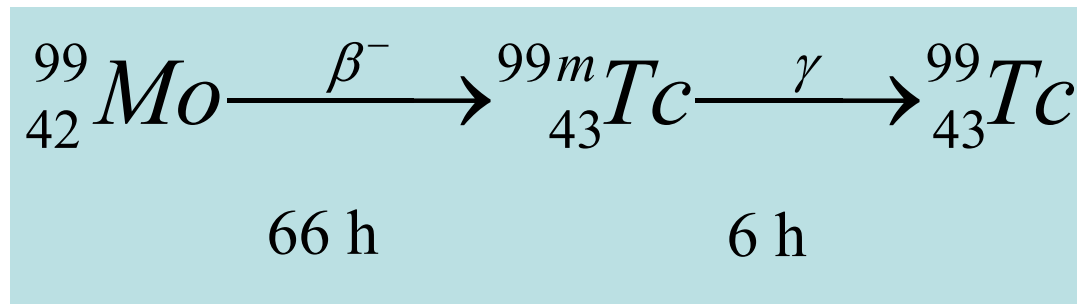
# Isomeric transition

In some rare cases the excited state of the daughter nucleus is metastable, the  $\gamma$ -radiation will be emitted later.

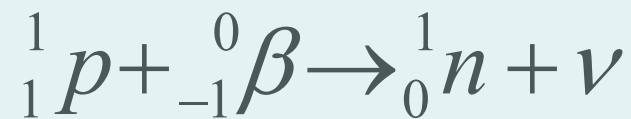
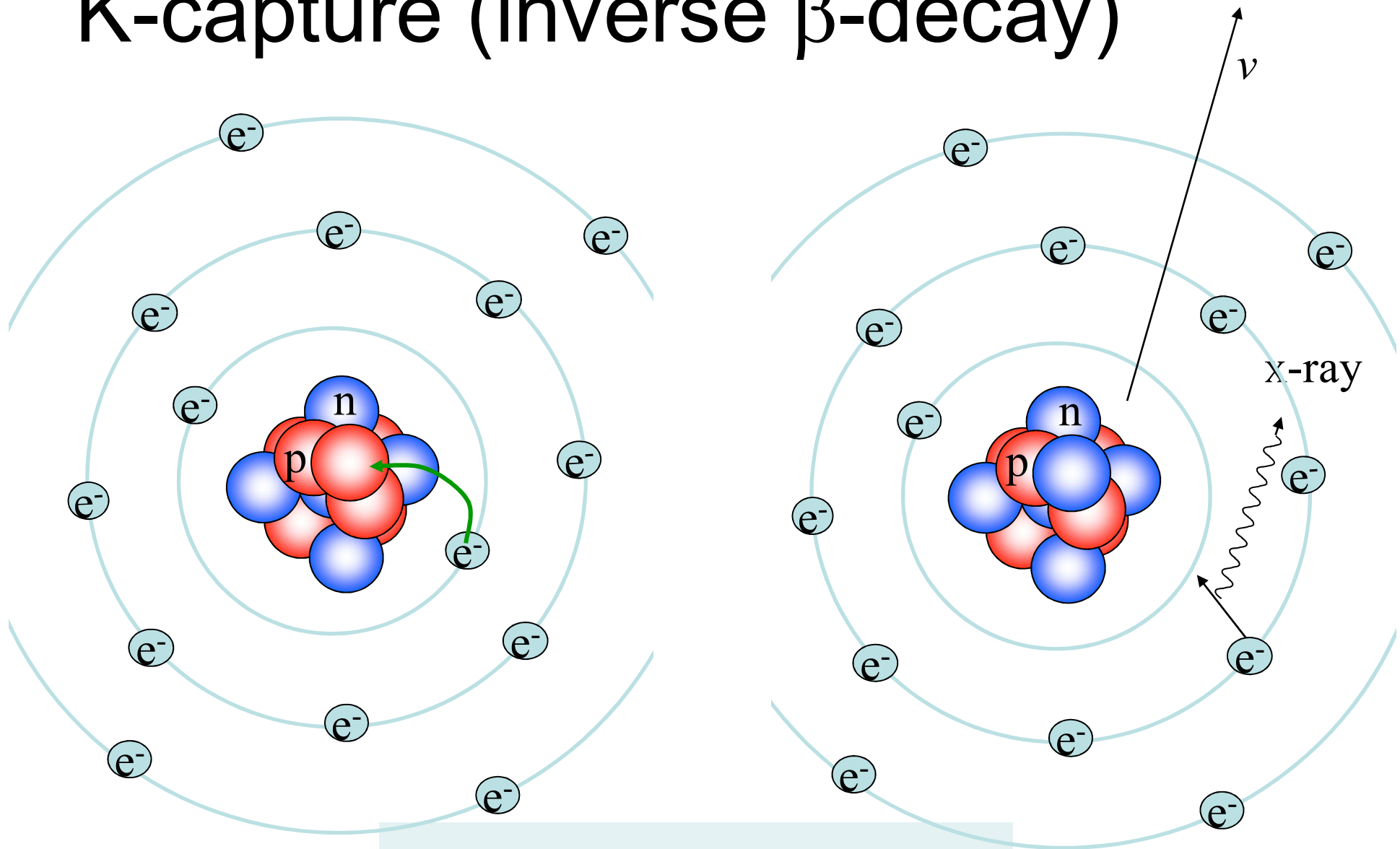
The parent and daughter atoms can be separated: the daughter atom emits only  $\gamma$ -radiation!

=> Isotope diagnostics (nuclear imaging)

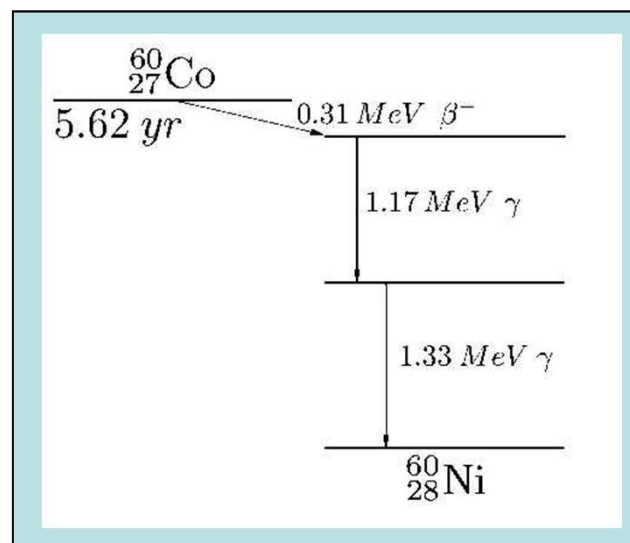
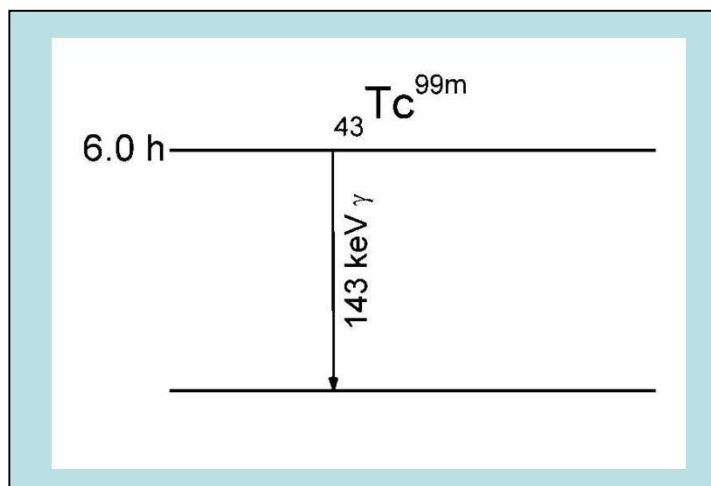
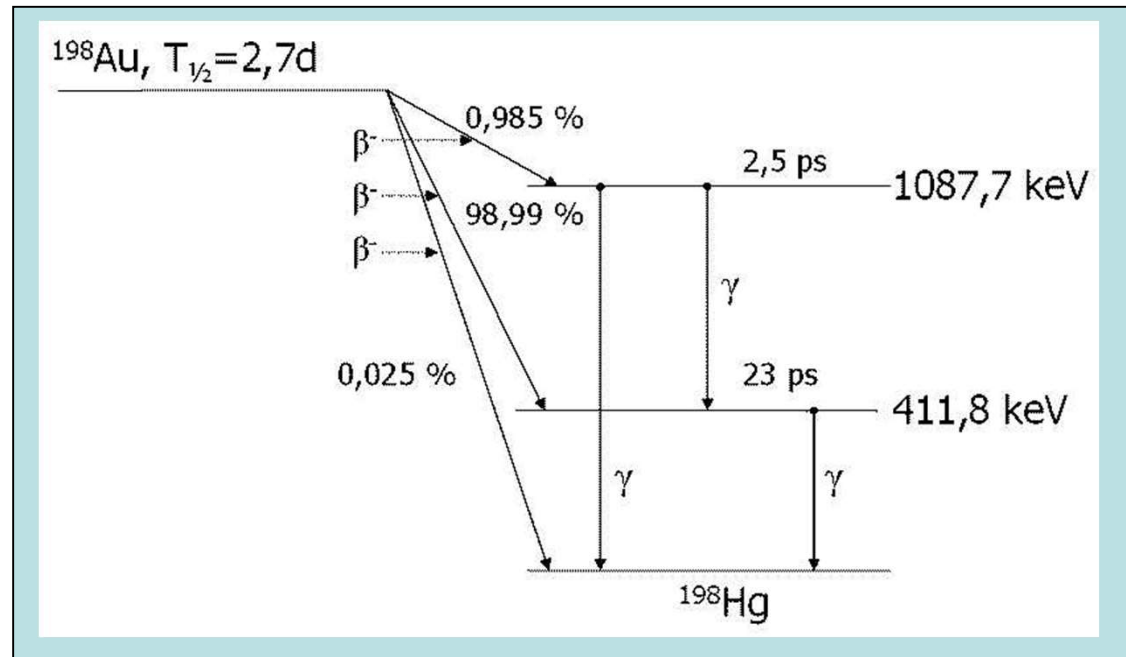
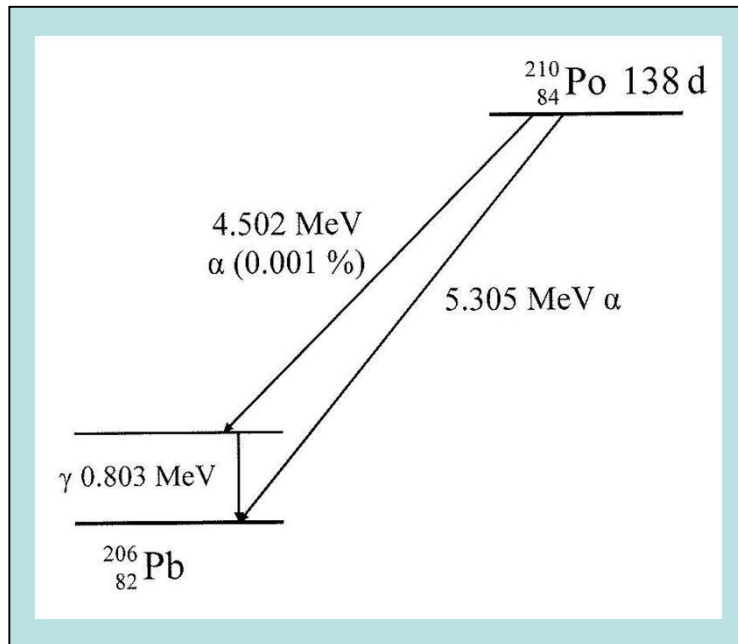
E.g.:  $^{99m}\text{Tc}$



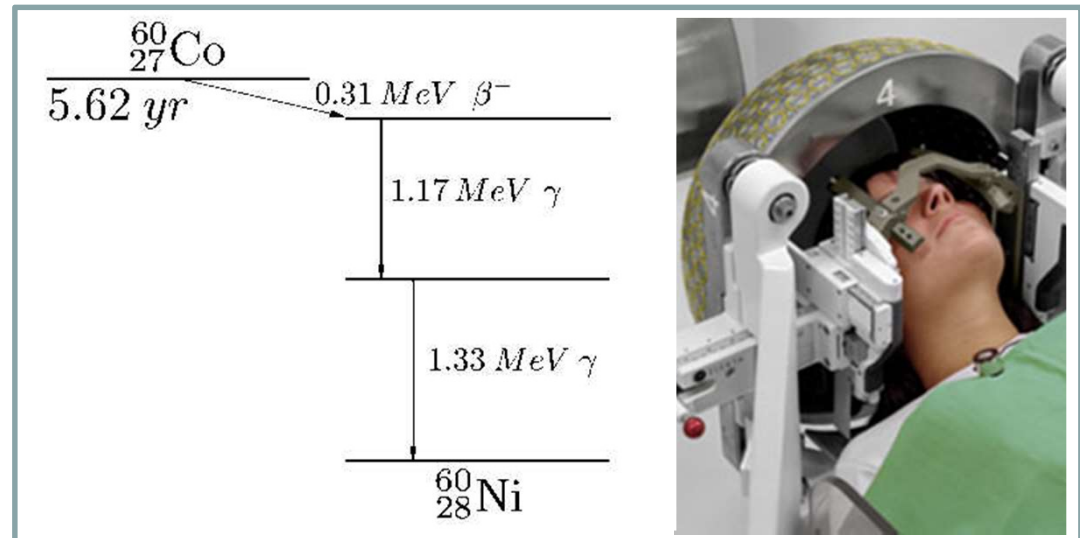
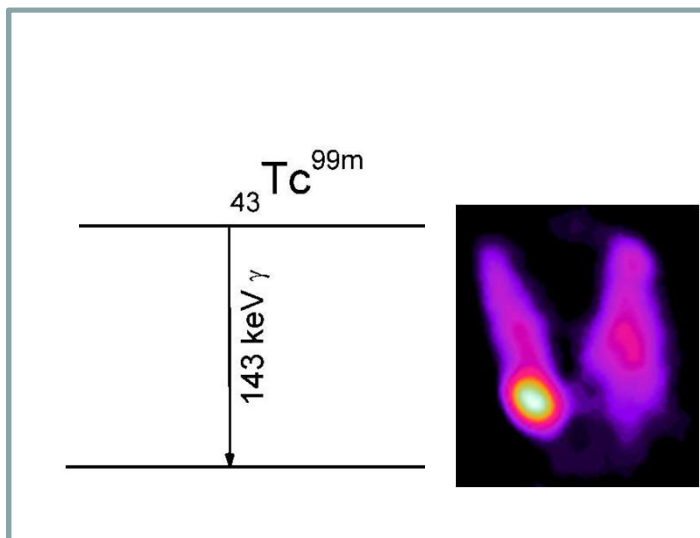
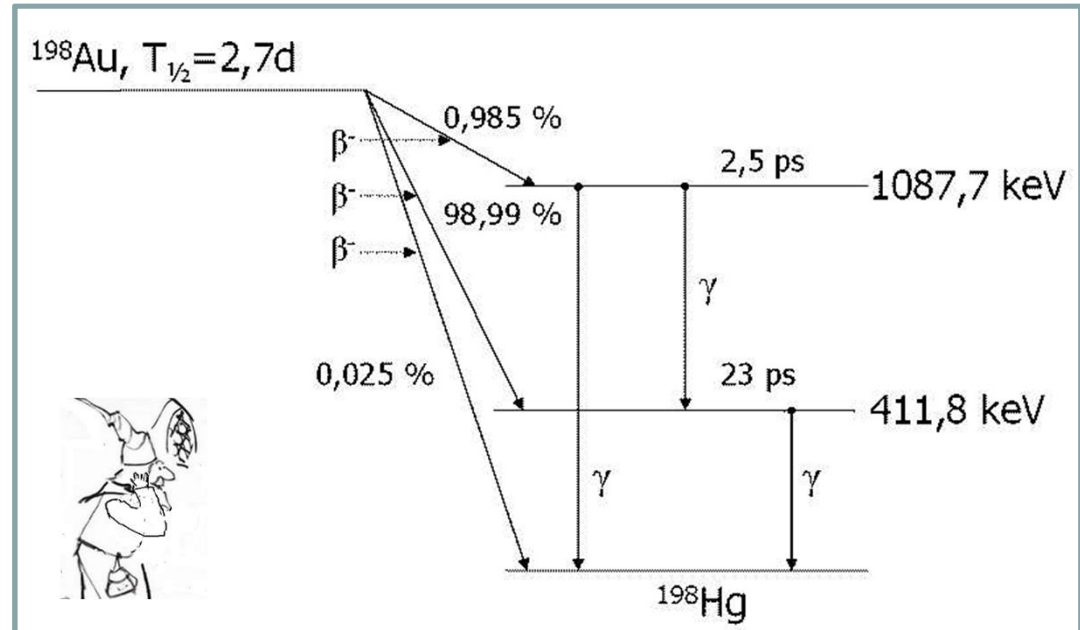
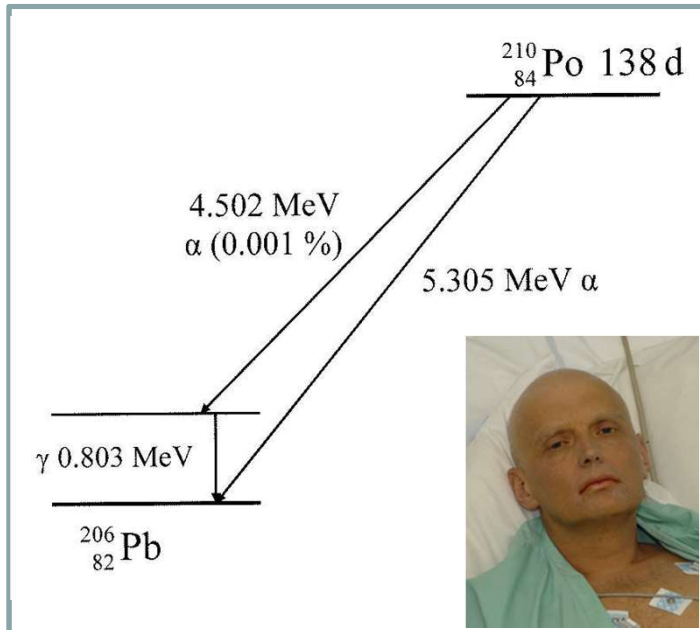
# K-capture (inverse $\beta$ -decay)



# Some examples of the decay paths



# Some examples of the decay paths



# How to produce radioactive isotopes?

$\beta^-$  decaying:

n surplus  $\rightarrow$  irradiate by neutrons  
(in a nuclear reactor)

$\beta^+$  decaying:

p surplus  $\rightarrow$  irradiate by protons

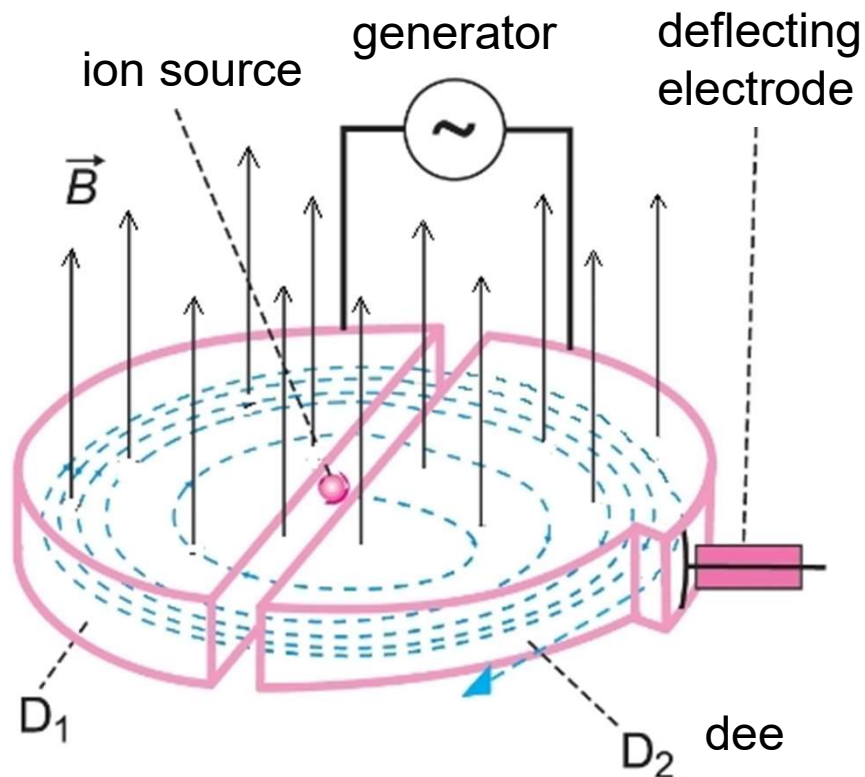
Coulomb repulsion  $\rightarrow$  you need accelerated protons!

$\rightarrow$  cyclotron

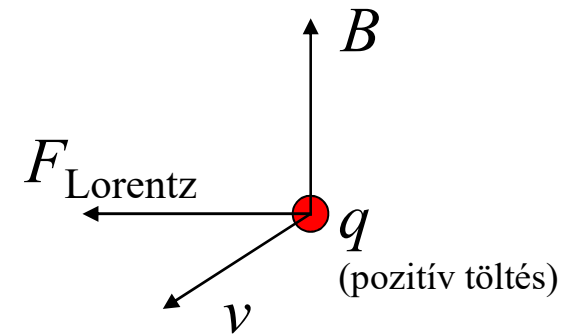
# Cyclotron

Protons or alpha particles are accelerated on a spiral path.

Typical energy: few 10 MeV (max 50 MeV)



$$\vec{F}_{\text{Lorentz}} = q\vec{v} \times \vec{B}$$



$$qBv = F_{\text{Lorentz}} = F_{\text{cp}} = mv^2/R$$

# Characteristics of radioactive decays in general

activity                      characterizes the source

decay type                      characterizes the source\* (see above)

half life time                      characterizes the speed of the decay\*

particle energy                      characterizes the radiation\*

\*depends on the type of the isotope

# Activity ( $\Lambda$ )

$$\Lambda = \left| \frac{dN}{dt} \right| \quad \left( = \left| \frac{\Delta N}{\Delta t} \right| \right)$$

$N$  = Number of undecayed atoms

$t$  = time

$\Delta N$  = Number of decays during  $\Delta t$  time

Activity = number of decays in a unit time

unit: becquerel Bq

1 Bq = 1 decay/sec

immeasurably  
small

level of natural activity

kBq, MBq, GBq, TBq

in vivo  
diagn.

careful  
work with  
it!

activity applied  
in radiotherapy

# Law of radioactive decay

$$\Delta N = -\lambda N \Delta t$$

N: Number of undecayed nuclei

$$\frac{dN}{dt} = -\lambda N$$

$\lambda$ : decay constant (probability of the decay [1/s])  
 $1/\lambda = \tau$  average lifetime

Differential equation

solution:

$$N(t) = N_0 e^{-\lambda t}$$

Exponential decrease

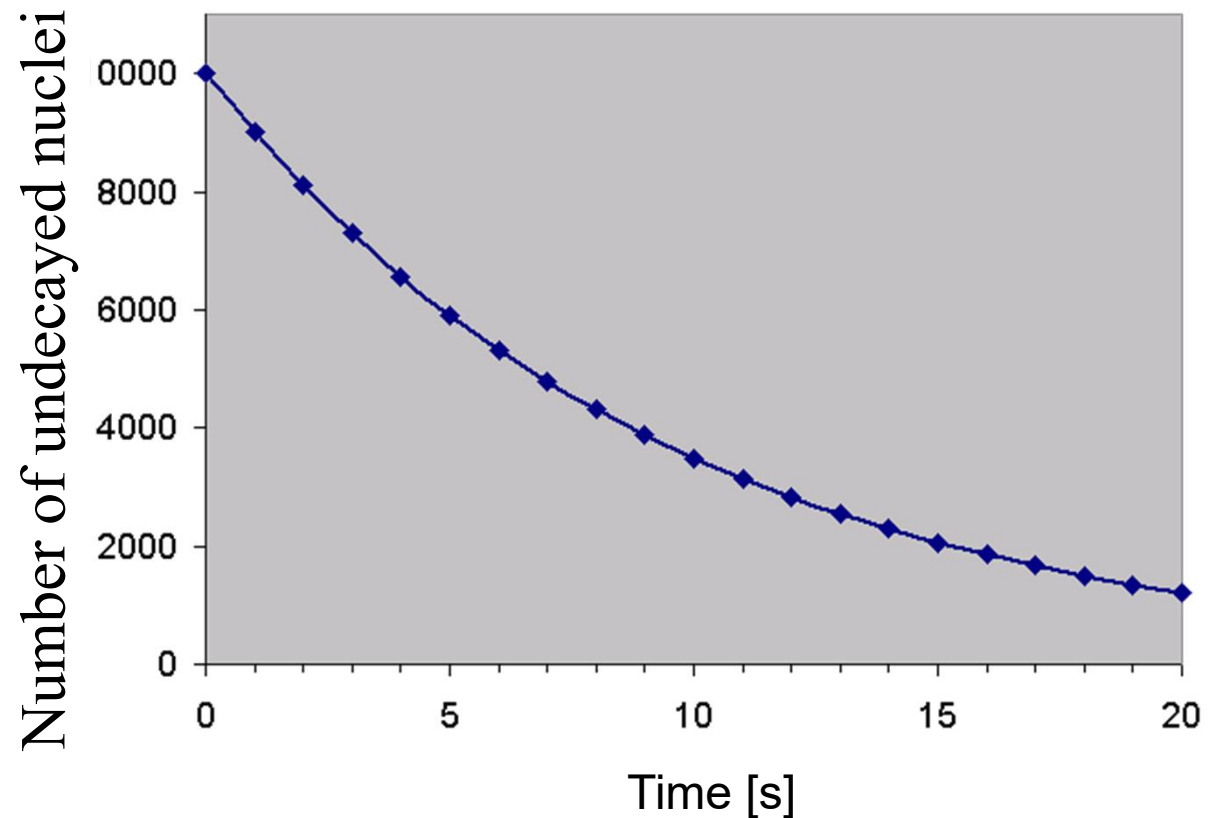
number of undecayed atoms at  $t=0$

# An example

- $N_0=10000$      $\lambda=0.1 \text{ } ^1/\text{s}$
- After 1 sec : 9000    ( $10000 \times 0.1 = 1000$  decayed)
- After 2 sec : 8100    ( $9000 \times 0.1 = 900$  decayed)
- After 3 sec : 7290    ( $8100 \times 0.1 = 810$  decayed)
- After 4 sec : 6561    ( $7290 \times 0.1 = 729$  decayed)
- ....

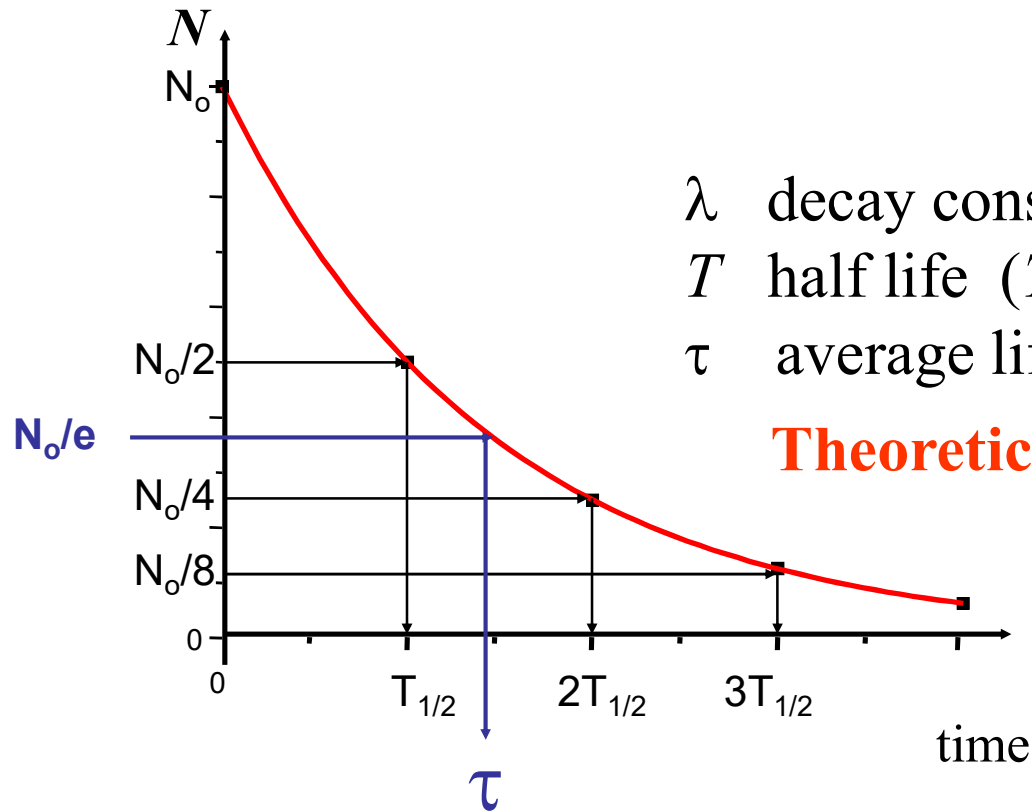
# An example

- $N_0=10000$        $\lambda=0,1 \text{ } ^1/\text{s}$
- 1 sec 9000
- 2 sec 8100
- 3 sec 7290
- 4 sec 6561
- ....



# Law of radioactive decay

$$N(t) = N_0 e^{-\lambda t} = N_0 2^{-\frac{t}{T}}$$



$\lambda$  decay constant (probability of the decay)

$T$  half life ( $T_{1/2}$ )

$\tau$  average lifetime

**Theoretically never decreases to zero !**

$$\lambda = \frac{\ln 2}{T} = \frac{0,693}{T}$$

# Decrease of the activity as a function of time

$$A = \left| \frac{dN}{dt} \right|$$

$$\frac{dN}{dt} = -\lambda N$$

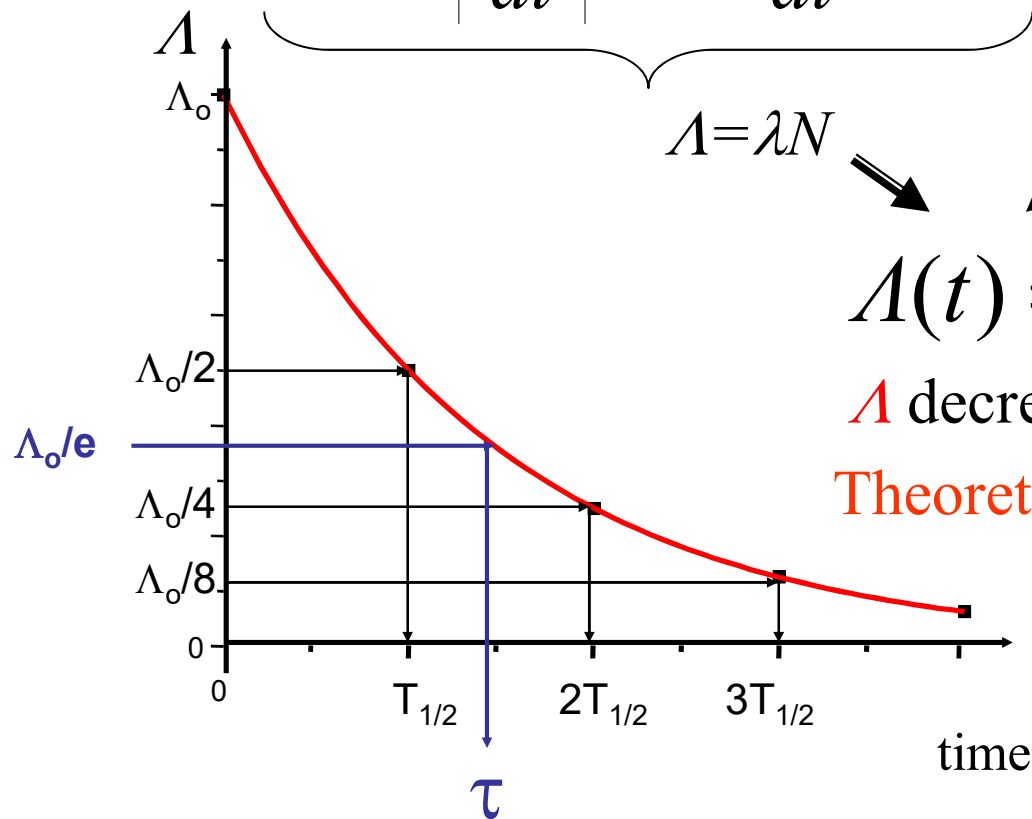
$$N(t) = N_0 e^{-\lambda t}$$

$$A = \lambda N$$

$$A(t) = A_0 e^{-\lambda t} = A_0 2^{-\frac{t}{T}}$$

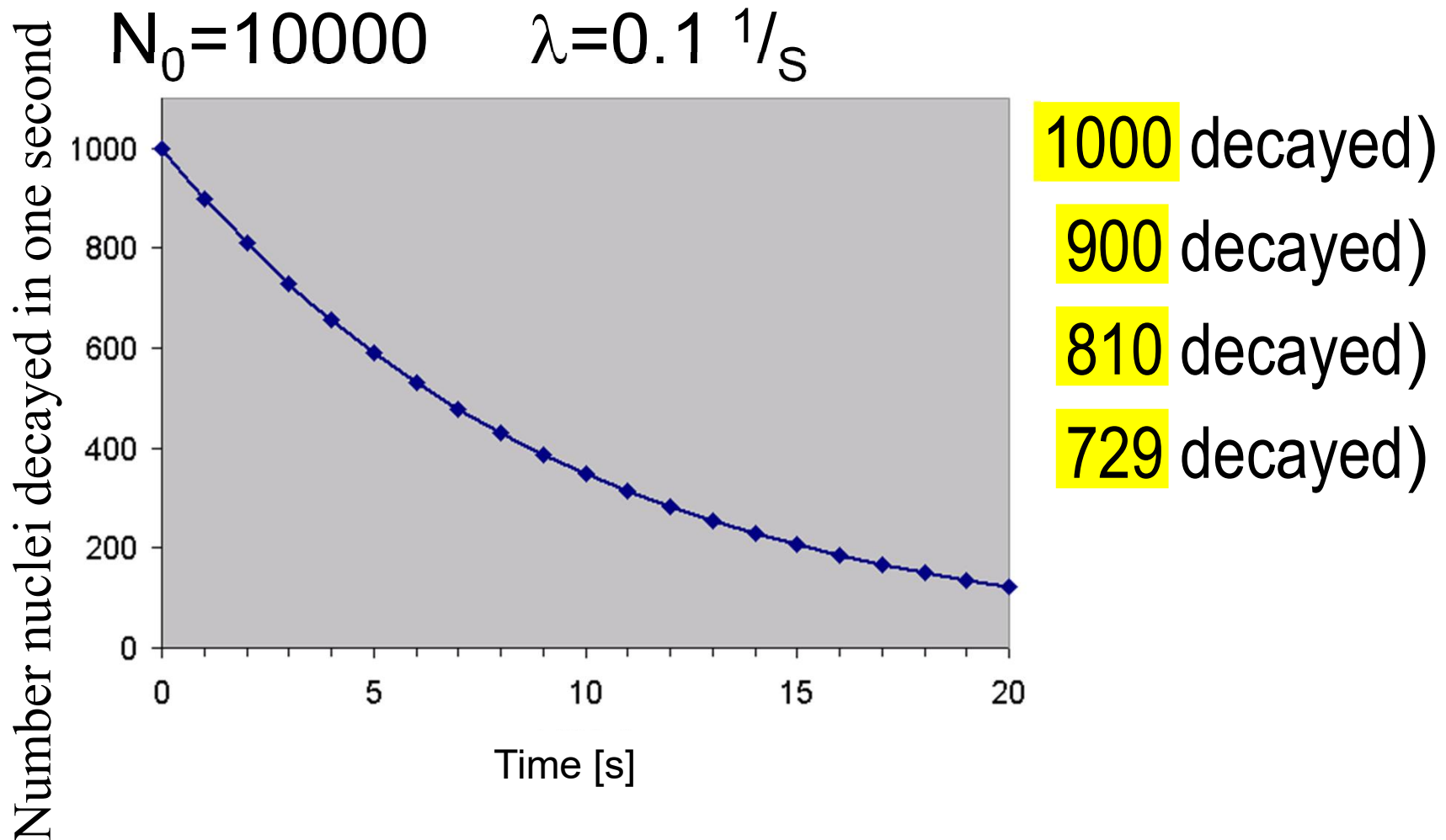
$A$  decreases on the same mode as  $N$ !

Theoretically never decreases to zero!



During about 10  $T$  the activity decreases to its 1/1000 (e.g GBq  $\rightarrow$  MBq)

# Example



# Few examples for half life

$^{232}\text{Th}$	$1,4 \cdot 10^{10} \text{ y}$
-------------------	-------------------------------

$^{238}\text{U}$	$4,5 \cdot 10^9 \text{ y}$
------------------	----------------------------

$^{40}\text{K}$	$1,3 \cdot 10^9 \text{ y}$
-----------------	----------------------------

$^{14}\text{C}$	$5736 \text{ y}$
-----------------	------------------

$^{137}\text{Cs}$	$30 \text{ y}$
-------------------	----------------

$^3\text{H}$	$12,3 \text{ y}$
--------------	------------------

$^{60}\text{Co}$	$5,3 \text{ y}$
------------------	-----------------

$^{59}\text{Fe}$	$1,5 \text{ m}$
------------------	-----------------

$^{56}\text{Cr}$	$1 \text{ m (28 d)}$
------------------	----------------------

$^{131}\text{I}$	$8 \text{ d}$
------------------	---------------

$^{99\text{m}}\text{Tc}$	$6 \text{ h}$
--------------------------	---------------

$^{18}\text{F}$	$110 \text{ min}$
-----------------	-------------------

$^{11}\text{C}$	$20 \text{ min}$
-----------------	------------------

$^{15}\text{O}$	$2 \text{ min}$
-----------------	-----------------

$^{222}\text{Th}$	$2,8 \text{ ms}$
-------------------	------------------

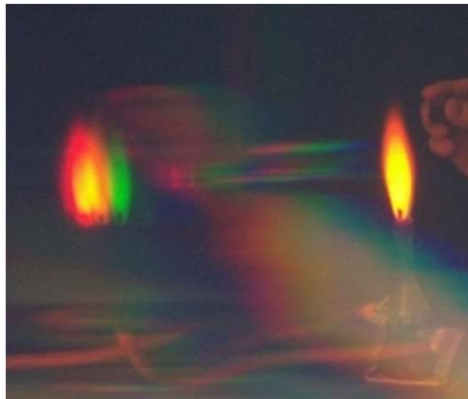
**Don't learn these numbers!**

# Typical energy levels in the microworld

Excitation of the  
outer electrons

eV (aJ)

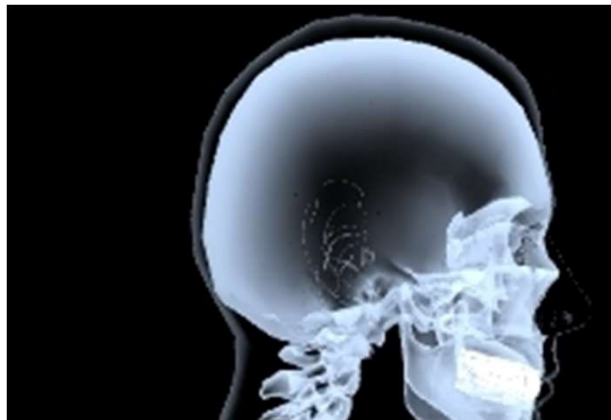
light



Electron transition  
between inner  
electrons

keV (fJ)

X-ray

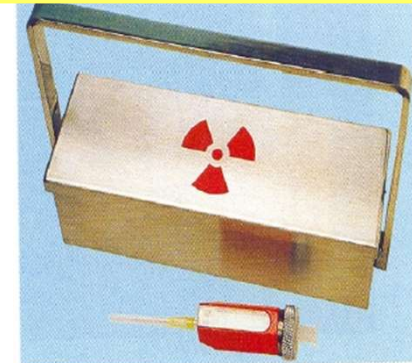


Transformation  
of the nucleus  
(decay)

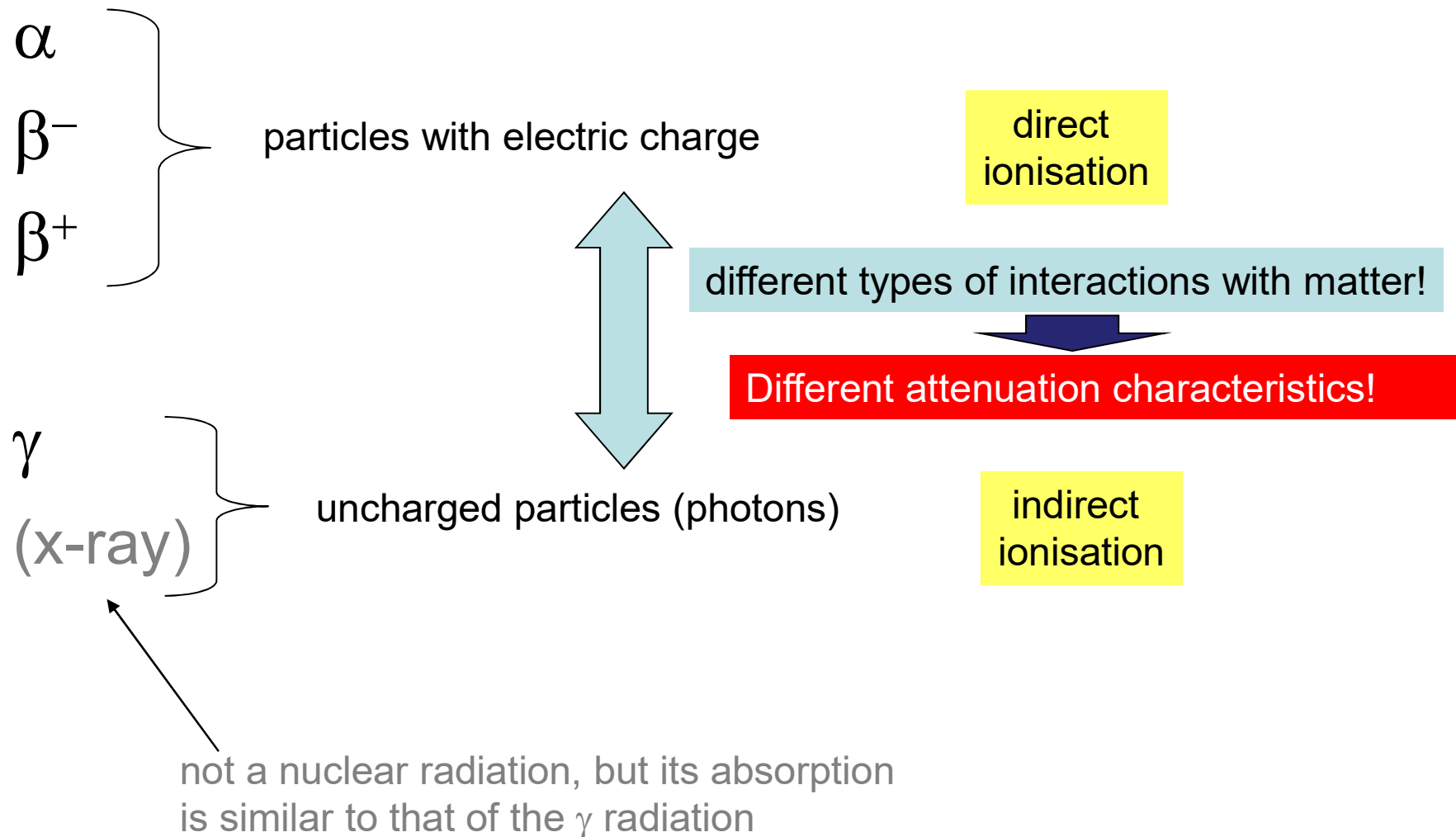
MeV (pJ)

Nuclear  
radiation

$\alpha$ ,  $\beta$ ,  $\gamma$



# Absorption of the nuclear radiation



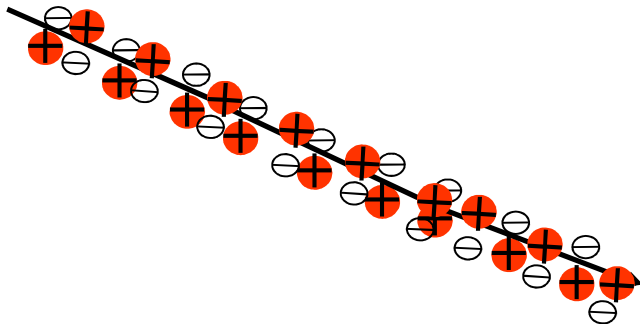
# Absorption of the charged particles

Ionizing during the path => continuous decrease of the particle energy  
The energy after a given path length decreases to the thermal value

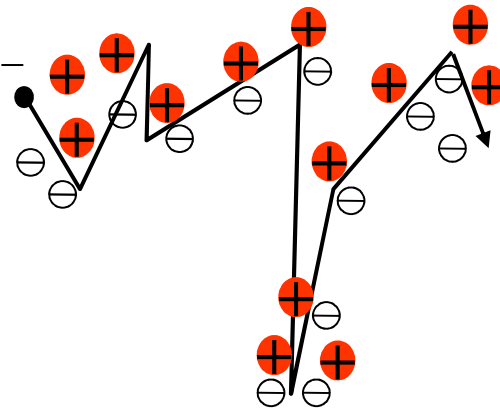


effective range

$\alpha$



$\beta^-$



# Effective range

$\alpha$ -particle

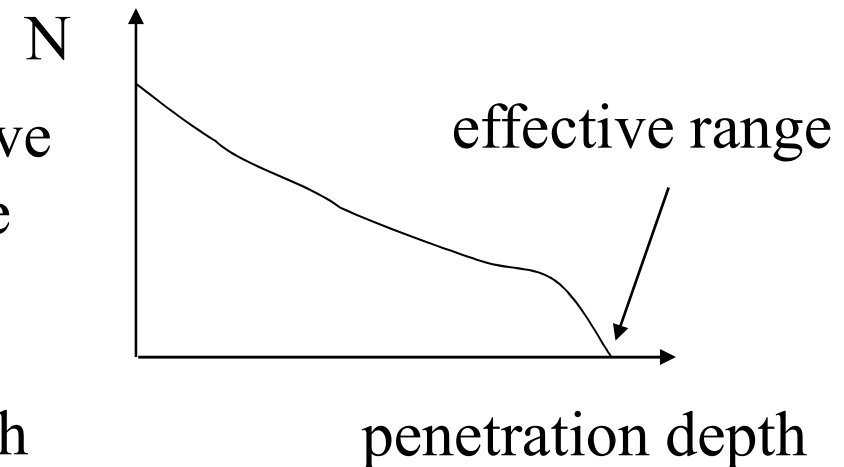
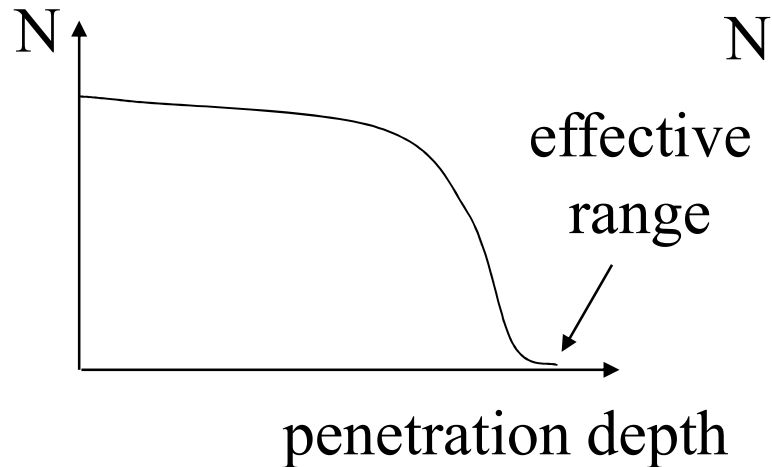
$\beta^-$ -particle

in air      **few cm**

**$\sim$  m**

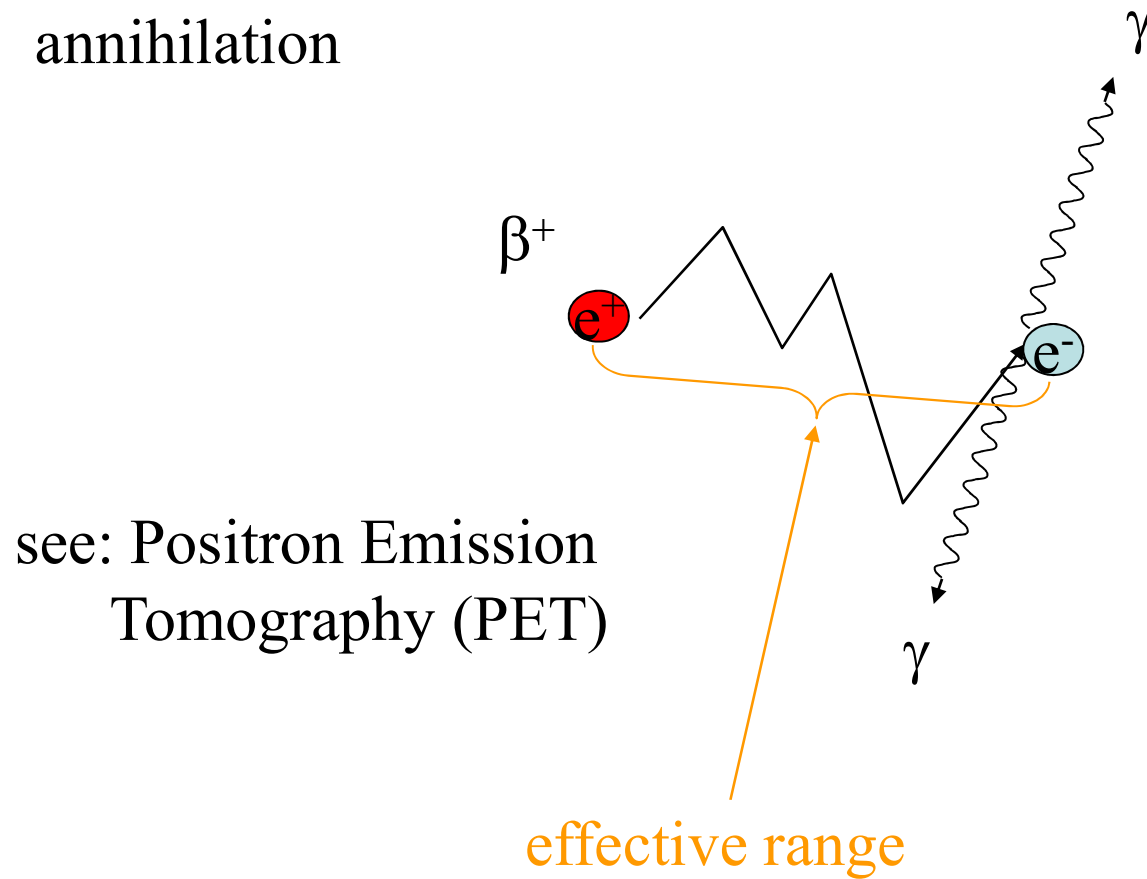
in tissue    **0,01-0,1 mm**

**$\sim$  cm**



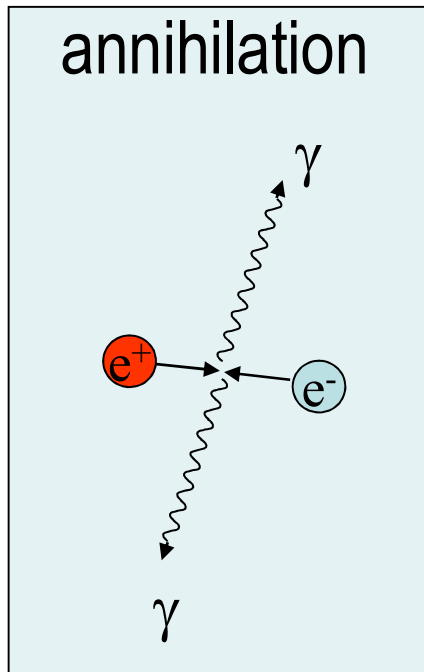
# $\beta^+$ -radiation

annihilation

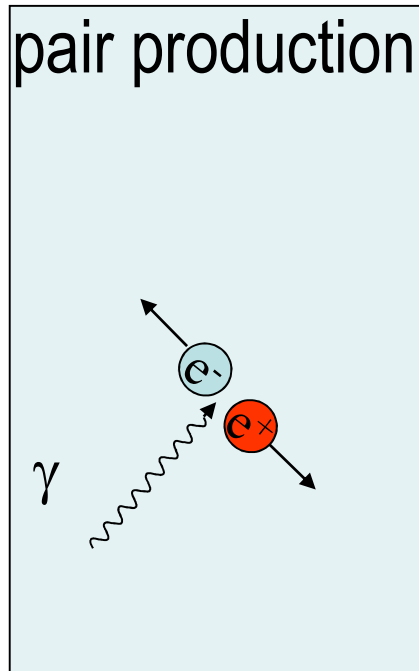


# Electron and positron

- particle - antiparticle
- same mass,
- charge: same value, but different



and pair production



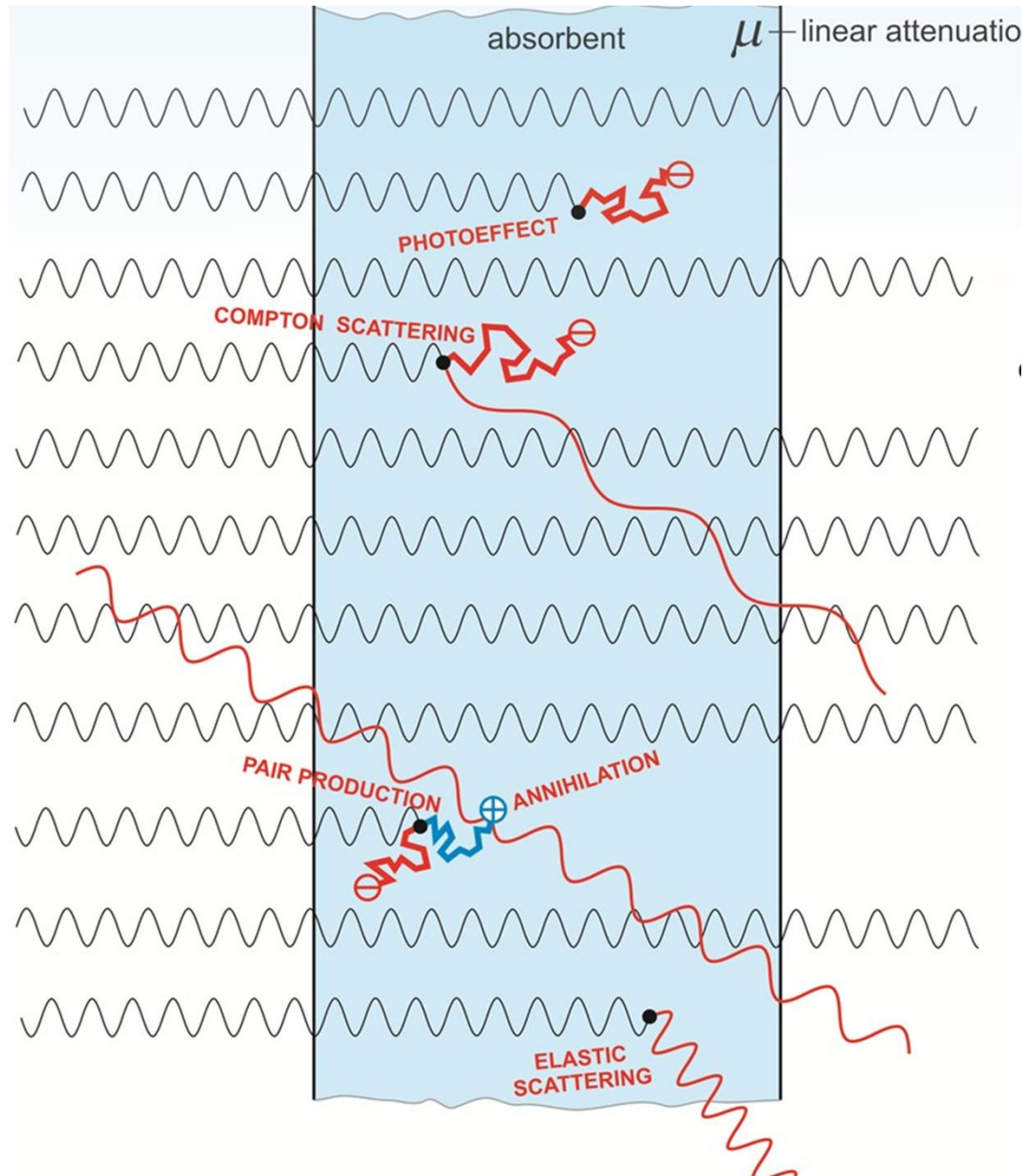
Einstein:

mass-energy  
ekvivalence

$$E=mc^2$$

$$m_e c^2 = 511 \text{ keV} \approx 0,5 \text{ MeV}$$

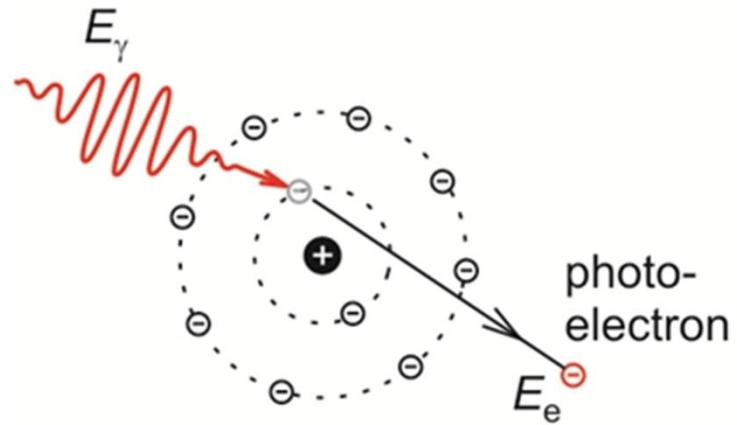
# Absorption of the $\gamma$ -radiation (and x-ray)



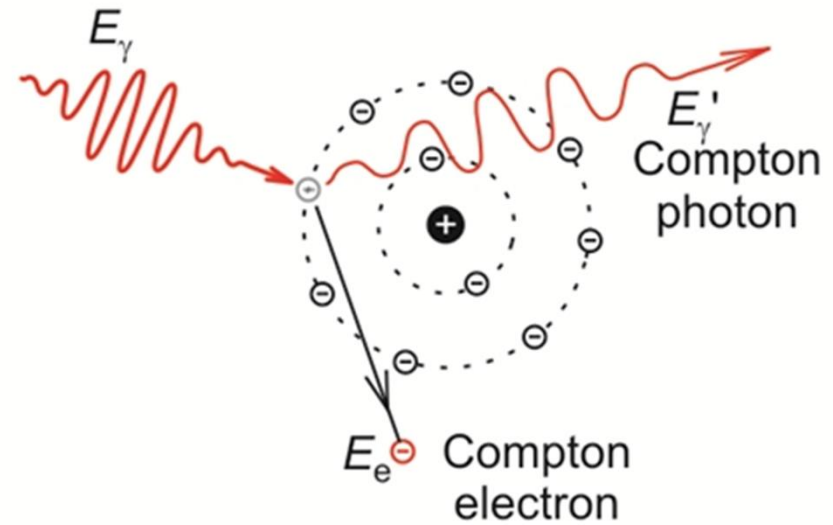
Absorption processes  
happen accidentally :

Photoeffect,  
Compton-effect,  
Pair production,  
(elastic scattering)

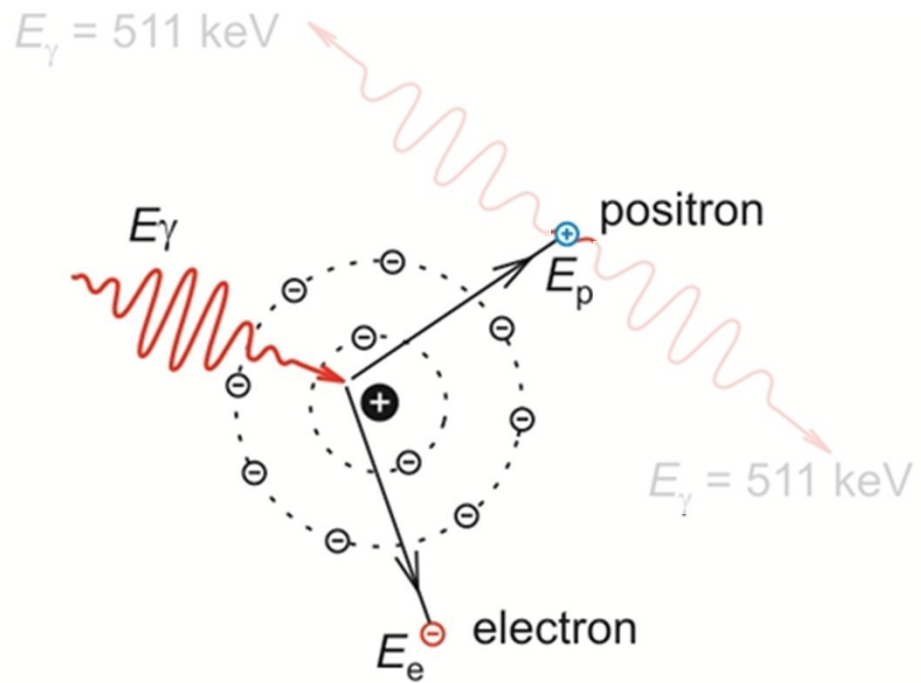
## Photoeffect



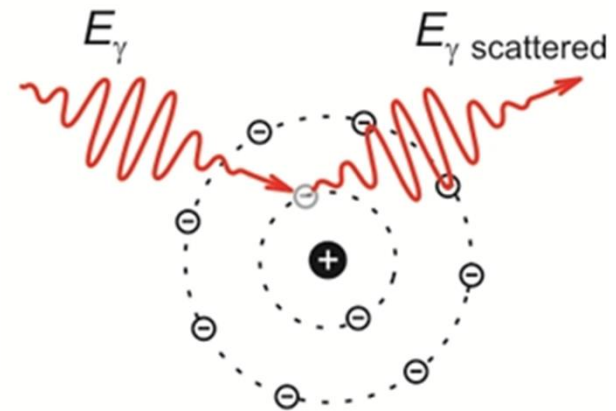
## Compton effect Compton-scattering



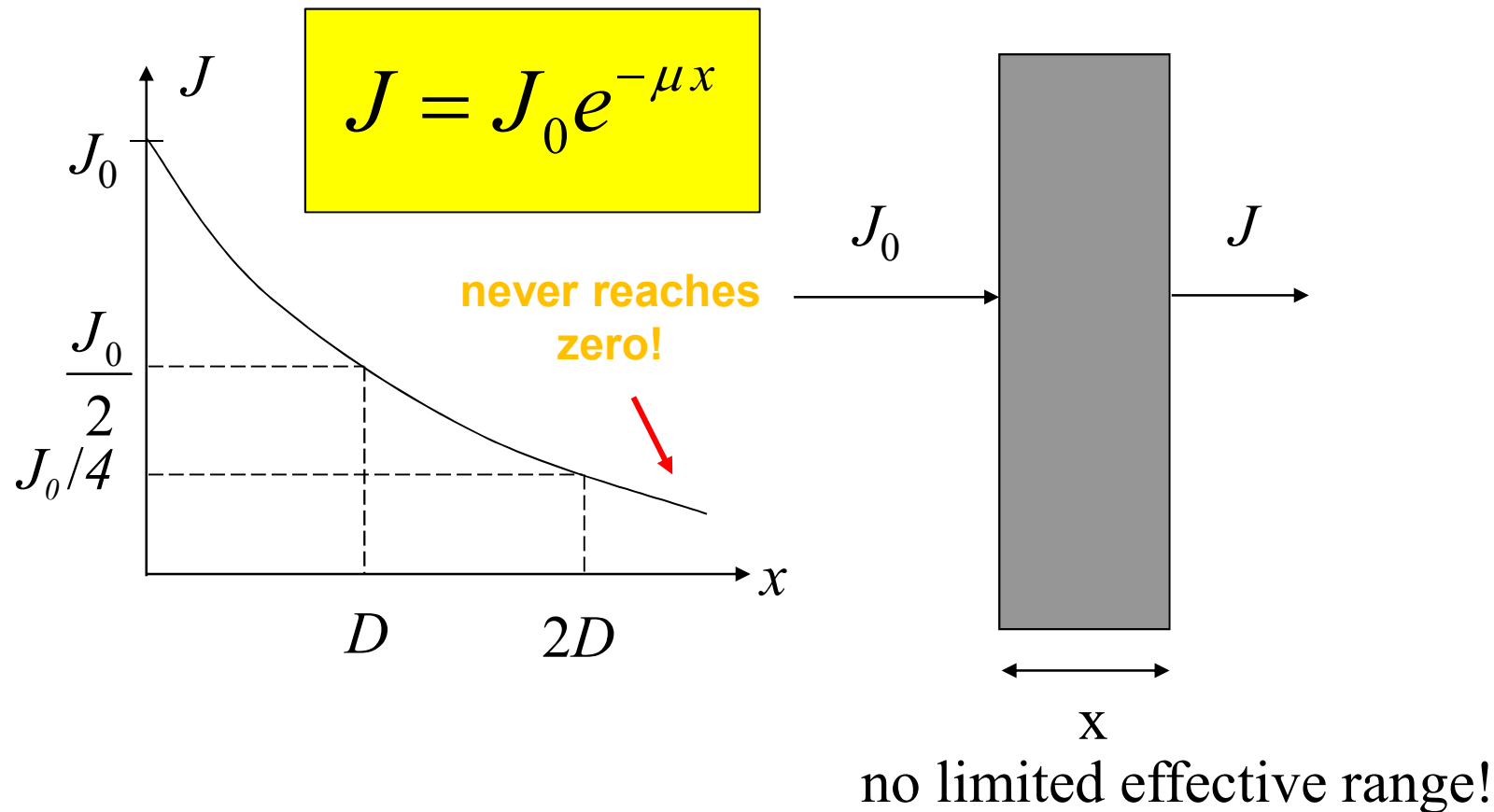
## Pair production



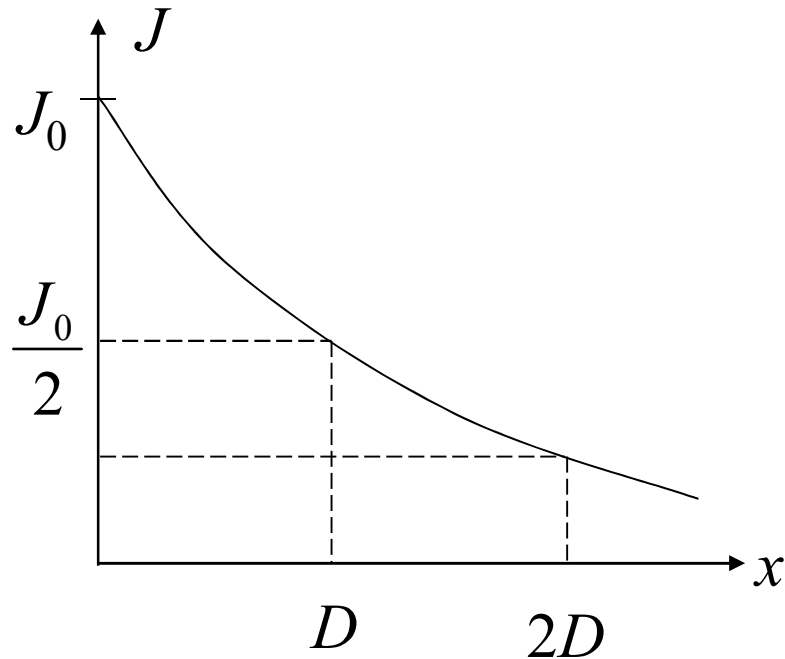
## elastic scattering



# Attenuation of the $\gamma$ -radiation and x-ray



few rules of thumb:  $x_{1/10} = 3,33 D$        $x_{1/1000} = 10 D$



$$J = J_0 e^{-\mu x}$$

$\mu$ : (linear) attenuation coefficient

its units are: 1/m, 1/cm

$$\delta = \frac{1}{\mu} \quad \text{„penetration depth”}$$

Intensity decreases to the e-th part (c.a. 37%)

$\mu$ (material, number of absorbing centers, energy of the radiation)

$$= \mu(\text{material}, \rho, E_{\text{photon}}) \sim \rho$$

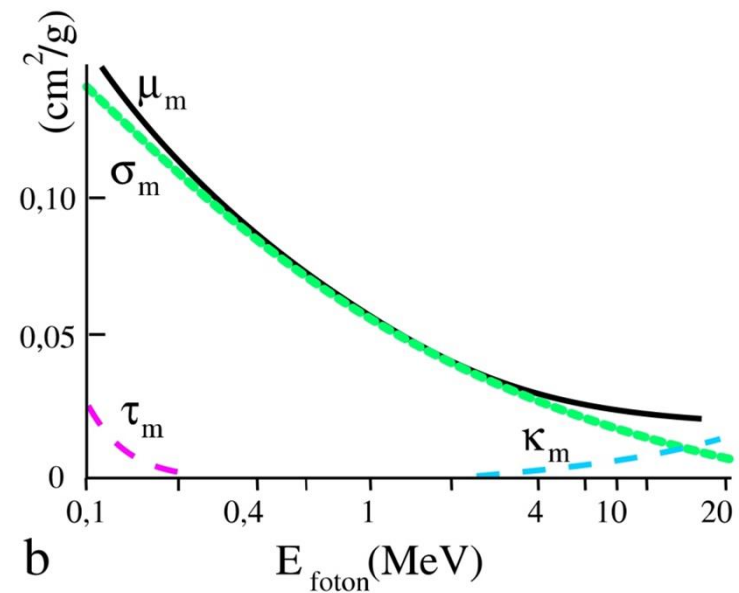
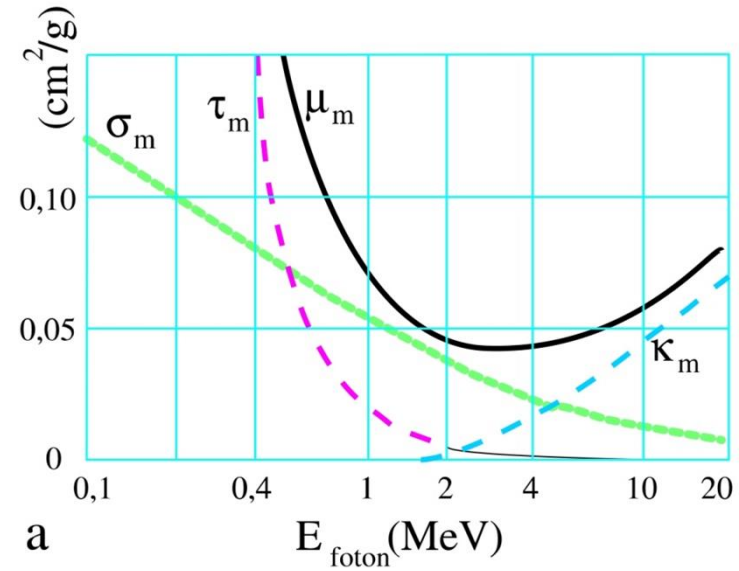
$$\mu_m = \frac{\mu}{\rho} \quad \text{mass attenuation coefficient}$$

mass attenuation coeff.

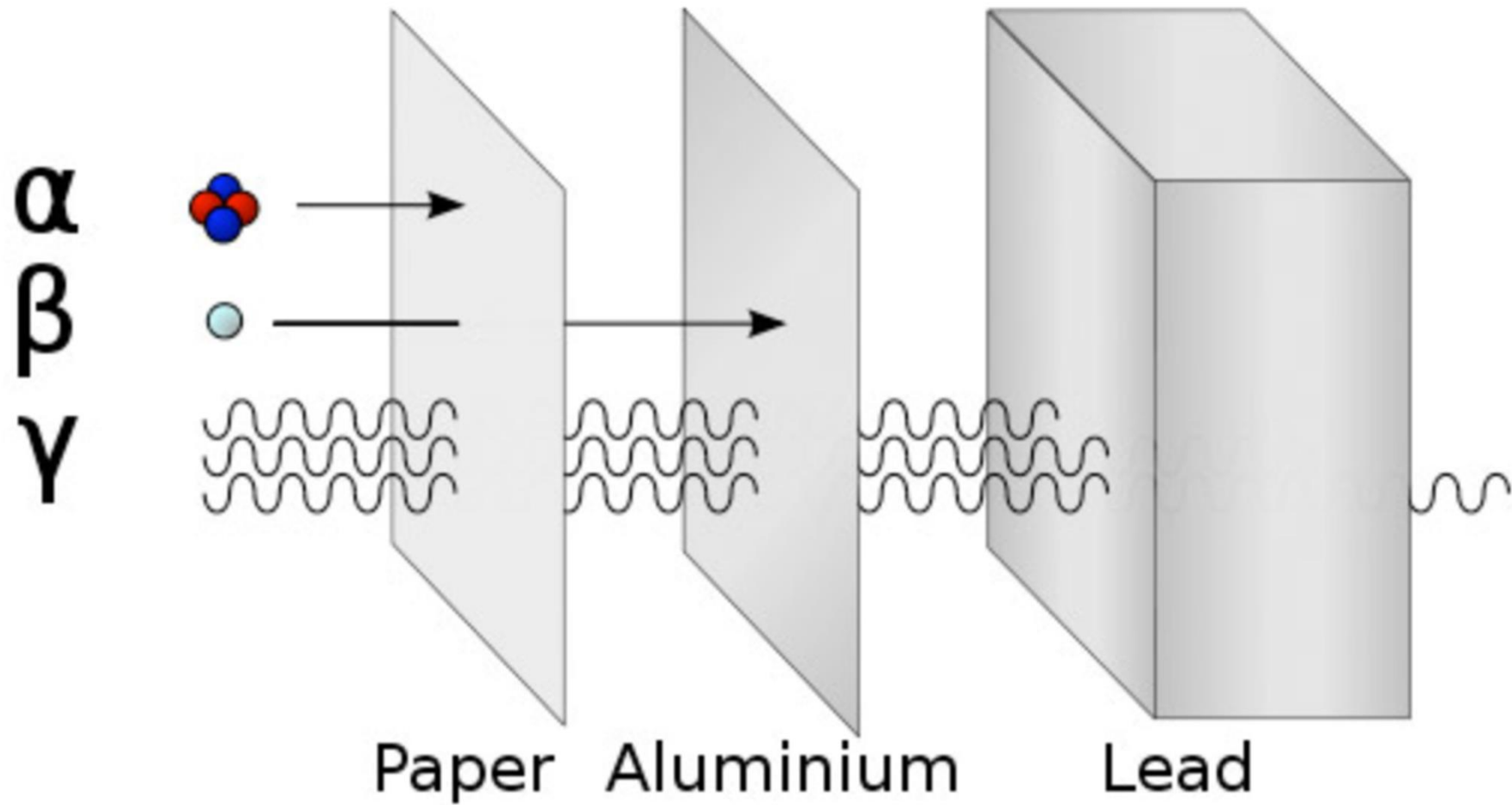
$$\mu_m = \frac{\mu}{\rho}$$

$$\mu_m = \tau_m + \sigma_m + \kappa_m$$

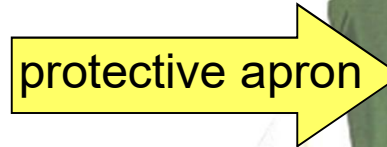
$$\tau_m = c\lambda^3 Z^3$$



# Summary of the absorption of $\alpha$ , $\beta$ and $\gamma$ radiation



# Applications (attenuation)



# Applications: isotopes and nuclear radiation

