

# Modelling of ligand-protein binding

## I. Computation of thermodynamic quantities

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## Outline

- Molecular dynamics
- Sampling in molecular dynamics
- Calculation of thermodynamic quantities along the pathway
  - Techniques for calculating free energy:
    - Thermodynamic integration
    - Free energy perturbation
    - Potential of mean force
    - Non-equilibrium work
  - Enthalpy and entropy
  - Examples



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## Molecular dynamics

2015.10.08

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## Introduction

- Molecular dynamics – link between microscopic and macroscopic quantities
  - structure
  - dynamics
  - thermodynamics



Frill\_10\_pse/MD/mpg

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## History

- Alder, B. J. and Wainwright, T. E.  
*J. Chem. Phys.* **27**, 1208 (1957)



- McCammon, J. A., Gelin, B. R., and Karplus, M.  
*Nature (Lond.)* **267**, 585 (1977)



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## Basic terms

- Thermodynamic/Macroscopic state
  - The system is characterized by few macroscopic parameters; e.g.: T, P, N
- Microscopic state
  - The system is characterized by the positions and momenta of atoms (phase space).
- Ensembles
  - Microscopic states corresponding to a macroscopic state
- Molecular dynamics simulations
  - Generation of microscopic states of an ensemble as a function of time

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## Thermodynamic ensembles

- Microcanonical – NVE (isolated system)
- Canonical – NVT (thermal equilibrium)
- Isotherm-izobar – NPT
- Grand canonical –  $\mu$ VT (equilibrium with a reservoir of particles)

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## Ergodic hypothesis

- Measurable quantities: ensemble average  $\langle A \rangle_{ensemble}$   
– e.g. (non-covalent) binding of two molecules in solution
- Molecular dynamics: time average  $\langle A \rangle_{time}$

$$\langle A \rangle_{time} = \langle A \rangle_{ensemble}$$

- „long enough” MD – appropriate sampling

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## Force field

- Molecular mechanics
  - basic entities: atoms
  - „classical”
  - simple, fast computations
  - includes parameters
    - Can be applied within the validity of the parameter space
    - Chemical reactions are typically outside the validity
- Quantum mechanics
  - basic entities: nuclei and electrons
  - accurate
  - time intensive computations

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## Molecular mechanics

Atoms are pointlike objects with mass and interactions

$$E = E_{str} + E_{bend} + E_{tors} + E_{vdw} + E_{el} + E_{cross}$$

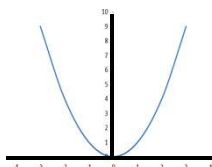
intramolecular
intermolecular

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## Bond stretching energy

$$E_{str} = k(r - r_0)^2$$

$$F_{str} = -2k(r - r_0)$$



good approximation in the vicinity of  $r_0$   
 $k$  and  $r_0$  are atom dependent parameters

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## Bond stretching energy - parameters

$$E_{str} = k(r - r_0)^2$$



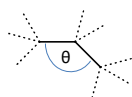
general class	atom type	description
hydrogen type		
H		aromatic or aliphatic hydrogen
HC		explicit hydrogen attached to carbon
HO		hydrogen on hydroxy oxygen
HS		hydrogen attached to sulfur
HW		hydrogen in water
H2		aromatic hydrogen in H <sub>2</sub>
H3		hydrogen of lysine or arginine (positively charged)
all-atom carbon [typeC]		
C	sp <sup>2</sup>	aromatic carbon and aromatic carbon with hydroxy substituent in tyrosine
CA	sp <sup>2</sup>	aromatic carbon in 6-membered ring with 1 substituent
CB	sp <sup>2</sup>	aromatic carbon at junction between 5- and 6-membered rings
CC	sp <sup>2</sup>	aromatic carbon in 5-membered ring with 1 substituent and next to a nitrogen
CK	sp <sup>2</sup>	aromatic carbon in 5-membered ring between 2 nitrogens and bonded to 1 hydrogen (in purine)
CM	sp <sup>2</sup>	cyclic and C=O bond one substituent
CN	sp <sup>2</sup>	aromatic carbon in 5- and 6-membered rings
CQ	sp <sup>2</sup>	carbon in 6-membered ring of purine between 2 NC nitrogens and bonded to 1 hydrogen
CR	sp <sup>2</sup>	aromatic carbon in 5-membered ring between 2 nitrogens and bonded to 1 H (in His)
CT	sp <sup>2</sup>	carbon with 4 explicit substituents
CV	sp <sup>2</sup>	aromatic carbon in 5-membered ring bonded to 1 H and bonded to an explicit hydrogen
CW	sp <sup>2</sup>	aromatic carbon in 5-membered ring bonded to 1 H and bonded to an explicit hydrogen
CC	sp <sup>2</sup>	aromatic carbon in 5-membered ring with 1 substituent

Bond Stretching Potential Parameters			
Bond	used for	$k_e$ kcal mol <sup>-1</sup> Å <sup>-2</sup>	$R_{eq}$ Å
CT-CT	BMMF_EMT	310.0	1.526
CT-H1	BMMF_EMT	340.0	1.090
CT-HC	BMMF_EMT	340.0	1.090
CT-HA	BMMF_EMT	317.0	1.475
CB-H3	BMMF_EMT	367.0	1.080
CB-NA	BMMF_EMT	477.0	1.343
CW-H4	BMMF_EMT	367.0	1.080
CW-NA	BMMF_EMT	427.0	1.391
CW-CW	BMMF_EMT	549.0	1.350
AL-CL	FF14	116.1	2.170
P-F	FF14	260.3	1.646
NN-ON	ND1	300.0	1.260

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## Bending energy

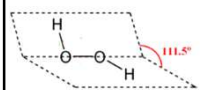
$$E_{bend} = k(\theta - \theta_0)^2$$



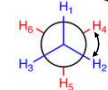
$k$  and  $\theta_0$  are atom dependent parameters

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## Torsional/dihedral angle energy



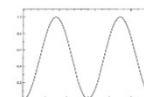
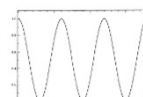
$$E_{tors} = \frac{V_n}{2} [1 + \cos(n\phi - \phi_0)]$$



$V_n$  – barrier height  
 $n$  – periodicity

Dihedral angle (atom types)	$V_n$ , kcal/mol	$n$	$\phi_0$	comments
C CT1 NH1 C	0.2000	1	180.00	backbone phi
NH1 C CT1 NH1	0.6000	1	0.00	backbone ksi
CT1 C NH1 CT1	1.6000	1	0.00	backbone omega
CA CA CA CA	3.1000	2	180.00	Phe side chain
H OH1 CT2 CT1	0.4200	3	0.00	Ser side chain

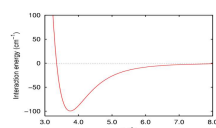
~15000 parameters



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## van der Waals energy



short range: repulsive;  $\exp(-r)$  or  $r^{-12}$  Pauli repulsion  
middle range: attractive;  $r^{-6}$  dispersion  
long range: disappears

$$E_{vdw} = 4\epsilon \left[ \left( \frac{\sigma}{r} \right)^{12} - \left( \frac{\sigma}{r} \right)^6 \right]$$

$$\sigma_{ij} = \frac{1}{2}(\sigma_i + \sigma_j)$$

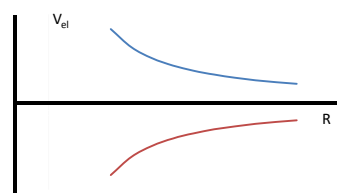
$$\epsilon_{ij} = \sqrt{\epsilon_i \epsilon_j}$$

$$c * \exp\left(\frac{-r}{\sigma}\right)$$

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## Electrostatic energy

$$V_{el} = \frac{q_i q_j}{\epsilon r_{ij}} \quad \text{Coulomb}$$



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## MM parameters

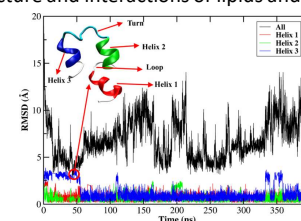
- Derivation
  - Quantum mechanical calculations
  - Experimental data
  - Extension based on analogy
- Validation by comparing computed and experimental data
  - Macromolecular structure
  - NMR data
  - Structure and energy of van der Waals complexes
- Error compensation; mutual interdependence of parameters

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## Quality of MM force field

- Protein structure
- DNA, RNA structure
- Conformation of organic molecules
- Ligand-protein interactions
- Structure and interactions of lipids and membranes
- ...



RMSDs of backbone atoms  
from the native structure  
as a function of MD  
simulation time

Duan J Mol Model 2014,20,2195

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## Selected MM force fields

- Charmm (Chemistry at HARvard Macromolecular Mechanics)
- AMBER (Assisted Model Building with Energy Refinement)
- OPLS (Optimized Potentials for Liquid Simulations)
- GROMOS (GROningen Molecular Simulation)
- MMFF (Merck Molecular Force Field)

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## MD algorithm

Newtonian mechanics

$$r^N(r_1, r_2 \dots r_N) \quad p^N(p_1, p_2 \dots p_N)$$

$$U(\underline{r}) \quad K(\underline{p}) = \sum_i \frac{|p_i|^2}{2m_i}$$

$$H = K + U \quad \dot{r}_i = \frac{p_i}{m_i} \quad \dot{p}_i = f_i$$

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## Verlet algorithm

Calculation of  $p_i$  and  $r_i$  at  $\delta t$  time steps

$$p_i(t), r_i(t) \rightarrow p_i(t + \delta t), r_i(t + \delta t) \rightarrow f_i(t)$$

$$p_i\left(t + \frac{1}{2}\delta t\right) = p_i(t) + \frac{1}{2}\delta t \cdot f_i(t)$$

$$r_i(t + \delta t) = r_i(t) + \frac{\delta t \cdot p_i\left(t + \frac{1}{2}\delta t\right)}{m_i} \rightarrow f_i(t + \delta t)$$

$$p_i(t + \delta t) = p_i\left(t + \frac{1}{2}\delta t\right) + \frac{1}{2}\delta t \cdot f_i(t + \delta t)$$

Typical  $\delta t$  for simulation of biochemical systems: 1-4 fs

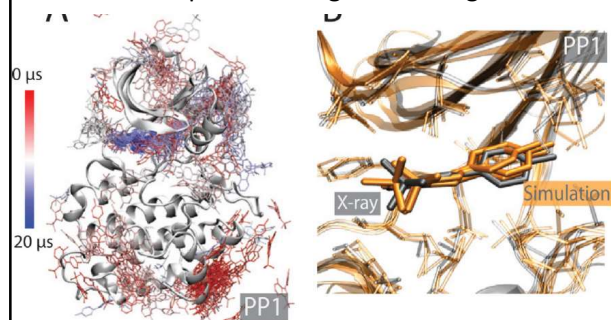
1  $\mu$ s MD  $\sim 10^6$  evaluations

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## Selected applications

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### process of ligand binding

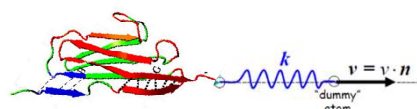


PP1 molecule finds the binding site of Src kinase in a 15 $\mu$ s simulation  
JACS 2011 133 9181

Application 1

### Steered MD

- Constant velocity pulling



$$U = \frac{1}{2}k[v t - (\vec{r} - \vec{r}_0) \cdot \vec{n}]^2$$

- Constant force pulling

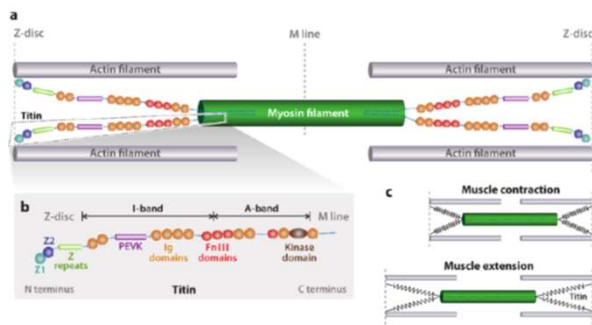
Application 2

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Application 2

## Titin structure and function



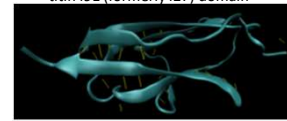
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Application 2

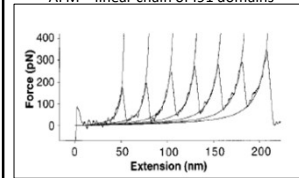
## Unfolding and force

titin I91 (formerly I27) domain



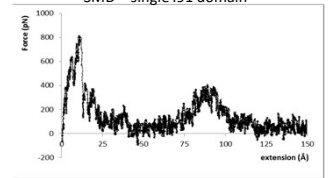
Constant velocity pulling

AFM – linear chain of I91 domains



Rief et al. (1997 Science 276 1109)

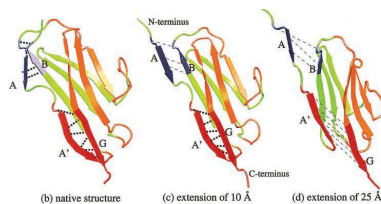
SMD – single I91 domain



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Application 2

## Unfolding and structure

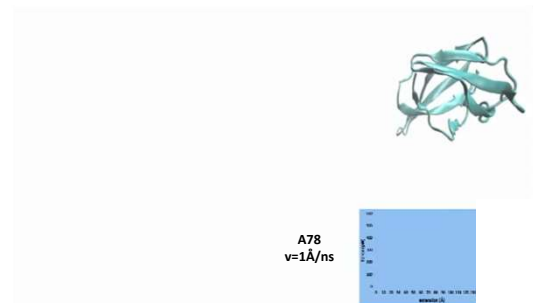


Lu, H., and Schulten, K. (2000). Biophys. J. 79, 51–65

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Application 2

## Titin Fn domain unfolding mechanism



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## MD scope and limitation

- Scope
  - Structural study; structure refinement
  - Dynamics
    - conformations, ligand-protein binding, steered processes,...
  - Thermodynamics see later
    - Free energy changes
      - solvation, ligand-protein binding,...
- Limitations
  - Sampling see later
  - Accuracy of force field
  - Chemical reactions cannot be routinely studied

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