

Dynamics

Motion, deformation and their background

One of the fundamental physical quantities of the topic is the **momentum** (p).

In classical case it is the product of the **mass** (m) and the **velocity** (v) of the body.

$$p = mv$$

vector quantity

Newton's laws of motion

II. For the **change of momentum** needs **force** (F).

$$\frac{\Delta mv}{\Delta t} = m \frac{\Delta v}{\Delta t} = ma = F$$

If no forces are exerted (or $F = 0$)

$\Delta mv = 0$, means **$p = mv = \text{constant}$** .

I. **Momentum is conserved** (momentum conservation)

law of inertia

III. $F = -F_{\text{versus}}$ interaction

A single force cannot exist.

Forces are always directed to contrary parts.

law of action and reaction

Application e.g.:

at the pressure of ideal gases, (see later)

at the explanation of annihilation (see in PET).

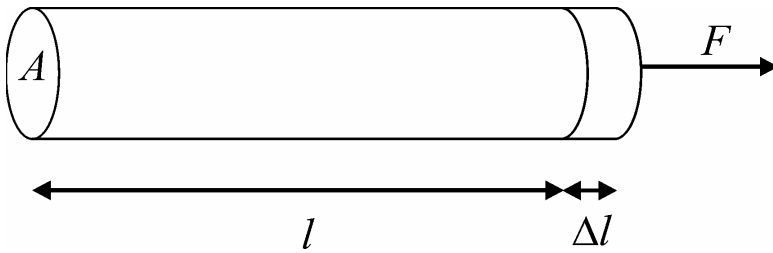
But

force may result in **deformation**.

Simplest deformation is the **elongation**.

tensile **strain**: $\Delta l/l$

Hooke's law



$$F = AE \frac{\Delta l}{l}$$

$$\frac{F}{A} = E \frac{\Delta l}{l}$$

F/A the **stress** (tensile stress), but

it could be compressive stress or **pressure** ($p[\text{Pa}]$)

Coefficient: **Young's modulus** ($E[\text{Pa}]$)

Similar to the case of spring: $F_{\text{spring}} = Dx$ (if $x \equiv \Delta l$, and $D \equiv AE/l$)

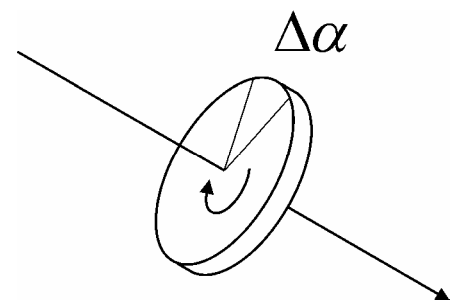
Newton's laws for rotation

similarly to the momentum ($m\mathbf{v}$) here the fundamental physical quantity is the **angular momentum** ($\Theta\omega$), where

Θ is **moment of inertia**, rotational analog of the mass,
 ω is **angular velocity**,

$$\omega = \frac{\Delta\alpha}{\Delta t} = \frac{2\pi}{T} = 2\pi f$$

period (T), **frequency** (f)
 (ω **angular frequency**)



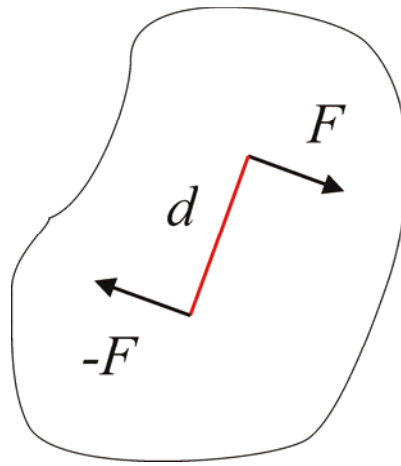
- I. $\Theta\omega = \text{constant}$ (conservation of angular momentum)
 (see: **rotating skater**)
- II. For the **change of angular momentum** needs **torque** (M).

$$\frac{\Delta\Theta\omega}{\Delta t} = M$$

Equilibrium, if

$F_{\text{resultant}} = 0$ **and** $M_{\text{resultant}} = 0$
simultaneously.

Then: $m\mathbf{v} = \text{constant}$
and $\Theta\omega = \text{constant}$



$$F_{\text{resultant}} = 0$$

$$M = Fd$$

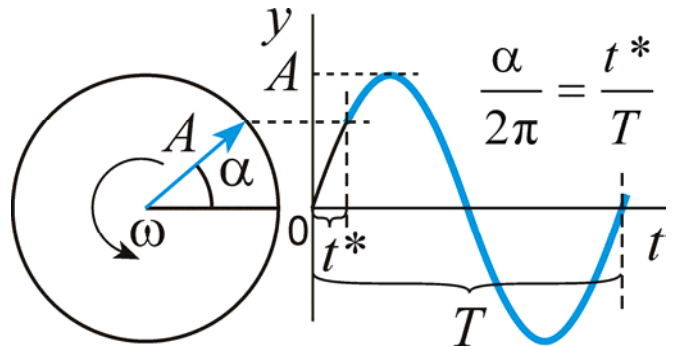
Statics

See: orthopaedy

Harmonic motion

Projection of
uniform circular motion
($\alpha = \omega t = 2\pi t/T = 2\pi f t$)

$$y = A \sin \omega t$$

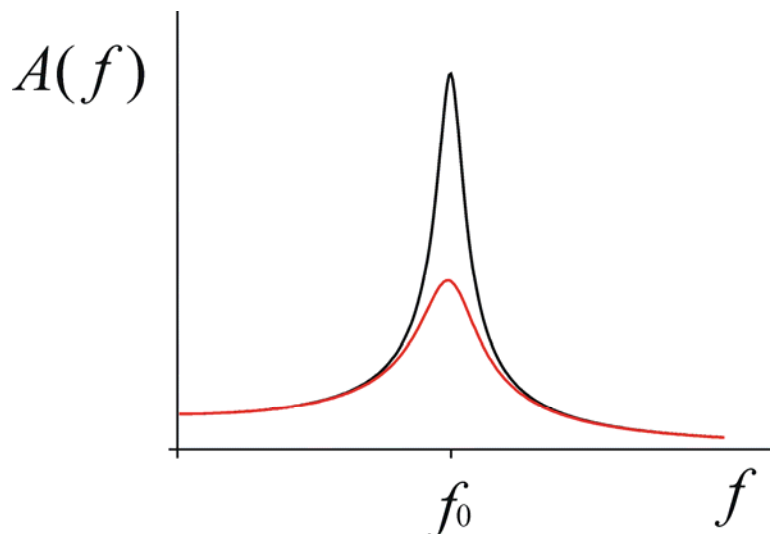


Dynamical condition: $F = -Dx = ma$

$$\omega = \sqrt{\frac{D}{m}}$$

Forced oscillation, resonance

($\omega = 2\pi f$)



$$A(f) \sim \frac{1}{(f - f_0)^2 + K}$$

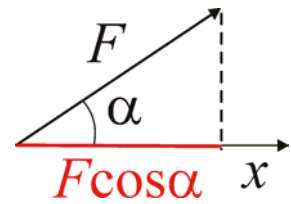
K is characteristic for
damping

Application e.g.: at the interpretation of different spectra
(ESR, NMR); at the explanation of AFM and MRI

Work

Work (W) is a product of **displacement** (Δx) and the projection of force (F) to the direction of displacement.

$$W = \Delta x F \cos \alpha \quad [\text{Nm}] \text{ or } [\text{J}]$$



Permanent acting force without displacement ($\Delta x = 0$);
or $\alpha = \pi/2$ (means $\cos \alpha = 0$), then $W = 0$ (in mechanics)

Work-energy theorem

Force is constant (and $\alpha = 0$).

$$W = F \Delta x = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 = \Delta E_{\text{kin}}$$

kinetic energy (E_{kin})

Result of work \rightarrow bigger E_{kin} .

Application e.g.: at the discussion of x-ray tube or electron-microscope.

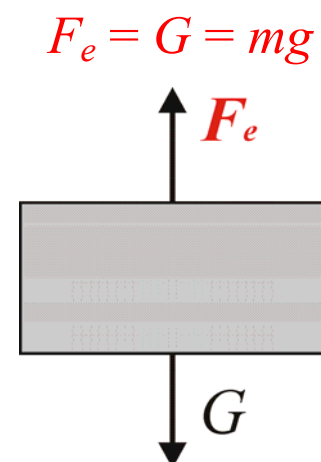
Work done against another force

e.g. elevation against **force of gravity** (G)
(g acceleration of gravity)

Result of work \rightarrow „storable”

potential energy (E_{pot})

In gravitational field: $\Delta E_{\text{pot}} = mg \Delta h$;



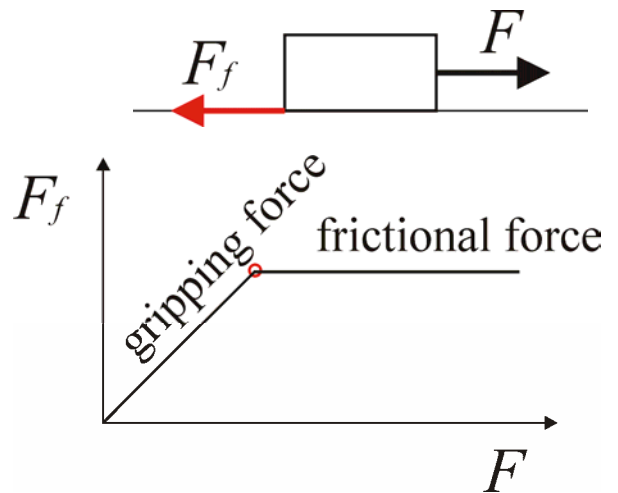
Power (“speed” of work done):

$$P = \frac{W}{\Delta t} = \frac{\Delta E}{\Delta t} \quad [\text{W}] = [\text{J/s}]$$

Friction

Is **not** the energy conservation valid?

(see [thermodynamics](#))



Static fluids (and gases)

→

hydrostatics

Pascal's principle

Pressure is transmitted undiminished in fluids because they are incompressible.

(hydraulic jack, brakes)

Hydrostatic pressure (originates from the weight of fluid)

In a static fluid on the Earth (simplest case):

$$mg = V\rho g = Ah\rho g = F_{\text{weight}}$$

(ρ density)

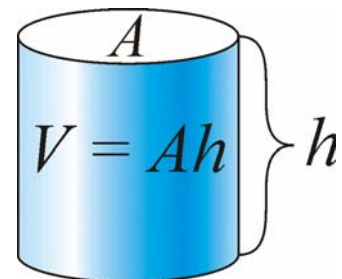
$$p = F_{\text{weight}}/A = \rho gh$$

Its consequence is the buoyant force (F_b):

Archimedes' principle

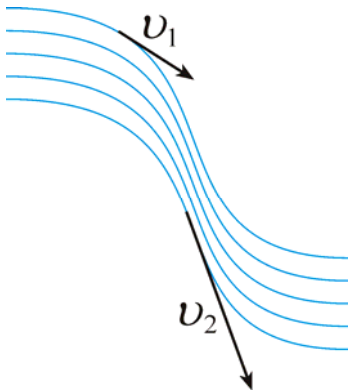
A body that is submerged in a fluid is buoyed up by a force:

$$F_b = \rho_{\text{fluid}}gV$$

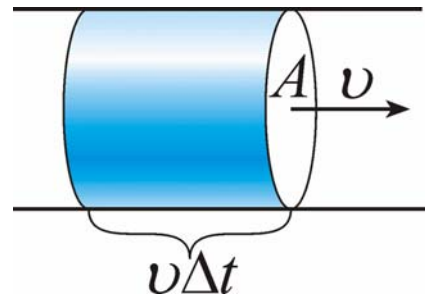


Flow of fluids (and gases) → **hydrodynamics**
 (constant **streamlines** in time: **stationary flow**)

Flow velocity and volumetric flow-rate:



$$I_V = \frac{\Delta V}{\Delta t} = \frac{A v \Delta t}{\Delta t} = A v$$



Law of continuity → $I_V = \text{constant}$ (conservation of matter)
 e.g. blood vessels

blood vessel	diameter (cm)	number of branches	A_{total} (cm ²)	v (cm/s)
aorta	2.4	1	4.5	23
arteries	0.4	160	20	5
arterioles	0.003	$5.7 \cdot 10^7$	400	0.25
capillaries	0.0007	$1.2 \cdot 10^{10}$	4500	0.022
venules	0.002	$1.3 \cdot 10^9$	4000	0.025
veins	0.5	200	40	2.5
venae cavae	3.4	2	18	6