Electromagnetism

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Charge

symbol of charge: Q

unit: coulomb (C)

Electric charge is a physical property of matter which causes it to experience a force when near other electrically charged matter.

charge is quantized:

1 C = $6.24 \cdot 10^{18}$ elementary charges

the elementary charge is: **$1.6 \cdot 10^{-19}$ C**

Discovery of electricity

amber = η λ ε κ τ ρ ο ν

ancient Egyptians: 2750 B.C.
mention electric fish in written text

Thales of Miletos: 600 B.C.
amber rubbed with fur attracted small objects, e.g. feather or leaf

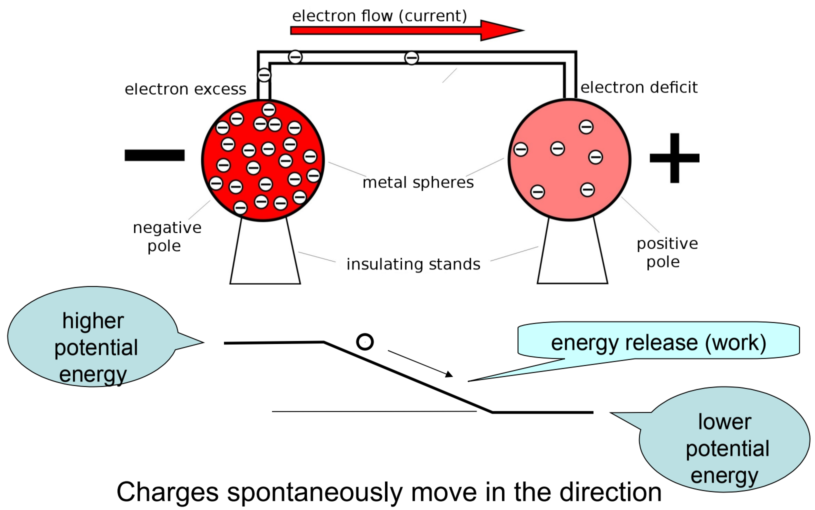
William Gilbert: 1600
coined the term: electric for those materials which attracted small objects after rubbing

Charles F. de Cisternay du Fay, Benjamin Franklin: 1737
discover that there are two forms of electricity
introduce the concept of positive and negative charge

Joseph John Thomson: 1897
the elementary unit of electric charge is the charge of the electron

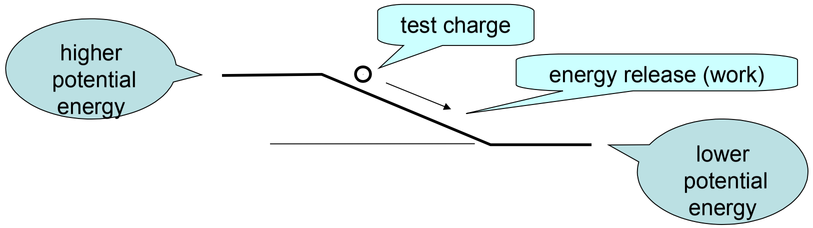
Robert Millikan: 1909
determined the charge of the electron:
$-1.6 \cdot 10^{-19}$ coulomb

Movement of charges

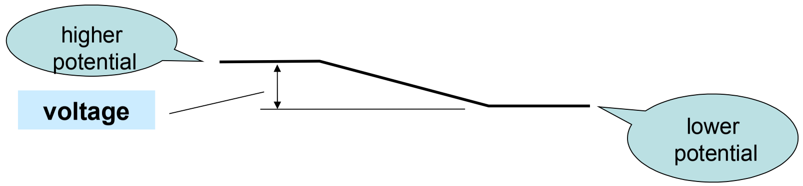


Charges spontaneously move in the direction that decreases potential energy.

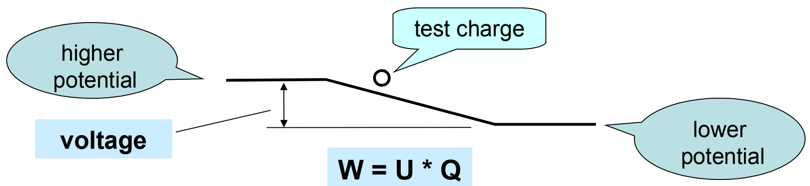
Electrostatic potential energy, electrostatic potential



electrostatic potential = electric potential energy / charge
Electrostatic potential is the potential energy of positive unit (1 coulomb) charge.

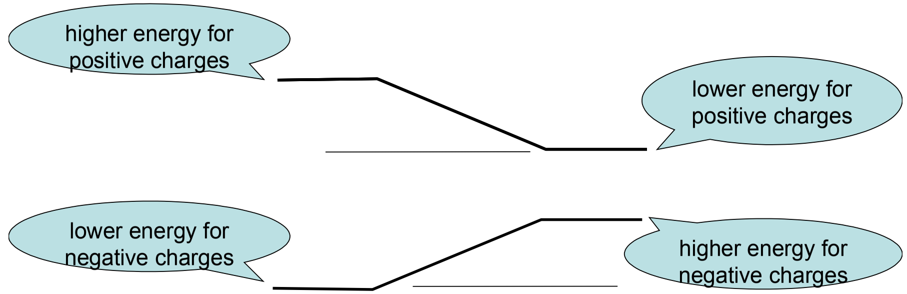


Electrostatic potential, electrostatic potential energy



W = U * Q

potential energy = electric potential * charge



Voltage

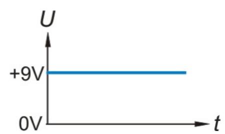
voltage: electric potential difference between two points

symbol of voltage: U

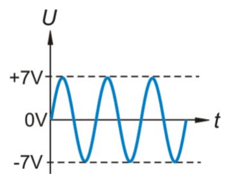
unit: volt (V)

$$U=\frac{W}{Q}\left[\frac{J}{C}=\frac{V\cdot A\cdot s}{A\cdot s}=V\right]$$

DC voltage



AC voltage



Electric current

electric current: amount of charge crossing the cross section of the wire in unit time

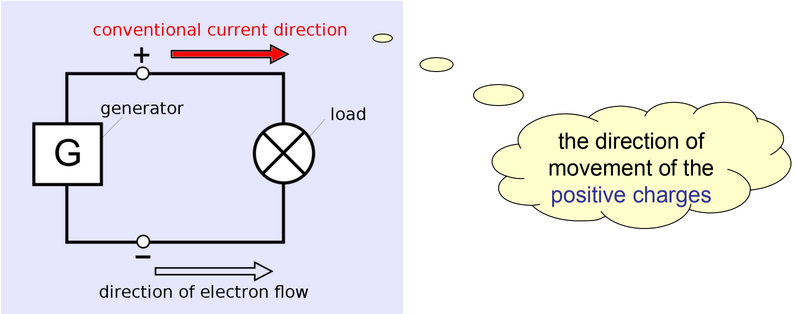
symbol of current intensity: I

unit: ampere (A)

$$I=\frac{\Delta Q}{\Delta t}\left[\frac{C}{s}=\frac{A\cdot s}{s}=A\right]$$

1 A means that 1 C charge flows through the cross section of the wire in 1 s.

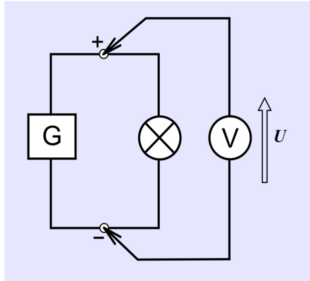
Direction of current



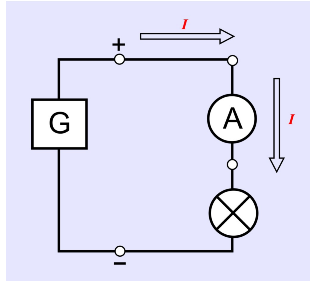
Charge carriers in different materials:

1. metals — electrons
2. electrolytes — positive and negative ions
3. gases — ions, electrons

Measuring voltage and current



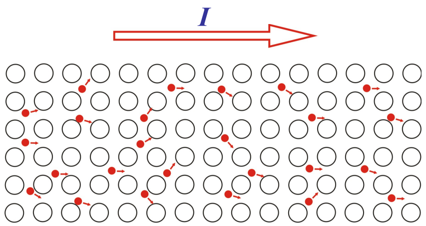
Very small (ideally zero) current flows through the voltage meter.



There is very small (ideally zero) voltage drop on the ampere meter.

Electrical resistance

Moving electrons bump in the metal atoms.
The material shows **resistance** to the electron flow.

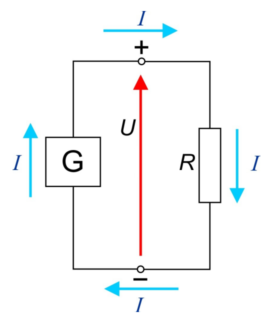


Work is needed to maintain current because of the resistance of the conductor. The conductor heats up.

Ohm's law

experiment:

current intensity flowing through different objects measured at different voltages



finding:

the current intensity I and the voltage U are proportional

$$U \sim I$$

$$\downarrow$$

$$U = R \cdot I$$

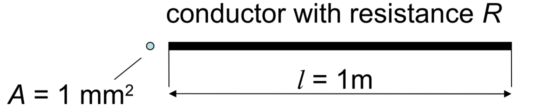
The proportionality constant R in the Ohm's ($U = R{\cdot}I$) law is called electrical resistance.

Electrical resistivity

The electrical resistivity is the resistance of a uniform specimen of material with unit length and unit cross section.

symbol of resistance: R
unit: ohm (Ω)

$$\Omega = \frac{V}{A}$$



$R \sim l$

$R \sim 1/A$

$$R = \rho \cdot \frac{l}{A}$$

resistance (Ω) — R; length (m) — l; cross-section (mm²) — A; resistivity — ρ

$$\rho = R\frac{A}{l}$$

depends on the matrial

Electrical conductance

The electric conductance is the reciprocal value of the resistance. It shows how easily an object conducts electric current.

symbol of conductance: G

unit: siemens (S)

$$G = \frac{1}{R} \qquad 1\ S = \frac{1}{\Omega}$$

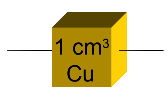
Electrical conductivity

The electric conductivity is the reciprocal value of the resistivity.

symbol of conductivity: σ

unit: siemens per metre (S/m)

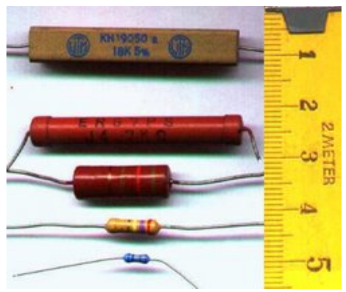
conductors	semiconductors	insulators
many free electrons per unit volume	few free charges per unit volume	no free charges per unit volume



1 cm³ of copper contains 10^{23} free electrons

Resistors

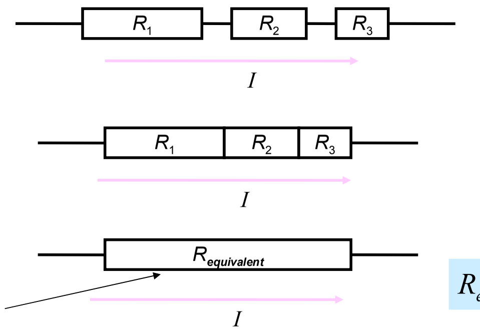
resistors in electronics



symbol of resistor

Connecting resistors in series

Resistances are connected in series if the same current flows through them.



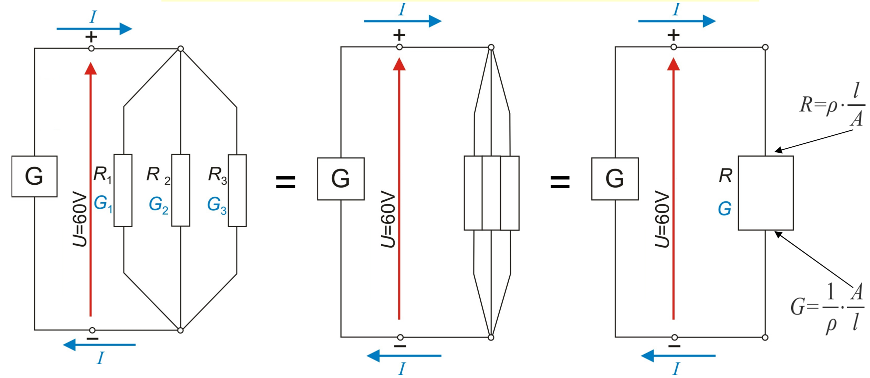
$R=\rho \cdot \frac{l}{A}$

$R_{equivalent}= R_1+R_2+R_3$

The resistance of the resistors connected in series adds up.

Connecting resistors in parallel

Resistances are connected in parallel if their ends are connected to the same voltage.



$R=\rho \cdot \frac{l}{A}$

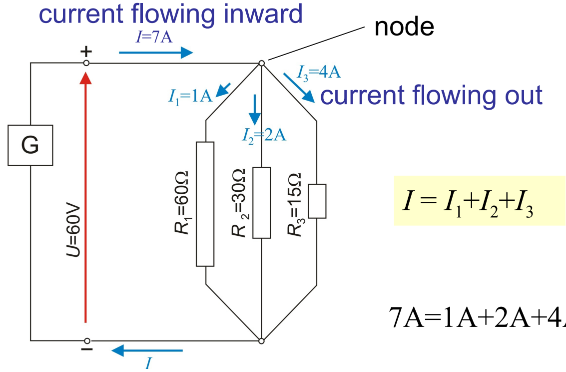
$G=\frac{1}{\rho} \cdot \frac{A}{l}$

The conductance of the resistors connected in parallel adds up.

$G_1+ G_2+ G_3= G_{equivalent} \longrightarrow \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}=\frac{1}{R_{\text{equivalent}}}$

Kirchhoff's current law (Kirchhoff's first law)

The **conservation of charge** implies that: at any node, the sum of currents flowing into the node is equal to the sum of currents flowing out of that node.



$I_1 = U / R_1 = 1\text{A}$

$I_2 = U / R_2 = 2\text{A}$

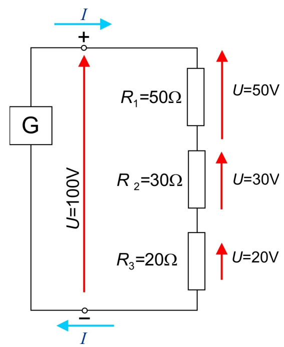
$I_3 = U / R_3 = 4\text{A}$

$I = I_1+I_2+I_3$

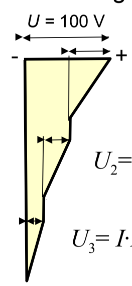
7A=1A+2A+4A

Kirchhoff's voltage law (Kirchhoff's second law)

The **conservation of energy** implies that: the sum of the electron motive forces in any closed loop is equivalent to the sum of the potential drops in that loop.



generator voltage: voltage drop on the resistors:



$U_1= I{\cdot}R_1= 50\text{V}$

$U_2= I{\cdot}R_2= 30\text{V}$ $\quad I = 1\,\text{A}$

$U_3= I{\cdot}R_3= 20\text{V}$

$U=U_1+U_2+U_3$

100V= 50V+30V+20V

Voltage source

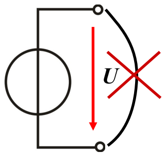
An ideal voltage source is a circuit element that generates constant voltage independently of the load current through it.

e.g.: $U = 10$ **V = constant**

If $R = 1|$, then $I = U/R = 10$ A.
If $R = 10|$, then $I = U/R = 1$ A

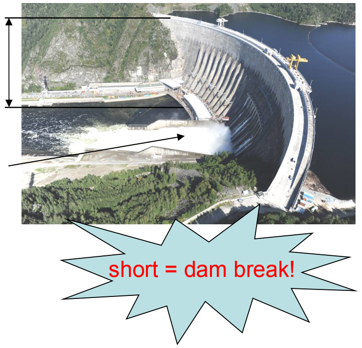
A voltage source has very small (ideally zero) internal resistance. It is forbidden to short it!

symbol:



(R=0); $I = U/R = \infty$

In practice most electric power sources can be considered voltage source.

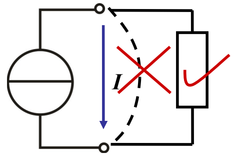


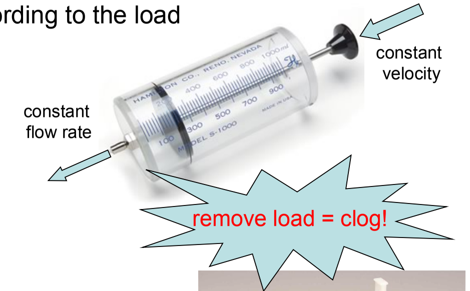
Current source

An ideal current source is a circuit element that generates constant current independently of the resistance of the load.

voltage changes according to the load

symbol:

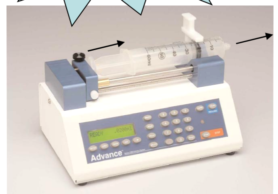




e.g.: $I = 1$ **A = constant**

If $R = 1|$, then $U = I{\cdot}R = 1{\cdot}1 = 1$ V.
If $R = 10|$, then $U = I{\cdot}R = 1{\cdot}10 = 10$ V

The internal resistance of the current source is very large (ideally infinite). A load always has to be applied.



Electrical work

Electrical work is done if charges move between two points with different potential.

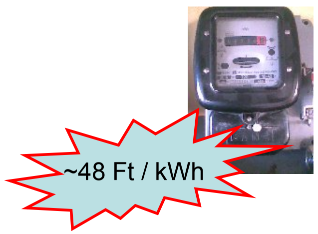
symbol: W

$$dW = U{\cdot}dQ \xrightarrow{Q = I{\cdot}t} W = U{\cdot}I{\cdot}t$$

$$V{\cdot}C = V{\cdot}A{\cdot}s = W{\cdot}s = J$$

Units of electric work: joule (J), Ws, Wh, kWh (kilowatt hour)

1 kWh = 1000·W· 3600s = 3.6 MJ



Electrical power

Electrical power is the amount of electrical work done in unit time.

symbol: P

$$P = W/t = U{\cdot}I{\cdot}t/t = U{\cdot}I$$

$$J/s = V{\cdot}A{\cdot}s/s = V{\cdot}A = W$$

unit: watt, (W = J/s)

$$P = U{\cdot}I = I^2{\cdot}R = U^2/R$$

100 W continuous

1000 W continuous

one trillion watts - one "terawatt" (10^{12} W)
pulsed: stroke lasts for about 30 microseconds

Electric field, Coulomb's law

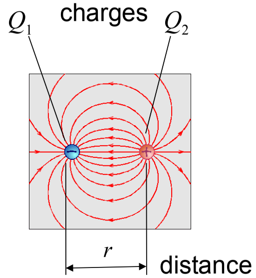
An electric field surrounds electric charges and time-varying magnetic fields. This electric field exerts a force on other electrically charged objects.

Electric field lines can be used to represent the electric field.

Opposite charges attract each other.

Same charges repel each other.

$F \sim Q_1, Q_2$ $\qquad F \sim 1/r^2$



Coulomb's law: $F = k\dfrac{Q_1 \cdot Q_2}{r^2}$

constant

Electric field strength

The electric field strength at a given point is defined as the force that would be exerted on a positive test charge of 1 coulomb placed at that point.

symbol: E

unit: V/m

$$E = F/Q$$

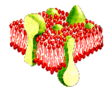
$$\mathrm{N / C = (J/m) / C = V{\cdot}A{\cdot}s / m{\cdot}A{\cdot}s = V/m}$$

examples for field strengths

break down limit in air: $3\cdot10^6$ V/m

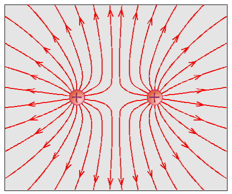
radio waves in the antenna: 1-100 ∝V/m

across the cell membrane: ~10^7 V/m

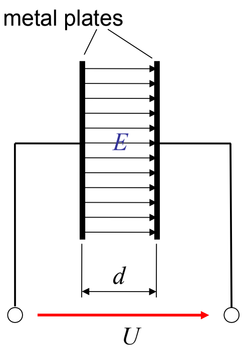


Inhomogeneous and homogeneous electric fields

The density of the field lines is different in every point of space → inhomogeneous field



Between two parallel charged metal plates the electric field is homogeneous.

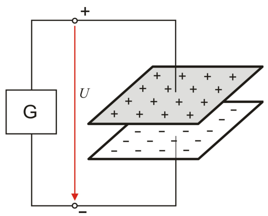


$E \sim U$ $\qquad E \sim 1/d$

$$E = U/d$$

Capacitor, capacity

The capacitor consists of two conductor plates separated by an insulator layer.



Capacitors store charge proportional to the applied voltage.

The proportionality constant is called capacity.

$$Q = C{\cdot}U$$

symbol: C

unit: F (farad)

$$C = Q/U$$

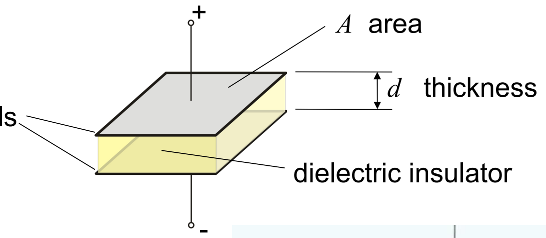
$$\mathrm{F = C/V}$$

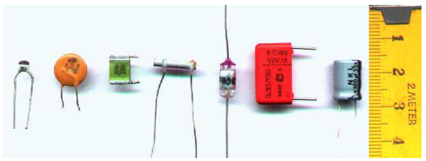
1F is the capacity of a capacitor that stores 1C charge when it is charged to 1 V.

leyden jar

Capacitor, electric permittivity

Capacitors are usually built of two metal foils separated by an insulator dielectric layer.





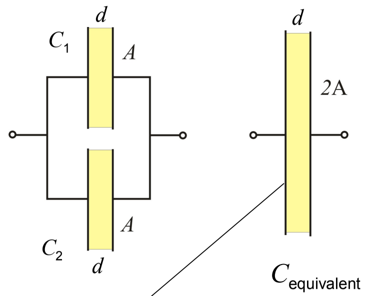
$C \sim A$

$C \sim 1/d$

$C = \varepsilon \cdot A/d$

The proportionality constant ε is called electric permittivity.

Capacitors connected in parallel

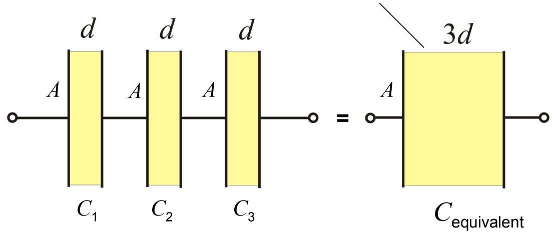


$C = \varepsilon \cdot \frac{A}{d}$

d = unchanged

$C_1 + C_2 = C_{equivalent}$

Capacitors connected in series



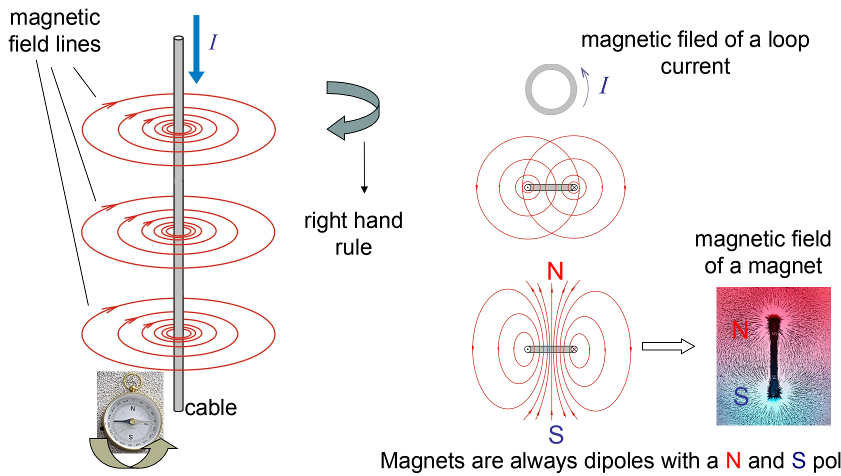
$C = \varepsilon \cdot \frac{A}{d}$

A = unchanged

$$\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{C_{equivalent}}$$

Magnetic field

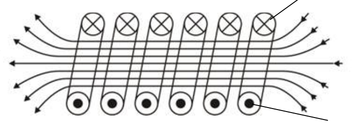
A magnetic field is a force field produced by ferromagnetic materials, moving charges, and changing electric fields.



Magnets are always dipoles with a N and S pole.

Magnetic flux

Flux is the total number of magnetic force field lines passing through a specified area.



symbol: Φ

unit: weber, Wb

Magnetic induction

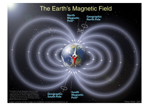
Magnetic induction (flux density) is the flux that crosses unit area perpendicularly.

symbol: B

unit: T (tesla)

$$\frac{\text{Wb}}{m^2} = T$$

the magnetic induction of the Earth in Hungary: ~ 50 ∝T

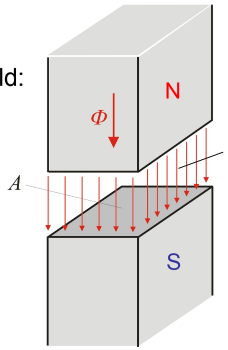


In homogeneous magnetic field:

$B \sim \Phi$

$B \sim 1/A$

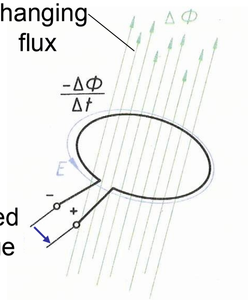
$$B = \frac{\Phi}{A}$$



magnetic induction inside an MRI: ~ 1T

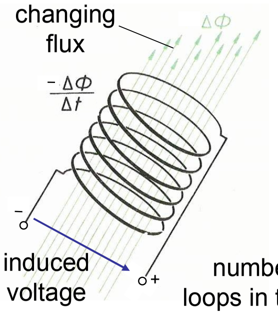
Faraday's law of induction

The electromotive force (voltage) produced around a closed path is proportional to the rate of change of the magnetic flux through the surface bounded by that path.



flux change

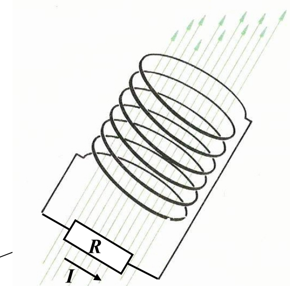
$U_i = - \Delta\Phi/\Delta t$ — time during which the flux change happened



$U = - N\cdot\Delta\Phi/\Delta t$

Lenz's law

An induced current is always in such a direction as to oppose the motion or change causing it.
(this is what the "-" sign means in Faraday's law)



R resistance, $I = U / R$ induced current

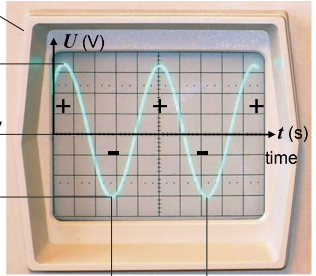
Alternating current (AC)

Parameters describing a sinusoidal alternating voltage:

$U = U_{peak} \cdot \sin \omega t$

voltage (V) — voltage amplitude (V) — angular frequency (rad/s) — time (s)

oscilloscope screen



U_{peak}, $U_{peak\text{-}peak}$, 0V, T

voltage amplitude (V)

period (s)

peak-to-peak voltage (V)

$\omega = 2\pi \cdot f = 2\pi / T$ — frequency (Hz=1/s) — period time (s)

$f = 1/T$

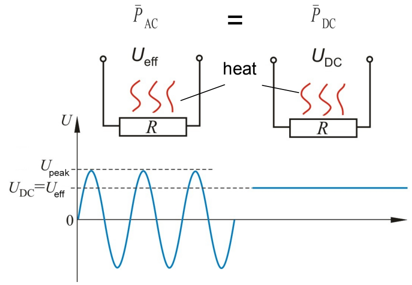
$U_{peak} = U_{peak\text{-}peak}/2$ $f = 50$ Hz

e.g. for the voltage of the mains: $T = 1/f = 1/50\text{Hz} = 0.02\text{s} = 20\text{ms}$

Effective value

The effective value of an AC current (voltage) is equivalent to the DC current (voltage) that on average produces the same amount of heat on a resistor.

For sinusoidal voltage and current:



$U_{eff} = \frac{U_{peak}}{\sqrt{2}}$

$U_{eff} = 0{,}707 \cdot U_{peak}$

$U_{peak} = 1{,}41 \cdot U_{eff}$

$I_{eff} = \frac{I_{peak}}{\sqrt{2}}$

Effective value of the mains electricity

230V sinusoidal voltage

Effective value is used and not the peak voltage!

$U_{eff} = 230$ V

$U_{peak} = 1{,}41 \cdot 230 \text{ V} = 324 \text{ V}$

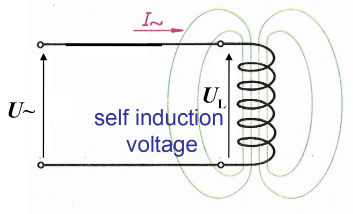
The advantage of using effective value rather than peak value is that the effective values can be directly used for calculating power.

$\bar{P} = U_{eff} \cdot I_{eff}$ but $\bar{P} \neq U_{cs} \cdot I_{cs}$

Self induction

- We connect an AC voltage to a coil.
- A constantly changing magnetic field is generated around the coil.
- The changing magnetic field induces a voltage in the coil that opposes the change of the external voltage.

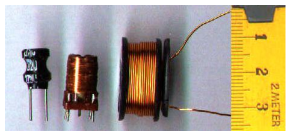
self induction voltage — $U_L \sim \frac{\Delta I}{\Delta t}$ — current change — time during which the current change happened



$U_L = -L \cdot \frac{\Delta I}{\Delta t}$

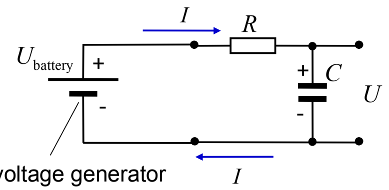
The proportionality factor L is called self inductance

unit of self inductance: henry, H



1H is the self inductance of the coil which responds to a change of 1A that happens uniformly during 1s with 1V self induction voltage.

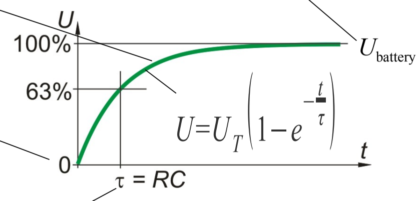
Charging a capacitor through a resistor



DC voltage generator
(e.g. battery)

After long time the capacitor will be charged to the voltage of the battery:
$U = U_{battery}$
(Equilibrium is reached and no current will flows.)

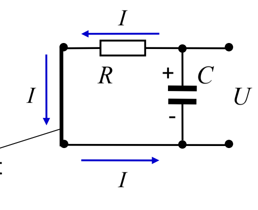
exponential charging



time constant

At t=0 the capacitor is uncharged ($U = 0$).
(In this moment the charging current is maximal: $I = U / R$.)

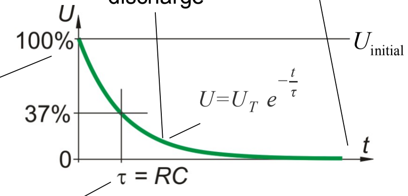
Discharging a capacitor through a resistor



short circuit
($R = 0$)

After a long time the capacitor loses the stored charge, the voltage on it becomes $U = 0$, and the discharge current stops.

exponential discharge



time constant

At the first moment of the discharge the voltage on the capacitor is $U = U_{initial}$, the current is maximal $I = U_T / R$.

Capacitive reactance

Reactance is the opposition of a circuit element to a change of current, caused by the build-up of electric or magnetic fields in the element.

symbol: X_C unit: ohm, | $X_C = U / I_C$

Although capacitive reactance is analogous to the resistance, no heat is produced on an ideal capacitor because $R = \infty$, and $P = U^2/R = 0$.

$X_C \sim 1/f$ $X_C \sim 1/C$ $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f \cdot C}$

If $f = 0$ (DC case), $X_C = \infty$ → The capacitor can be substituted with a breakage.

If $f = \infty$, $X_C = 0$ → The capacitor is equivalent with a short circuit.

Inductive reactance

symbol: X_C unit: ohm, | $X_L = U / I_L$

Although inductive reactance is analogous to the resistance, no heat is produced on an ideal coil because $R = 0$, and $P = I^2 R = 0$.

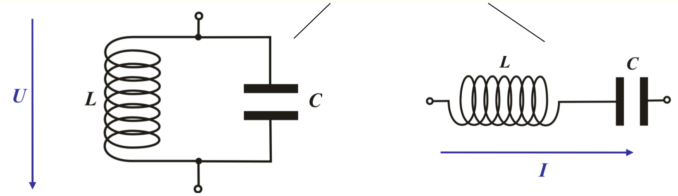
$X_L \sim f$ $X_L \sim L$ $X_L = \omega \cdot L = 2\neq f \cdot L$

If $f = 0$ (DC case), $X_L = 0$ → The coil can be substituted with a short circuit.

If $f = \infty$, $X_L = \infty$ → The capacitor is equivalent with a breakage.

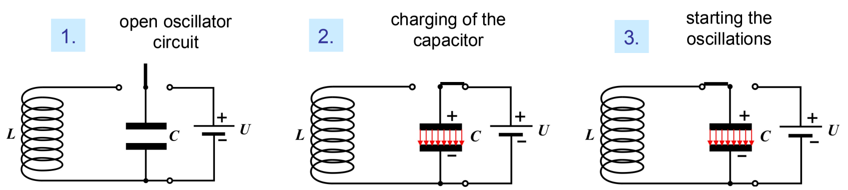
Oscillator circuit

The oscillator circuit consists of a coil and capacitor connected parallel or in series.



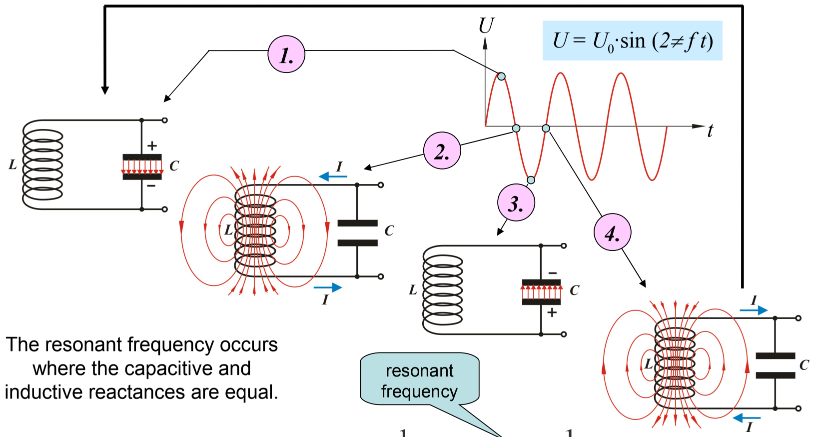
To start the oscillation we provide energy in the capacitor or in the coil.

The example below shows the start of a parallel oscillator circuit.



Oscillator circuit

The current and the voltage of a released ideal oscillator circuit will follow sinusoidal oscillations.



The resonant frequency occurs where the capacitive and inductive reactances are equal.

$$X_L = X_C \longrightarrow 2\pi f L = \frac{1}{2\pi f C} \longrightarrow f = \frac{1}{2\pi\sqrt{LC}}$$