

Non-parametric tests

Distribution free methods.

advantage : independent from the distribution.
disadvantage: normally it's power is less.

Ranking tests:
Instead of original values we use the so-called **ranks**.

Ranks

Rank: numerical or ordinal data belonging to a value in data series sorted according to a certain rule.

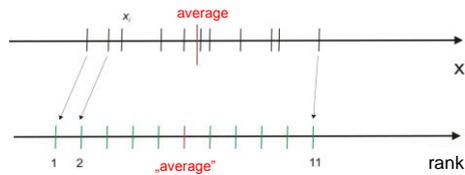
„tie”:
In the case of same values every value replaced by the average of the ranks.

e.g.:

- lieutenant
- major
- colonel
- etc.

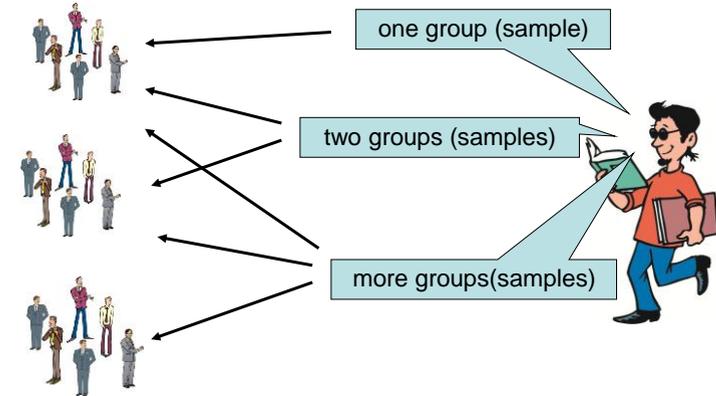
value:	1.2	2	2	3.5	4
rank:	1	2,5	2,5	4	5

the „average” of the ranks is the median



The median plays the role of the average.

according to the question

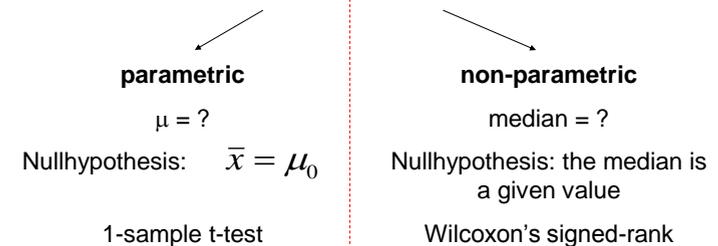


Summary table

	parametric	non-parametric
one group	1-sample t-test,	Wilcoxon's signed-rank test, sign-test
two groups	2-sample t-test	Mann-Whitney U-test
more groups	ANOVA	Kruskall-Wallis test

Examination in one group

Question: On the base of the sample the parameter of the population may be a given value?



1-sample t-test

example: The medicine effective or not?



Nullhypothesis: not! $\mu_0 = 0$. But the average is not 0!

sample	Average
1.	-0.2 °C
2.	-1 °C
3.	-1.5 °C



If the difference is bigger, it seems to be more probable being non random.

What does it mean big difference?

What is the measure of the difference?

Standard error: the average deviations of the averages from the μ .

$(\bar{x} \pm s_{\bar{x}})$ ~ 68% - confidence interval.

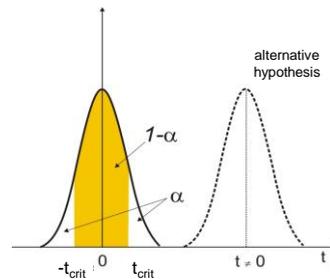
t-value

$$t = \frac{\bar{x} - \mu_0}{s_{\bar{x}}}$$

Compare the difference to the standard error!
(μ_0 very frequently = 0)

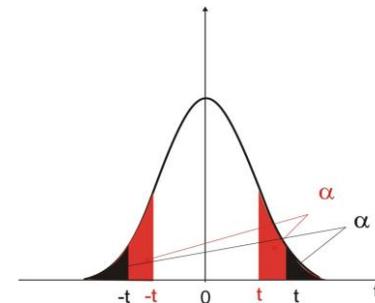
The averages fluctuate around the μ_0 so the t -values deviate around the 0.

(providing, that the nullhypothesis is true!)



Why the t-value is better?

We are able to calculate the probabilities on the base of this distribution!!! (Student- or t -distribution)



It describes only the **random deviations** of the t -values!

The shape of the distribution depends on the no. of elements.

Why has it t-distribution?

$$t = \frac{\bar{x} - \mu_0}{s_{\bar{x}}} \longrightarrow \bar{x}$$

The fluctuation of the average has normal distribution. The numerator is variable having normal distribution!

$$s_{\bar{x}} = \frac{s}{\sqrt{n}} \longrightarrow s = \sqrt{\frac{\sum_i (x_i - \bar{x})^2}{n-1}}$$

variance is the sum of squared probability variables.

remember!

t-distribution

$$\xi_n = \frac{\sqrt{n} \cdot \xi}{\sqrt{\sum_i \xi_i^2}}$$

(Quad erat demonstrandum)

the t variable has t-distribution.



Degree of freedom (d.f.)

I think 3 numbers! (sample)

The average of them: 8! (information!)



they must be!

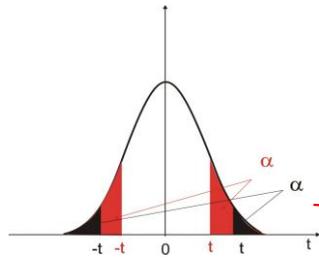
3, 12, 8 or 5, 7, 11 etc.

d.f. = n

3, 12, **9** or 5, 7, **12** etc.

d.f. = n-1

The t-table



t-table

d.f.	significance level			
	0.1	0.05	0.02	0.01
1	6.31	12.7	31.8	63.7
2	2.92	4.3	6.96	9.92
3	2.35	3.18	4.54	5.84
4	2.13	2.78	3.75	4.6
5	2.02	2.57	3.37	4.03

Different t_{crit} values belong to different significance level.

d.f.: n-1

Decision the base of t-table

t-table

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d.f.: n-1

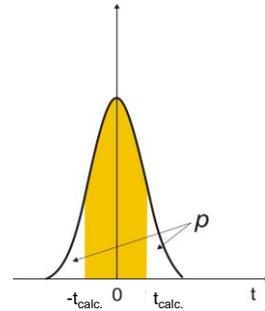
Select an appropriate significance level!

If ≥ 2.78 reject, if smaller accept the nullhypothesis.



Decision using computer

I am able to integrate!!!

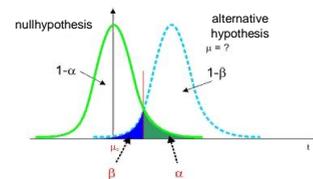
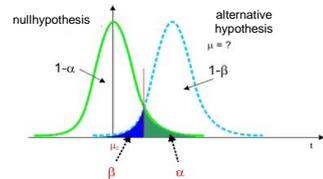


p : probability, that the $t_{calculated}$ is so large randomly.

Decision

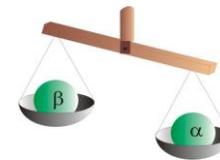
- 1. If the probability of the random deviation is small ($p(|t| \geq t_{crit}) \leq 5\%$) – **reject** the null hypothesis.
- 2. If the probability of the random deviation is large ($p(|t| \geq t_{crit}) > 5\%$) – **accept** the null hypothesis.

error?



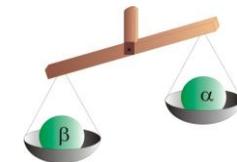
The error

Reject the null hypothesis



α is the measure of the error. Smaller p is better.

Accept the null hypothesis



β is the measure of the error. Larger p is better.

Condition for 1-sample t-test

- Task: Decision about the μ on the base of one sample.
- The variable must have **normal distribution**.



What can we do if it isn't true?

Wilcoxon's signed-rank test

Example: Is there an effect of an entertaining movie on the patients? (The numbers are scores)

n	before	after	Diff.
1	2	2	0
2	0	1	1
3	3	2	-1
4	2	4	2
5	1	3	2
6	3	3	0
7	1	4	3
8	1	5	4
9	5	2	-3
10	4	4	0

Normal distribution?



Ranking

Sort the absolute values of the differences (without 0-s)! Let the sign of ranks be same then the differences!
Calculate the averages and sd of signed-ranks!

Diff.	absolute value	rank	Signed-rank
0	0		
1	1	1.5	1.5
-1	1	1.5	-1.5
2	2	3.5	3.5
2	2	3.5	3.5
0	0		
3	3	5.5	5.5
4	4	7	7
-3	3	5.5	-5.5
0	0		



The nullhypothesis

There is no effect of the movie!

The median = 0!
The deviation is random!

$$H_0: \mu_0 = 0$$

$$H_1: \mu_0 \neq 0$$



known distribution



$$t = \frac{\bar{R} - 0}{s / \sqrt{n}}$$

If n is enough large!

\bar{R} - the average of the signed-ranks

s - the standard deviation

Remember!
„average” of the ranks = median



Decision

This is known!!!

Of course! This is similar to the 1-sample t-test!!!



Paired t-test

If the data may be paired according to a rule!

Observation on the same person, paired organ (e.g. kidney).

Rare, on the base of viewpoints (age, profession, etc.).

Look at: decreasing the fewer.



↓
Experimental design

Experimetal design

test from previously collected data?

practical order:
Question → experimental design → calculation.

Many problems: e.g. a few data are suitable only.



„real” 1-sample t-test

Is it possible that the μ is equal to a value?

$$t = \frac{\bar{x} - \mu_0}{s_x}$$



Rare case.



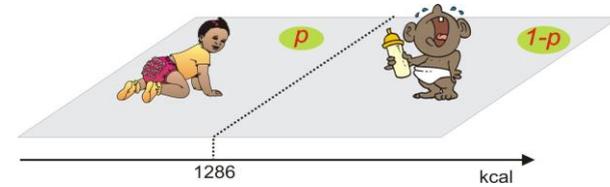
sign-test

Example: Energy uptake in the population of 2-year old children.

Question: May be the median

(This derives from an another test) is 1286 kcal?

Nullhypothesis: median = 1286 kcal (deviation is random).



Test

Small sample

$$P(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

binomial distribution

Large sample

$$z = \frac{|x - np| - 1/2}{\sqrt{np(1-p)}}$$

standard normal distribution

x – no. of children below 1286 kcal.

n – no. of children in test.

p – probability, that randomly smaller (look at: binomial distribution)

Decision

Calculate the probability of the random deviation. (binomial, or standard normal distribution)

End of this part!

